Development of the Virtual Testing Benchmarks (VTB)

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ABSTRACT

The SubProject 7 "Virtual Testing" [1] of the 7th FP Project APROSYS (Advanced PROtection SYStems) was aimed at development of a complete and consistent methodology for the implementation of the virtual testing of vehicles for safety improvement. Recall that by Virtual Testing we imply any analytical certification procedure which uses experimental and numerical simulation methods [2]. To achieve this goal, specific models, methods, and tools were developed. One of the final achievements relates to the future use of virtual testing in regulations, not only in the design of vehicles for safety [3].
The implementation of virtual testing in regulations would be a very complex process involving several steps [2], and concerning many different actors and stakeholders from car manufacturers to consumer organizations, and from regulatory bodies to experts group in automotive engineering. Among the many envisaged steps, which are being currently structured in a specific roadmap, there is the qualification problem. For both type of accreditation method, either the type approval scheme usual in the EU, or the US style self-certification scheme, a qualification process is required.
To this aim the authors propose to establish a series of benchmarks, the Virtual Testing Benchmarks (VTB), to be used for qualification at two different levels: codes and methods validation, and operators' qualification. These benchmarks consist of typical crash cases to be tested in the virtual environment: there are several different cases covering different topics of modeling (different element types, material models, contacts…). The code validation can be achieved by giving a well defined problem to be solved, whereas the operators qualification can be achieved giving a less defined framework and leaving more freedom to the operators to generate their own models of the problem.
At least 5 different cases are provided and described in the paper. Verification by means of experimental or theoretical solutions is given. Of course, this will not cover all possible modeling situations but is a first step towards this electronic certification.

1. INTRODUCTION

One of the main concerns about the applicability of VT in regulations is the validation of the VT predictions. In the APROSYS SP7 workshop, held in 2007 at the Delft TNO location, a presentation [4] was given putting in evidence possible downsides for the applicability: different codes were shown to give different results. A discussion was initiated during the period following that workshop, coming to a couple of main results [5]:

1. **VT is** the results of the application of **a model**: whatever the model and whatever sophisticated, detailed, and accurate the model **it will always have limits of validity** and, therefore, does not extend to any possible boundary conditions and scenarios unless with careful verification
2. It is necessary to have some validation procedures for VT

VT is a complex methodology, which has many of aspects, all examined in this APROSYS subproject:

- Virtual models
- Virtual methods
- Virtual tools
- Virtual testing procedures

As a consequence, it involves several "components" and steps within the preparation and development of a VT approach:

- The model data (materials data, dimensional data and geometry, contacts...) and the scatter

involved in the set of data (dispersion, distribution...)

- The code (with its implemented material models, element models, and their options, contact models and definition, boundary conditions...)
- The computer used in the analysis, including the interface with the codes and with the operator
- The procedures and scalars used for the analysis and evaluation stated either internally in the company or the research centre, or externally by some institutional reference (national or international standard, EU directive or any applicable national or international law, other institution or organization like NAFEMS, APROSYS standards, etc.)
- The operator himself, with his skills and experiences, including the possibility of human errors during the manual work

Thus, several validation levels should be taken into account and different benchmarking procedures should be defined. Already within the ADVANCE FP5 project, a multi-level validation approach for crash models was proposed [6]. The validation of a model was, in this case, at the component, subsystem, and full-vehicle or car compartment levels.

Consequently, within APROSYS benchmarks for Virtual Testing have been conceived. They are called Virtual Testing Benchmarks (VTB) and try to solve the previously discussed problems by treating most of the proposed items. In the current definition of the proposed simple models to be used as a basis for a benchmark procedure, two validation phases have been selected:

1. Human factor influence or operator validation
2. Instrumental certification or code validation (including the validation of the models implemented in the codes, the data input/output and the computer used in the analysis)

In phase 1 only generic definition are given since it is exactly the human factor that has to be assessed. That is, the test case is only defined in general terms, and although fully detailed, no mesh or input cards are given. It is the operator's job to prepare his/her own model, selecting the types of elements and material models that he/she thinks most appropriate for the purpose, etc.

In phase 2 all that can be associated to the human factor has to be avoided or at least limited as much as possible. This is done not only by precisely defining the test case with all its input data and parameters, but also by giving the involved operator the explicit inputs in terms of geometry (that is, even the mesh is given in some widely used format or some general purpose definition style), materials data (in terms of input values or curves in some reference format universally accepted or precisely documented), contact definitions, etc.

