

# YAW RATE ERROR – A DYNAMIC MEASURE OF LANE KEEPING CONTROL PERFORMANCE FOR THE RETROSPECTIVE ANALYSIS OF NATURALISTIC DRIVING DATA

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Paper Number 09-0326

## ABSTRACT

The aim of this work is to define and evaluate a “yaw rate error” (YRE) derived from naturalistic driving data to quantify driver steering performance during lane keeping. This measure of lane keeping performance is based on the predicted kinematic control error at any instance. Scope is limited to the demonstration that such a quantity exists, that can be computed from naturalistic driving data, and that it correlates with instantaneous control performance in real-world driving. The YRE is defined as a measure of conflict: the difference between current vehicle yaw rate and kinematic values required to be consistent with forward lane boundary crossing. A second, well-known measure is computed for comparison: the predicted time to lane crossing (TTLC). All data is obtained from naturalistic driving databases containing detailed information (over 200 signals at 10 Hz.) on driver input and vehicle response as well as aspects of the highway and traffic environment. As a continuously updated measure of the control correction required by an alert driver, it is expected that the YRE will be more informative of driving situations than the simpler kinematic measure TTLC. This latter measure is only loosely related to the closed loop control of vehicle motion. For example a very small TTLC can represent either a critical case where the vehicle is about to depart the lane and requires a large correction, or it could be a case where the vehicle is close to the lane boundary but with small lateral velocity requiring only a small correction. The YRE represents the severity of the possible lane departure in a natural way, accounting for current position, path direction, and path curvature. While no in-depth statistical analysis is conducted for YRE, it is proposed as a new tool for post-hoc analysis of driver steering performance during lane keeping.

## INTRODUCTION

Driving is a control task based on visual input; it includes filtering of input for relevance, extracting signals or patterns from that visual information, and hence provides a reference to guide steering and speed control. Control action then involves manual effort by the driver to modulate vehicle motion using further force and acceleration cues [1-3]. Here we focus on the visual reference for lane keeping in terms of a conflict measure or error criterion. In broad terms we seek a simple measure of the control reference for when the driver is concerned with staying in the lane but less concerned with some optimal path within that lane. To this end we introduce and evaluate a suitable measure of “yaw rate correction required” or yaw rate error. Since no preferred path is computed, the YRE is computed for multiple lane boundary points and the most critical of these will represent the overall correction required. This metric has been used previously in driver modeling [4] and applied to collision avoidance [5]

The approach is analogous to longitudinal speed control in traffic, where control action required can be found in terms of the vehicle deceleration required to avoid a collision with the vehicle in front. Again, this contrasts with the predicted time to collision (TTC), based on instantaneous positions and velocities of the vehicles [6]. While in the speed control problem there is essentially a single target point, the more complex lane keeping activity involves multiple conflict points and more complex vehicle kinematics.

We focus on yaw velocity rather than the related variables of path curvature and lateral acceleration because of the focus on visual reference. Yaw velocity is directly available to the driver as the perceived angular rate of distant or peripheral objects across the field of view. Path curvature by contrast

requires a constructive element as the driver “imagines” the path of the vehicle, something that is surely more appropriate to low speed maneuvering. Again, vehicle lateral acceleration is not a visual input, but rather a feedback for the lower level manual control of the vehicle. Thus the emphasis on yaw rate as the reference is based on its availability through visual feedback, analogous to what happens in vehicle stability control [7] – vehicle yaw rate is directly measured and compared to a reference – though in this case it is based on anticipated vehicle response to steering at the current speed. In this case path curvature is not directly measurable, and lateral acceleration is subject to disturbances such as body roll; also, the lateral acceleration is dependent on sensor location, unlike the yaw rate, which is only sensitive to sensor orientation.

It is also worth noting that under simple conditions of constant speed, minimal vehicle sideslip (i.e when the vehicle is in a normal stable condition) and negligible body roll angle, the three variables mentioned (path curvature, yaw rate and lateral acceleration) are actually proportional to each other. So under these simple conditions any one of these variables might be used for the present purpose. We now turn to the details of the yaw error criterion.

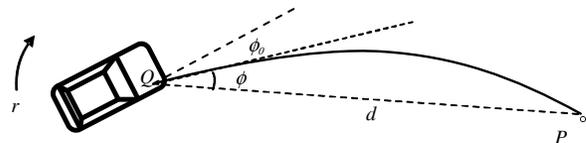
For any point on the road or lane boundary, we are to determine whether a yaw rate correction is needed to avoid going outside of the lane/road. If so the yaw correction required is a measure of conflict. The maximum magnitude of all such corrections (left or right) is to be our conflict measure, though it is often of interest to analyze “worst right boundary case” and “worst left boundary case” in parallel. Additional information is relevant, namely the distance and polarity (left, right) of any conflict point, as well as the horizon distance: the maximum distance or headway for which – under ideal yaw rate – no conflicts occur). The horizon distance is a combined measure of position and direction error, as well as road geometry, and arises naturally out of the YRE analysis.

As mentioned, YRE and these associated measures are related to TTLC, but are expected to incorporate a greater degree of continuity and relevance to the control task. Unlike TTLC, the “angle of attack” of the lane excursion is implicitly included, so it potentially attaches due significance to how severe the predicted lane excursion will be, not just when it will be. For this reason YRE is expected to be a superior combined metric of lane keeping performance analysis than TTLC.

This study was motivated by a more general problem of establishing surrogates for road departure crashes [8,9]. The idea is to find kinematic or other variables that respond to road, traffic and driver conditions in a way that mimics the pattern of crash occurrence. Provided the dependency is based on common cause (for example due to disturbances in the closed loop control of the vehicle), detailed analysis of surrogates and counter-measures is much more feasible than the corresponding analysis of crash occurrence. In this paper we restrict attention to the YRE metric of interest, and leave aside its factorial analysis relative to crash frequencies.

### YAW RATE ERROR DEFINITION

In Figure 1 we consider the lateral vehicle control relative to a single “conflict point”  $P$ . This is presumed to be on the right lane boundary, so the yaw rate (assumed positive in the case shown, with the vehicle curving to the right) should be no more than for the critical case shown; the vehicle point  $Q$  required to pass to the left of  $P$ , while here it just intersects with  $P$ . Using polar coordinates  $(\phi, d)$ ,  $\phi$  is the azimuth angle and  $d$  is the distance-to-target, both computed relative to the velocity vector at the reference point  $Q$ . This in turn is oriented at an angle  $\phi_0$  relative to the vehicle axes, and if we assume  $Q$  to be at the outside edge of the front right tire, then  $\phi_0$  is very roughly equal to the steering angle at the right front wheel.



**Figure 1. Turning kinematics – critical case where reference point  $Q$  intersects with boundary edge point  $P$  during a steady-state turn.**

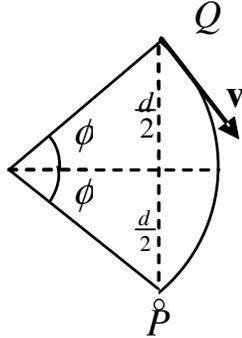
Assuming the vehicle path is in the form of a circular arc, the geometry is represented in Figure 2; we find that the critical case occurs when the turning radius  $R$  satisfies the equation

$$\sin \phi = \frac{d}{2R} \quad (1)$$

which is equivalent to the yaw rate condition

$$r = \frac{2U \sin \phi}{d} \quad (2)$$

$U$  being vehicle speed. Thus equation (2) defines the maximum yaw rate of the vehicle to avoid conflict with a right boundary point  $P$



**Figure 2. Basic geometry of steady turning motion.**

### ESTIMATION METHOD

When using driving data, we do not have direct information on all of the variables used in equation (2) – the absolute coordinates of the boundary points are unknown, as are their relative locations to the vehicle; hence they must be inferred from the lane tracker, which estimates of lateral position and lane width. Note that while in principle GPS could be used, it is far from being accurate enough to give useful estimation of the lane keeping performance, so this was not considered.

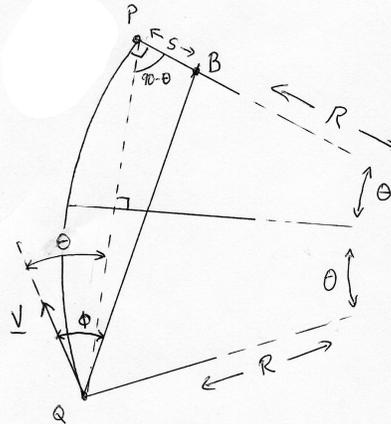
The idea is to use the vehicle itself, with known speed and yaw rate, to provide a reference for which to estimate the relative position of the lane boundary over time. From the variations in lateral lane position over time, the lane geometry is to be estimated, and variables such as  $\phi$  and  $d$  derived.