Reference results are given for the validations. The reference results are obtained either from experiments, from a theoretical model, or from validated simulations whenever neither experiments nor a sound theory are available.

In the following section, the various proposed cases are summarized. The cases are then described and commented in details. All the data that are not suitable to be written on a document will be available on files.

It was the aim of the authors in this activity to cover the main topics of interest for crash simulation. In this respect, the different cases consider different types of elements (solids rather than shell elements), different materials (polymeric foams rather than metals), and other essential modeling issues like, for example, contact definition (including modeling of friction, elastic constant...).

The family of simple cases or benchmarks for VT finalized up to now is considered a starting point. Since the development in APROSYS DIP3 and DIP4 were focused on pedestrian protection (mainly head and leg impacts), the partners chose to select a set of cases related primarily to this subject. The B-pillar case, and future proposals, will cover all the other aspects related to safety and crash simulations.

The VTB method should be, in the authors' idea, a common background for engineers approaching virtual testing, but also for all the people working in crash simulations. The VTB family should continuously improve and adapt to the ever changing requirements of simulations and evolution of modeling codes and techniques.

2. VTB OVERVIEW

Table 1 and Figure 1 summarize the main aspects taken into account with the VTBs. Each case is described synthetically here and will be addressed in details in the following section.

In phase 1 the modeling capacities have to be evaluated, and, therefore, most of the job is left to the operator under review. No mesh or material cards can be given, but a precise geometrical description together with material curves and indications of the type of material model to be used (i.e. elastic vs. plastic, hardening model, strain-rate sensitivity…). The choice of the type of elements is somewhat enforced, but freedom is still given for what concerns the element formulation (fully integrated vs. under-integrated, linear vs. parabolic…). Perhaps the most difficult problems, as is often with FE analyses, arise from boundary conditions modeling: contact is for example a critical issue to be addressed.

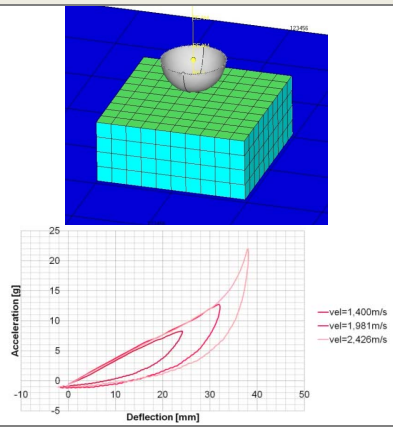
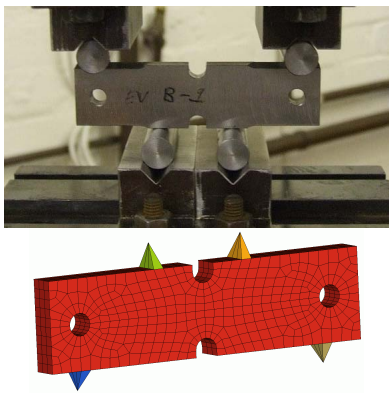
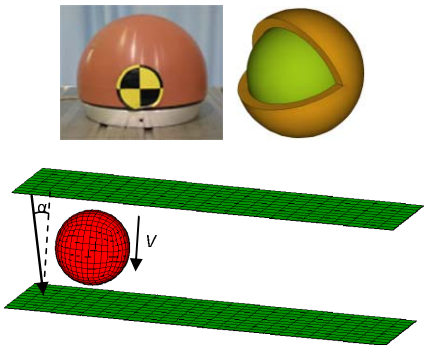
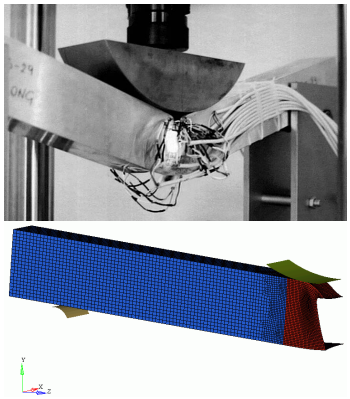
Table 1.
Summary of the VTB cases