We also need to estimate the azimuth offset  $\phi_0$  for the direction of the velocity vector at the front wheel relative to the vehicle longitudinal axis (at low speed this is the steer angle, but in general it depends on the front axle cornering stiffness). In the case where steer angle and cornering stiffness is not available, a simple estimate can be made based on general vehicle dynamics properties

$$\tan \phi_0 = \frac{rL}{U - rc} \quad (3)$$

where  $r$  is the yaw rate,  $L$  is the wheelbase,  $2c$  is the front track, and  $U$  is the instantaneous vehicle speed (this equation is based on the assumption of near-zero slip angle at the rear axle, but is expected to be reasonably accurate). Simple adjustments are to be made to this equation when considering left side boundary points.

Figure 3 shows the modified geometry when boundary point  $B$  is offset from the vehicle path. For simplicity assume a fixed preview time  $T$  to the boundary point, and an approximately constant curvature for the path of the reference point from  $Q$  to  $P$ . In the figure,  $\phi$  is the azimuth angle to the boundary point  $B$ , while  $\theta$  is now the critical azimuth angle corresponding to the motion from  $Q$  to  $P$ . (Again, both angles are defined relative to the velocity vector, not the vehicle longitudinal axis).



**Figure 3. Sketch of turning geometry for an offset boundary point  $B$ .**

During the vehicle motion from  $Q$  to  $P$ , the heading angle and direction of velocity vector  $V$  change by  $2\theta$ , so numerically integrating the yaw rate over the time interval  $T$  we have

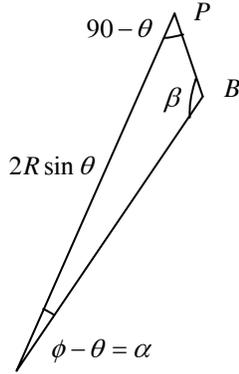
$$\theta = \frac{1}{2} \sum_i r(t_i) \delta t \quad (4).$$

The mean radius of turn,  $R$ , during the time interval can also be obtained from the yaw rate:

$$\frac{2\theta}{T} = \frac{\bar{U}}{R} \quad (5)$$

where  $\bar{U}$  is the mean vehicle speed during the interval, and both sides of this equation are estimates of the mean yaw rate during time interval  $T$ . Then, to determine  $\phi$ , we consider triangle  $BPQ$  in Figure 4. Angles at  $P$  and  $Q$  are known in terms of  $\phi$  and  $\theta$ , and hence the angle at  $B$  is given by

$$\beta = 180 - (90 - \theta + \alpha) = 90 + \theta - \alpha = 90 + 2\theta - \phi$$



**Figure 4. Geometry to determine  $\phi$**

Then from the sine rule

$$\frac{\sin \beta}{2R \sin \theta} = \frac{\sin(\phi - \theta)}{s} \quad (6)$$

which is a nonlinear implicit equation for  $\phi$  in terms of other known variables. For normal highway driving we expect  $\alpha = \phi - \theta$  to be sufficiently small (less than around  $5^\circ$ ) to allow the approximation  $\sin \alpha \approx \alpha$ ,  $\cos \alpha \approx 1$ . In this case

$$\begin{aligned} \sin \beta &= \sin(\theta + (90 - \alpha)) \\ &= \sin \theta \cos(90 - \alpha) + \cos \theta \sin(90 - \alpha) \\ &= \sin \theta \sin \alpha + \cos \theta \cos \alpha \\ &\approx \sin \theta \alpha + \cos \theta \end{aligned}$$

Substituting this into equation (6) then gives

$$\frac{\sin \theta \alpha + \cos \theta}{2R \sin \theta} = \frac{\alpha}{s}$$

Hence

$$\alpha \left( \frac{1}{s} - \frac{1}{2R} \right) = \frac{1}{2R \tan \theta}$$

giving the approximate expression for  $\phi$  (with all angles in radians)

$$\phi = \theta + \frac{s}{(2R - s) \tan \theta} \quad (6a).$$

Distance  $d = |QB|$  is also found from the geometry of Figure 4:

$$\begin{aligned} d &= |QP| \cos \alpha - |BP| \cos(90 - \theta) \\ &\approx 2R \sin \theta - s \sin \theta \end{aligned}$$

and hence

$$d = (2R - s) \sin \theta \quad (7).$$

Equations (4), (5), (6a) and (7) then determine all the relevant terms in the critical yaw rate expression

$$r_c = \frac{2U \sin \phi}{d} \quad (8)$$

where  $U$  is the instantaneous vehicle speed at  $Q$ , and now we use  $r_c$  to denote the critical yaw.

Multiple calculations can be performed for point pairs  $(P, Q)$  for values of  $T$  in a range of say 0.5-2 seconds, and the results referenced on the initial point  $Q$ . We are then interested in the minimum value of  $r_c(Q)$  and its corresponding distance  $d$  from  $Q$ . The yaw rate error is then

$$yre(t_Q) = r(t_Q) - r_c^*(Q) \quad (9)$$

where  $r(t_Q)$  is the vehicle yaw rate at time  $t_Q$ , and  $r_c^*(Q)$  was the minimum critical yaw rate at  $Q$ .

A second yaw rate error for left boundary points also has to be found, making similar calculations with relevant shift of reference point (to the outside of the left front tire) together with relevant sign changes.

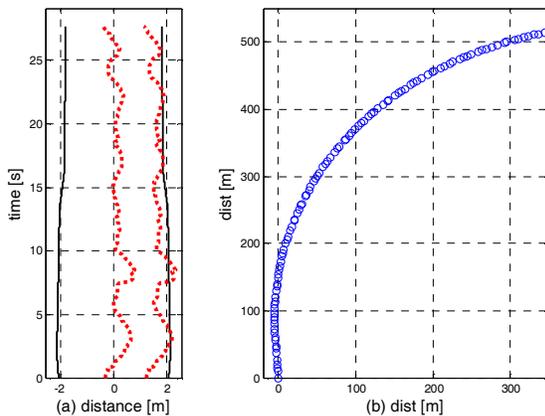
The above equations are obtained for computing YRE, but it is worth noting that with a minor adjustment they can be used to determine local road curvature from the on-board vehicle data (assuming lane position, speed and yaw rate are measured) removing the effects of vehicle lateral drift. The method is to estimate the critical yaw rate for a shifted point  $P$  that has the same lateral offset as current point  $Q$ : thus replace  $s = s(P)$  in the above, by  $s' = s(P) - s(Q)$ . The critical yaw rate  $r_c'$  is then the yaw rate that maintains equal lane deviation over time interval  $T$ , and hence provides the radius of curvature  $R_e$  (referenced at the right lane boundary) we obtain

$$R_e = \frac{\bar{U}}{r_c'} - s' \quad (10).$$

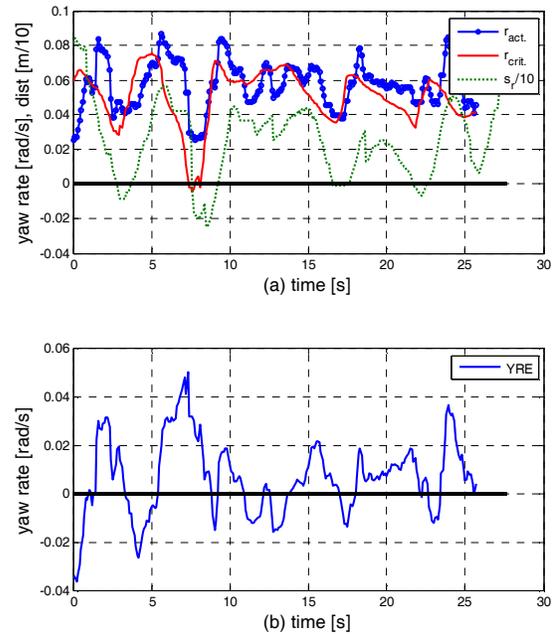
## RESULTS

The estimation method described above was used to determine the YRE for driving events recorded in the Road Departure Crash Warning (RDCW) field operational test database [10], which contains more than 200 data channels recorded at 10 Hz or 20 Hz, depending on the signal; included within these data is a wide range of information on driver input, vehicle response as well as aspects of the highway and traffic environment. As well as objective data from sensors, video images of the forward scene and drivers face were available to establish context. Here we present data from three events which appear quite typical or real-world lane keeping.

Event 1, depicted in Figure 5, was of a driver negotiating an on-ramp which is in the form of a right-hand curve. The left plot shows the location of the left and right front wheels relative to the lane boundaries (note that there is some variation in the lane width, but that most of the variations are in the dashed lines which depict the outside edges of the front tires). This event shows a situation where the driver maintained a position very close to the lane boundary with several excursions beyond the boundary. From video review, it appeared that the driver's attention was switching between reading a map and looking at the road ahead. Clearly the event represents an example of poor lane keeping. Figure 6(a) shows critical and actual yaw rate time histories, as well as lateral distance within the lane boundary (scale by a factor 0.1 so that scales are reasonably consistent).