Name	**VTB1**	**VTB2**	**VTB3**	**VTB4**	**VTB5**
	Foam	Legform	Bouncing	Bending	B-pillar
Description	Impact on foam	Legform knee	Bouncing ball	Bending beam	B-pillar impact
Scope	Evaluate modeling of soft foam materials for dummies and other EA components	Evaluate modeling of metals and of bending situations, solid elements	Evaluate modeling of contacts, energy management and stability	Evaluate performance of shell elements, simple contacts	Same as for VTB4
Material	Foam+rigid impactor	Elasto-plastic	Elastic	Elasto-plastic	Elasto-plastic
Elements	Solids (shell only rigid)	Solids (shell only rigid)	Solids	Shells	Shells
Other			Contacts		
Phase 1	Foam material stress-strain curves at different strain-rates (loading/unloading)	Geometry and description of the case. Material description	Geometry and description of the case. Main material properties (elastic)	Geometry and description of the case. Main material properties (elasto-plastic). Contact parameters.	Same as for VTB4
Phase 2	Mesh Foam material model specification and curves	Mesh Boundary conditions Material data (yield stress, yield curve, strain rate sensitivity)	Mesh Material (elastic) Contact definition Contact parameters	Mesh Boundary conditions Yield stress curves	Mesh Boundary conditions Yield stress curves
Validation	Experimental curves	Experimental curves	Theoretical results (for the 0° case only)	Experimental results	Simulations results

In phase 2 topics like material and element modeling are crucial. This activity is aimed to evaluate code functionality and ability to correctly address these features; therefore it is necessary to fix the other modeling issue: the geometrical description is given (mesh and boundary conditions) together with a detailed description of the material model and of the necessary curves.

The last issue in the definition of the VTB regards the evaluation of the results.

Comparison results have to be provided. Two approaches are possible: the comparison can be with experimental results or with some theoretical prediction. The first possibility is preferable, for obvious reasons. However, whereas most of the VTB come from real cases and have a physical counterpart, in some cases the problem is only "virtual", that is, is a generic problem with, possibly, a theoretical solution.

Name	Description
VTB1 Foam	
VTB2 Legform knee	
VTB3 Bouncing ball	
VTB4 Bending beam	

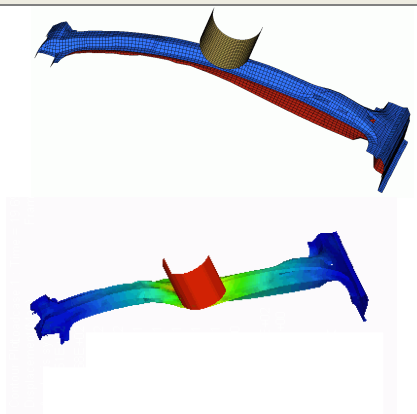
Name	Description (cont.)
VTB5 B-pillar	

Figure 1. Pictorial summary of the VTB cases

3. VTB DETAILED DESCRIPTION

Following is a description of the first five VTB defined. As discussed previously, the VTB family would be further increased and improved with more cases related to impact and crash analysis.
These five cases are as follows:

- VTB1: Foam
- VTB2: Legform
- VTB3: Bouncing
- VTB4: Bending
- VTB5: B-pillar

The impact of a ball on a piece of foam (VTB1) and the impact of a headform like spherical component between two rigid surfaces (VTB3) are inspired to the activities carried out in APROSYS SP7 (head protection demonstrator [3]). The knee ligament test (VTB2) is equally motivated by APROSYS SP7 (leg protection demonstrator [3]). The last two cases (VTB4 and VTB5) come from typical crash situations.
The various VTBs try to cover as many modeling topics as possible. As previously argued, future needs can be equally covered by defining more VTB cases.

3.1 VTB1: impact on foam

The first VTB can be summarized as follows:

- Description: hemispherical impactor on foam specimen
- Aim: evaluate modeling capacity for solids and soft materials
- Reference: experimental impact tests on an EPP foam

Highly compressible low density foam models are often used in passive safety numerical simulation. Main applications are for seat cushions and padding elements on biomechanics test devices. A compression test with a spherical impactor on an EPP 20 g/dm³ foam specimen is proposed as VTB test (Figure 2).

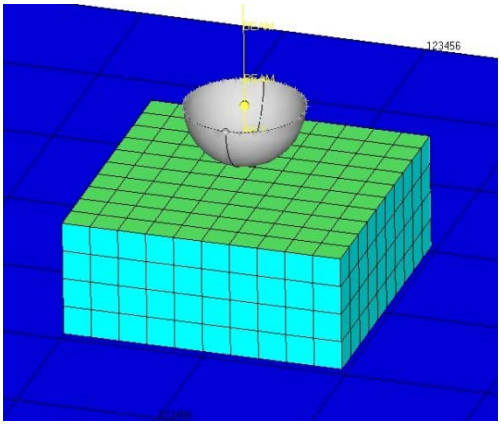


Figure 2. Impact of a spherical impactor on low density foam model

In phase 1 of the evaluation of this VTB only generic guidelines about geometry, boundary conditions and material have been given to the different operators that worked on the model. In particular:

- The foam specimen is 42 mm thick with a base of 110 mm × 110 mm
- The rigid spherical impactor has a 42.5 mm diameter, an 8.3 kg mass and different test velocities: 1.4 m/s, 2.0 m/s and 2.4 m/s.
- Some graphs (Figure 3) based on foam material experiments

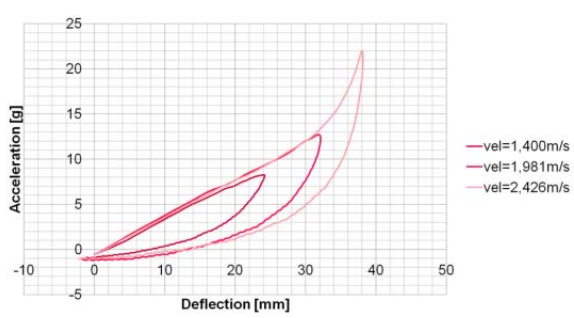


Figure 3. Impact of a spherical impactor on low density foam model

In phase 1 the model has been developed for three different codes (RADIOSS, LS-DYNA, and MADYMO).

In phase 2 the same mesh for the foam has been used for the different models and the foam material model has been chosen as similar as possible between the different codes in order to give to the imaginary operators not only the experimental foam results but directly the material defined for the used code.
In this case also, the model has been developed in RADIOSS, LS-DYNA and PAM-CRASH.
All the different data specified for phase two are summarized in Table 2.

Table 2.
Data specified for phase 2

Average element dimension	10 mm (10×10×4 elements)
Boundary conditions	Supported (not constrained) on a rigid plane
Data for material card	Depends on code
Young Modulus	1.89 MPa
Density	20 g/dm³
Poisson Ratio	0.02 (for compression)
Compression Curves	at 0, 1×10^{-6}, 1×10^{-4}, 2×10^{-1} and 1×10^{-1} strain-rate
Tension Curve	
Unloading Curve	

Numerical results have been compared between codes, but in this case with experimental results also.
A qualitative comparison of results is represented in Figure 4. The three codes analyzed are able to reproduce the experimental test at different velocities with sufficient fit of the experimental curves. Figure 5 reports some results of an unacceptable case of analyses performed. The scatter is not acceptable and more detailed modeling is required.

3.2 VTB2: legform knee

This second VTB can be summarized as:

- Description: knee joint element used in the EEVC WG17 (WG10) legform to bumper test
- Aim: evaluate modeling of steel materials
- Reference: experimental 4-point bending tests

The deformable, simulated knee joint element (Figure 6) used in the EEVC WG17 (WG10) legform to bumper test has been chosen for the second VTB [7-8].
The geometry has been obtained from the sketch shown in Figure 6.