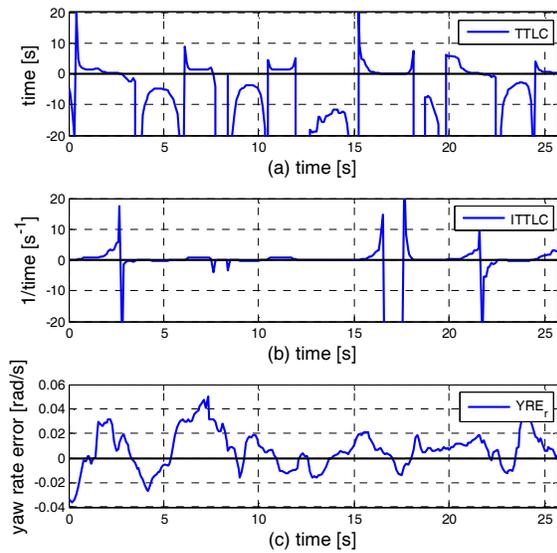
**Figure 5. The vehicle path for event 1. (a) The dotted lines represent the left and right edges of the vehicle with respect to the center of the lane markings (solid lines). (b) The X-Y position of the vehicle in space.**



**Figure 6. The yaw rate parameters for event 1; (a) the actual yaw rate, critical yaw rate, and distance to the right lane boundary, and (b) the calculated YRE through the curve.**

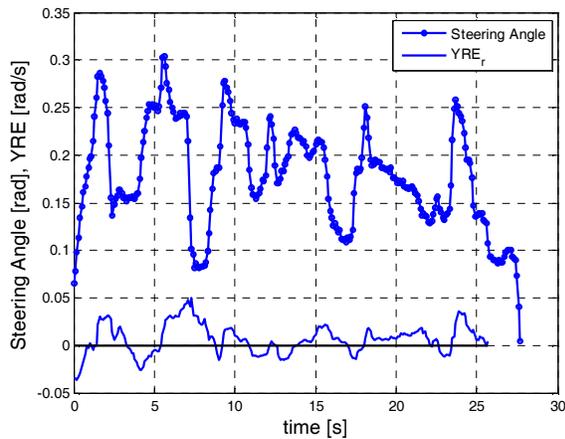
All conflicts for this event appear to be “right side only”, so the yaw rate error in Figure 6(b) is positive whenever the current location and path predict at least on lane boundary conflict within the chosen time horizon (0.5 – 2 sec). We see that YRE is always positive at the start of a lane excursion, and actually always becomes positive before a lane excursion occurs. In this sense, as would be expected, it is predictive of each lane excursion.

Figure 7 shows YRE again (lower plot) together with the time to lane crossing (TTL) in the upper plot and also its reciprocal (inverse TTL, or ITTLC) in the center plot. ITTLC might be preferred as a conflict metric since large values indicate proximity to a lane excursion, in contrast to TTL which is large when the vehicle is tracking the lane well. The main features seen in Figure 7 are the great variations and major discontinuities in TTL and ITTLC, as compared to the much more continuous form of YRE. This suggests that YRE may potentially connect more directly to the continuous steering control behavior of the driver, especially since lane crossing is not generally a catastrophic event and does not generate a panic response from the driver.



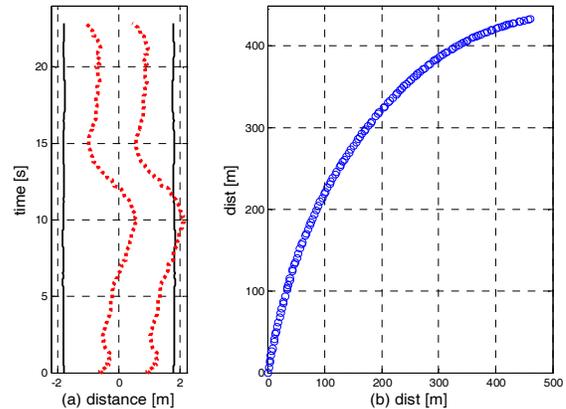
**Figure 7. A comparison of the driver risk parameters for event 1; (a) the TTL, (b) the ITTL, and (c) the YRE.**

In Figure 8 this is tested informally by plotting steer response (upper curve) as well as YRE (lower curve). Each local peak of the YRE curve seems to coincide with a sharp negative slope in the steering, and this is clearly the case at the YRE peaks at around  $t=2, 7, 16$  and  $24$  seconds – these sharp reactions seem to correlate with corrective actions by the driver in a way that TTL, ITTL and even lane crossing in Figure 6(a) do not. The distracted driver in this event is not responding to YRE as it reaches positive values, but arguably when attention to the road coincides with a positive value of YRE.