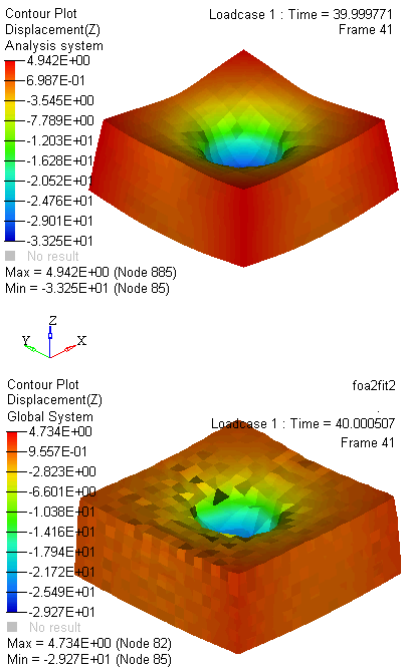


Figure 4. Comparison of the results obtained from two of the used codes

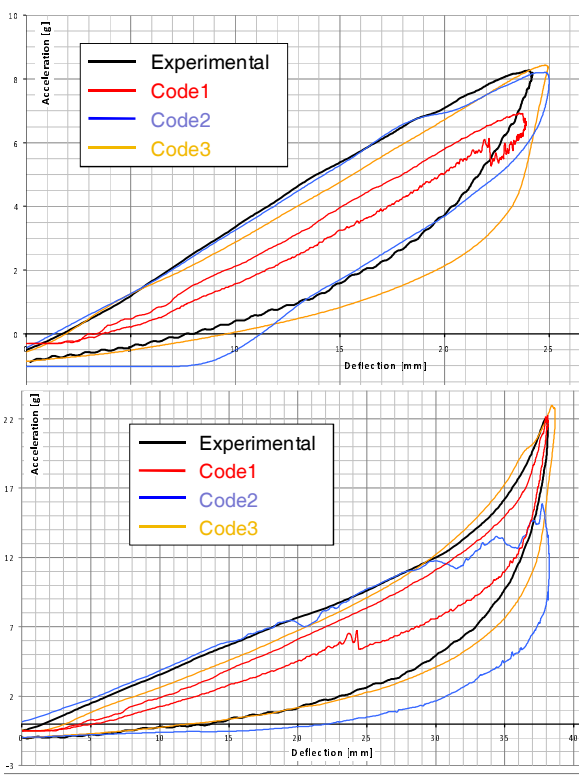


Figure 5. Comparison of some unacceptable results

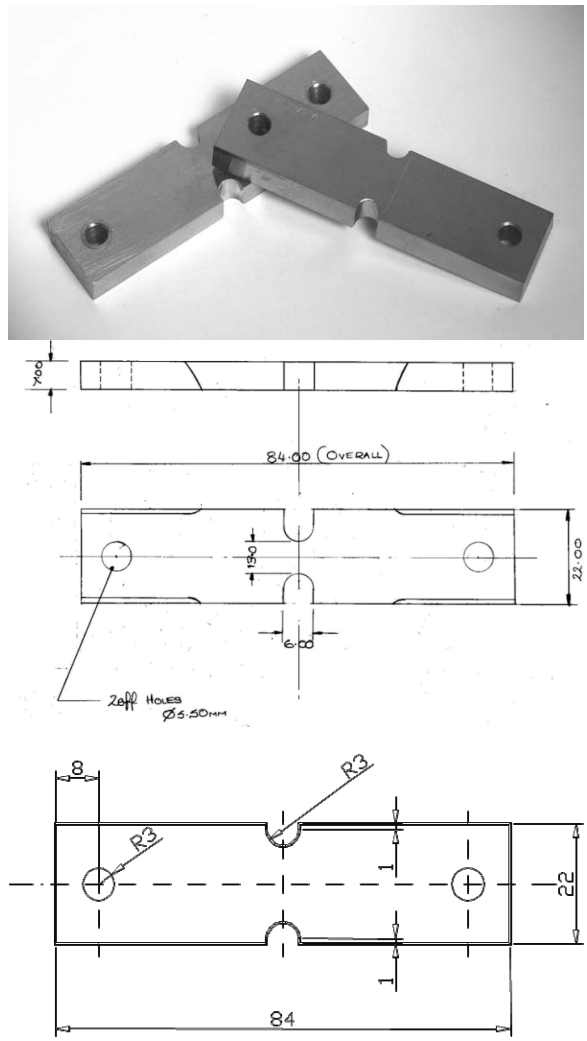


Figure 6. The EEVC knee ligament: photo of the component, sketch, and detailed drawing

The aim here was to obtain an easily reproducible test, avoiding the experimental and numerical complexities and uncertainties of a "real" case. Therefore, the ligament only was tested, and not the entire legform. Testing the ligament in bending can be made at least in two configurations (Figure 7). The first is the usual 4 point bending test. A second case is the 4 point asymmetrical loading, shown in the same Figure 7. In this case the middle section is loaded in pure shear instead of pure bending. This loading case has not been considered yet, but it is certainly a future possible step ahead in VTB method development.

The knee specimen has been modeled by using a mild steel material (Table 3).

The behavior in terms of moment vs. rotation and force vs. displacement histories has been used as reference for the comparison between the different

codes. In particular, in order to compare numerical results with experiments also, boundary conditions schemes represented in Figure 7 have been considered and the force vs. displacement history of the impactor has been used. The experimental results are reported in Figure 8.