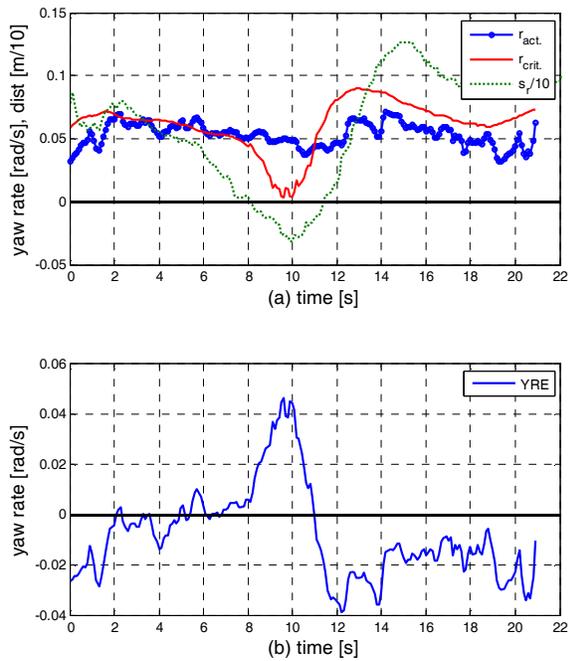
**Figure 8. A comparison of the driver controlled steering angle and the calculated YRE for event 1.**

The second event from the RDCW data was a single boundary crossing followed by a correction back to the middle of the lane. The vehicle trajectory data can be seen in Figure 9. The event is somewhat simpler than event 1, in that only one major excursion exists. Figure 10 shows the event in terms of yaw rate and critical yaw rate, and it's interesting that the conflict most heavily dominated by variations in the critical yaw rate rather than the actual yaw rate. In the upper plot, the yaw rate exceeds its critical value at around 7 seconds, while the first lane excursion takes place around 1 second later, again showing the predictive nature of YRE. In the lower plot, the YRE undergoes a correction at  $t=10$  seconds and from the previous analysis we would expect to see a sharp negative slope in the steering angle then.

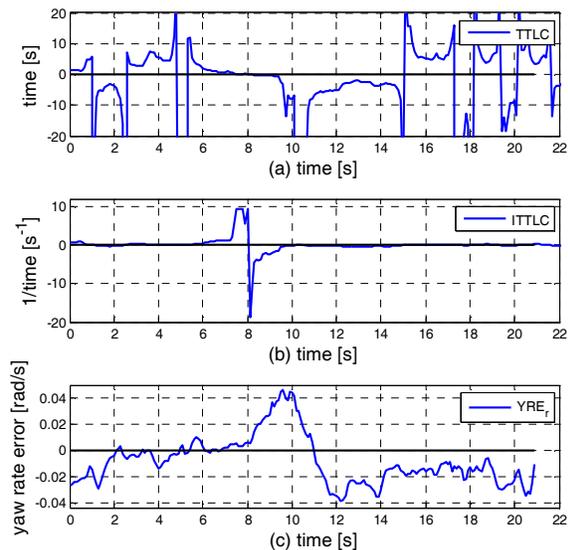


**Figure 9. The vehicle path for event 2. (a) The dotted lines represent the left and right edges of the vehicle with respect to the center of the lane markings (solid lines). (b) The X-Y trajectory of the vehicle.**

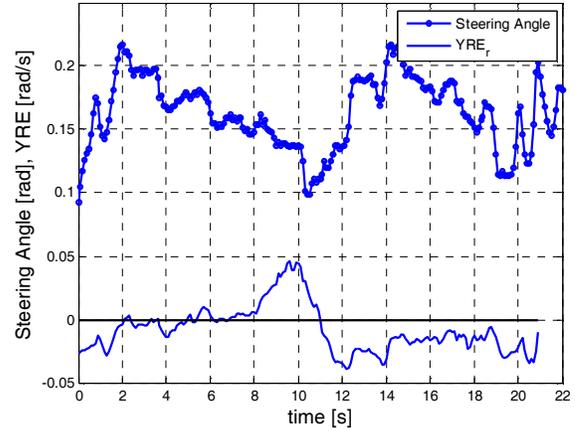
First we note however that in Figure 11, the previous comparisons with TTL and ITTL are repeated, the time-based metrics showing large discontinuities, while YRE varies continuously and in a simple way during the event – it grows at a very roughly uniform rate until the correction is presumably applied at  $t=10$  seconds, then decays uniformly until at around 12 seconds it is corrected again in the opposite sense. Turning to Figure 12 a sharp negative slope is seen at  $t=10$ , and a positive slope steering correction takes place at  $t=12$ , as expected. Of course there are other steering corrections visible in Figure 12, and not all are directly predicted by conflicts with the right lane boundary, but perhaps some involve the right lane boundary. To this end we consider below a modified plot of vehicle yaw rate plotted over the pair of critical boundary cases.