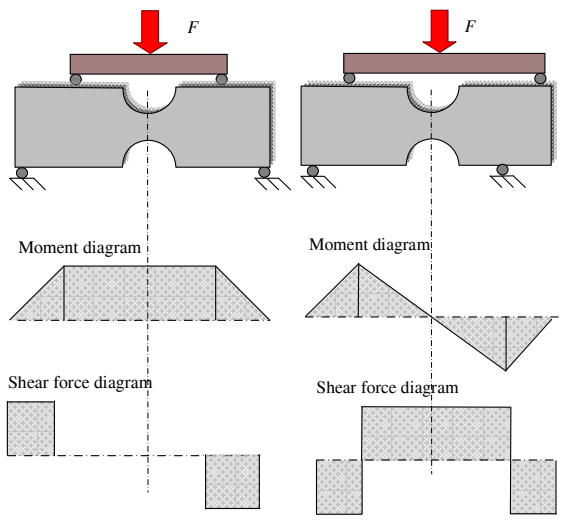


Figure 7. Loading schemes for the knee ligament: left, classical 4 points bending test; right, 4 points asymmetrical loading

Table 3.
Material data VTB2

Material model	Bilinear
Density	7.8×10^{-6} kg/mm3
Young modulus	210 GPa
Poisson ratio	0.3
Tangent modulus	6 GPa
Yield stress	150 MPa
Element property	Reduced integration
Impactor velocity	0.06 m/s
Support material	Rigid

The VTB results, given here for phase 2 only, are in good accordance with the experimental results. The comparison with the experimental results is shown in Figure 9, whereas Figure 10 shows a simple comparison of the deformed shapes obtained from two of the codes already mentioned (LS-DYNA and RADIOSS). It is important to notice, the level of details necessary despite the simplicity of the problem. Some preliminary investigations demonstrated that simpler boundary conditions' modeling is insufficient to obtain the correct behavior.

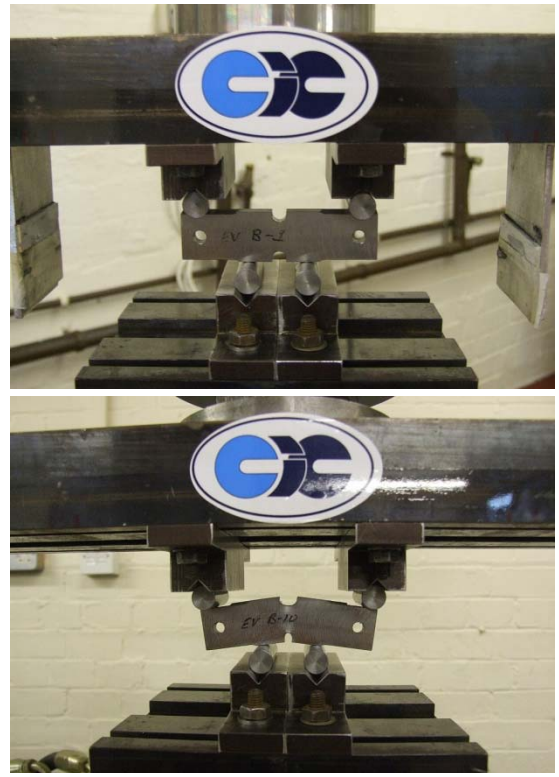


Figure 8. Experimental results for comparison

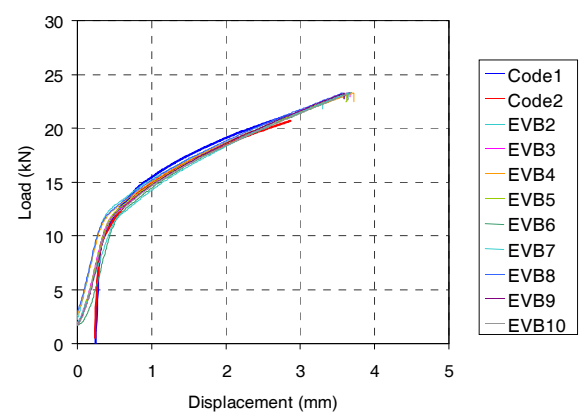


Figure 9. Comparison of the experimental curves with the numerical simulations of two codes used

3.3 VTB3: bouncing ball

This third VTB also inspired by the headform experiment for pedestrian protection has the following characteristics:

- Description: a spherical head, similar to the adult headform of the pedestrian, bouncing between two rigid parallel surfaces
- Aim: evaluate modeling for contacts
- Reference: analytical/numerical solution