**Figure 10. The yaw rate parameters for event 2; (a) the actual yaw rate, critical yaw rate, and distance to the right lane boundary, and (b) the calculated YRE through the curve.**

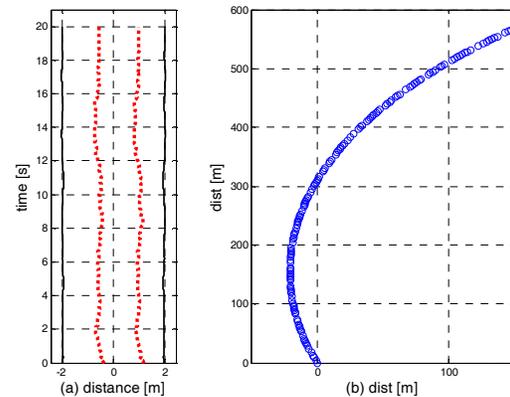


**Figure 11. A comparison of the driver risk parameters for event 2; (a) the TTLC, (b) the ITTLC, and (c) the YRE.**

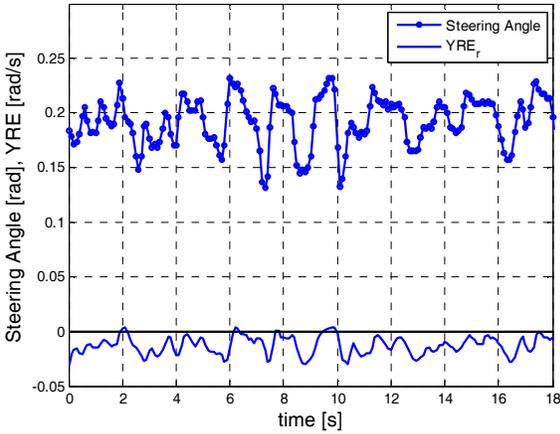


**Figure 12. A comparison of the driver controlled steering angle and the calculated YRE for event 2**

First however we consider a third example, also on a curved road section, but where there are no obvious lane boundary conflicts – see Figure 13 – which shows a nearly uniform distance from the car to the lane boundaries while negotiating the right-hand curve. Surely in this case the control loop is “inactive”, meaning the driver has found a stable line and has no need to make multiple corrections to avoid boundary conflicts. Figure 14 appears to show otherwise. Again we are plotting YRE for the right boundary and steering control actions. Far from being random or disconnected from the boundary conflict, the driver appears to be making regular steering corrections (negative slope interventions) whenever YRE approaches a critical (zero or positive) value



**Figure 13. The vehicle path for event 3. (a) The dotted lines represent the left and right edges of the vehicle with respect to the center of the lane markings (solid lines). (b) The X-Y position of the vehicle in space.**

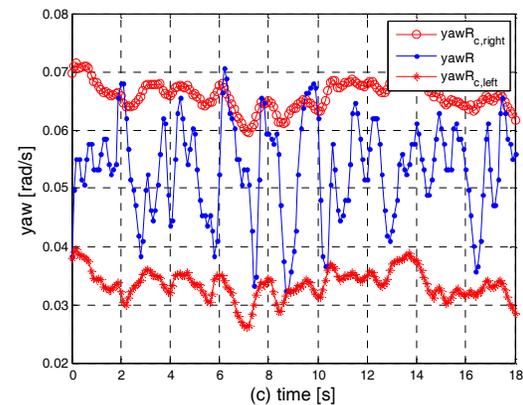
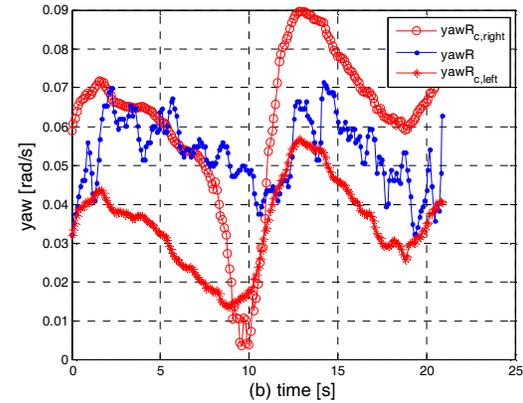
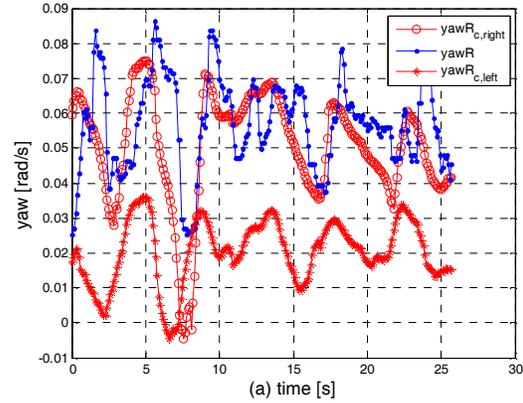