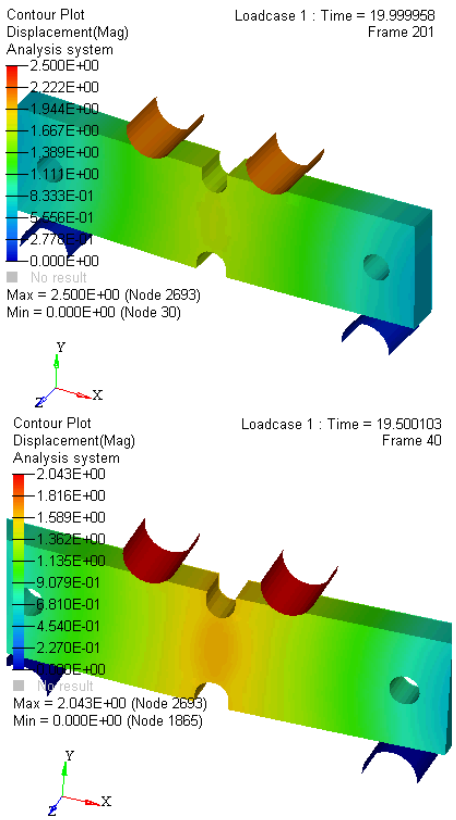


Figure 10. Comparison of the numerical results from two explicit simulation codes

The case is explained by Figure 11: a ball made of aluminum covered by rubber, is constrained to bounce between two rigid surfaces moving at an initial given speed v_0 and angle α.

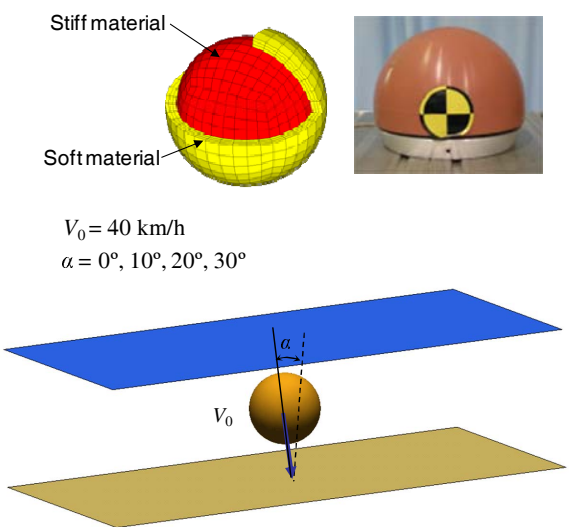


Figure 11. The VTB3 case

In VTB3 focus is on the modeling of contacts rather than on materials that are simplified as linear elastic. This is to avoid that possible difficulties in material modeling overshadow the contact modeling aspects.
The sphere is launched normally against the first wall with a v_0 velocity equal to 40 km/h (as in Figure 11). The angle α between v_0 direction and the surface normal varies (0°, 10°, 20° and 30°) to take into account contact friction. Tables 4 and 5 give additional details about the problem.

Table 4.
Definitions for VTB3: phase 1

	Elastic modulus (GPa)	Density (kg/mm^3)	Poisson coefficient
Inner sphere, 80 mm radius	70	2.8×10^{-6}	0.3
External layer, 12 mm thick	0.05	1.05×10^{-6}	0.3

Table 5.
Definitions for VTB3: phase 2

Fixed planes	Meshed with elements (not defined as rigid planes); E = 210 GPa (for contact stiffness)
Contact properties	Surface to surface contact
Solid element	Full integration
Solid elements covered with skin of null shells or segments	
Static friction coefficient	0.6
Dynamic friction coefficient	0.6
Contact thickness (gap) for both slave and master side	0.5 mm

Contact force histories and sphere trajectories have been considered for the comparison between the different models.
For this VTB also, the work has been divided in the two phases. In addition to the general description given to the operators during the first phase, mesh density and contact definition and parameters have also been given in the second phase. The different requirements for phase one and two are summarized in Tables 4 and 5.
Figure 12 shows a comparison of some results obtained at different moving angles. Together two images of two different models, obtained from different operators as in phase 1, are reported. Figure 13 reports two comparisons in terms of contact forces vs. time: first the phase 1 comparison is reported, and the phase 2 results follows.

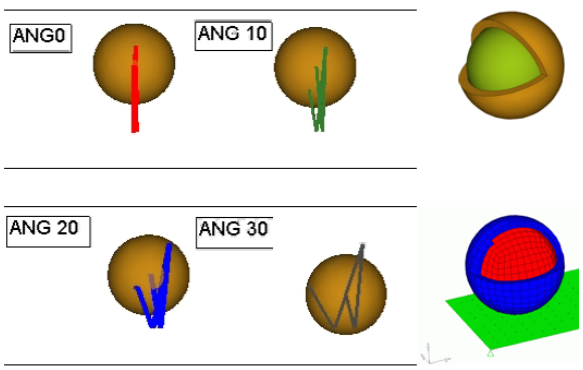


Figure 12. Some VTB3 results at different angles: lines overlaid to the balls images represent the trajectories. On the left different models obtained from a couple of different operators in phase 1 analysis