**Figure 14. A comparison of the driver controlled steering angle and the calculated YRE for event 3.**

Figure 15 now shows the yaw rate versus its two critical limits, where conflict avoidance takes the form

$$r_{c,left} < r < r_{c,right} \quad (11).$$

All three events are shown, but the most striking is for event 3 in the lower plot: the vehicle appears to be controlled very precisely within the critical boundaries, with minimal overshoot but using the full range. Far from a stable “on center” steering control tracking the lane center, in “YRE space” the vehicle is “bouncing” quasi-periodically between its limits. If this interpretation is correct, the YRE provides a simple picture of lane-keeping control actions by the human driver. Turning to the center plot, where a single excursion event was seen, the degradation in control appears to be initiated as early as  $t=3$  seconds when the more stable “bouncing between limits” is interrupted. After the lane excursion is corrected, normal effective control appears to be regained at around 14 seconds. Turning back to Figure 9(a) this same interpretation seems reasonable from the within lane drift – intuitively the driver is drifting right from about  $t=3$ , and only recovers full control at around 15 seconds. The point here is that YRE seems to provide a direct measure of lane keeping performance, and may even correlate with the error criterion active in the control loop of the human driver. In Figure 15(a) it appears that the driver does not regain effective control of the vehicle throughout the 15 seconds, and this is consistent with the distracted nature of the driving event. Finally in the upper two plots we see that left and right boundaries actually cross over, so no “solution” to (11) actually exists! We briefly consider this intriguing situation in the discussion below.



**Figure 15. Comparison between the critical yaw rate for left and right boundary conflicts and the actual yaw rate for (a) event 1: riding the right boundary, (b) event 2: single boundary crossing with correction, and (c) event 3: good lane following.**

## DISCUSSION

In the above we have defined a yaw rate error (YRE) criterion that is motivated by the potential shortcomings of time to lane crossing (TTL) as a measure of steering control performance during lane keeping. The main features have been seen above, but in summary:

- YRE behaves in a continuous way, even when lane boundary crossings take place, and this is not the case for TTL and its reciprocal
- YRE excursions correlate strongly with rapid steering interventions by the driver, especially when the driver is providing effective control of lane position
- When left and right critical yaw rate boundaries are considered simultaneously, the normal effective control of lane position appears to operate to constrain between the crucial limits
- YRE may be a useful predictor of actual lane excursions, but more importantly it seems to provide a strong indicator of degraded or ineffective lane keeping

In events 1 and 2, the lane excursions appear to induce an “impossible” situation for the driver – the left and right limits cross over. This is most easily seen in Figure 15(b), where crossover takes place between approximately  $t=8$  and  $t=12$  seconds. From Figures 9(a) and 10(a) this corresponds to the vehicle being outside the lane boundary – clearly the steering task changes from lane keeping to lane recovery, though from Figure 12 the reaction seems to be consistent with a single sharp correction to “divert” the YRE to a correct linear rate of descent, followed by a second sharp correction in the opposite direction at around  $t=12$  seconds. Thus it seems the crossover is not a major factor to the driver, who perhaps applies focus to one boundary at a time.

## CONCLUSIONS

A simple yaw rate error criterion has been proposed for the analysis of steering control behavior. It can be used for the post-hoc analysis of naturalistic driving data, and with suitable development is likely to be feasible for real-time evaluation on the vehicle. It offers a number of simple advantages in terms of continuity and correlation to steering response, and offers a potential means of distinguishing between normal and degraded steering control while lane-keeping. In this paper there has been no attempt to analyze a large number of driving events, or establish a formal relationship between YRE and particular

driving situations or other measures of control (e.g. eyes off the road, secondary tasking). The results above were based on randomly chosen events, and there was no selection procedure other than to find events from lane position typical of (1) an extended period of degraded lane control (2) a single event lane excursion error (3) well controlled lane keeping.

Further work will expand the number of events and attempt to more formally and accurately quantify the relationships hinted at in the three events presented. Also, further work is anticipated to evaluate YRE as a potential surrogate for crashes that happen due to disturbed control during lane-keeping. This particularly refers to lane-departure crashes and single-vehicle road departure crashes. Surrogate validation is to be based on factor analysis that link the statistical properties of both crash and surrogate.

## ACKNOWLEDGEMENT

The authors are grateful to the SHRP 2 Safety Program [9] for funding and collaborative guidance in the conduct of this research.

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