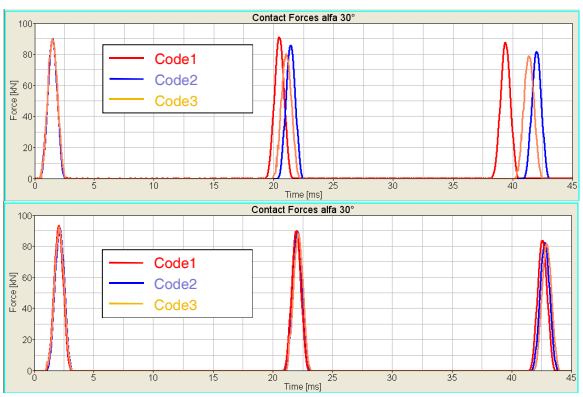


Figure 13. Results from phase 1 and phase 2 analyses: contact forces vs. time; angle 30°

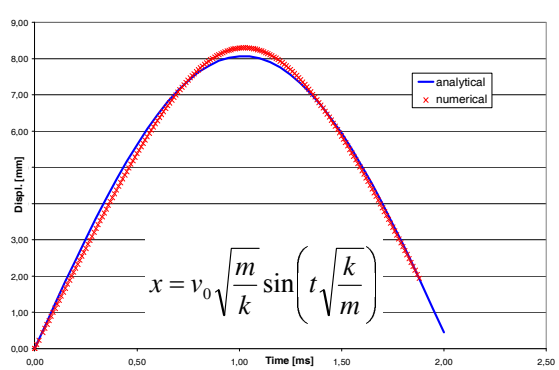


Figure 14. Theoretical approximation of the ball impact

As expected, there is a certain amount of scatter between the results from phase 1: the uncertainty on the data is slightly greater. However, phase 2 comparison is excellent and the three used codes already mentioned give more or less the same results. Some minor discrepancy appears after repeated impacts.
Finally an approximate theoretical solution is available and shown in Figure 14.

3.4 VTB4: thin walled beam bending

The VTB4 problem is summarized as follows:

- Description: formation of plastic hinges in a thin-walled square cross section aluminum beam
- Aim: evaluate shell modeling capability
- Reference: experimental bending tests [9]

Deep bending collapse is often the main failure mode in the thin walled tubular members of vehicle structures. This occurs because this energy absorption mechanism requires a lower specific energy (dissipated energy/material volume) than the axial crushing. This VTB is related to the formation of plastic hinges in a thin-walled square cross-section aluminum beam. The numerical model tries to reproduce the experimental test represented in Figure 15.

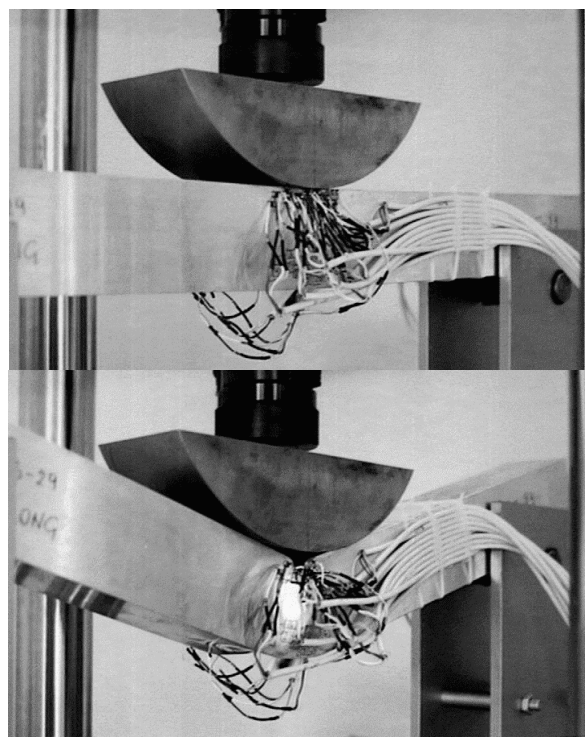


Figure 15. The VTB4 experiment [9]

For this case, only phase 2 has been considered. The model has been developed in LS-DYNA, RADIOSS and MADYMO and the mesh in Figure 16 (taking

into account double symmetry of the problem, only one quarter has been modeled). The main problem data are summarized in Table 6.

Table 6.
Definitions for VTB4

Property	Value
Shell thickness	2 mm
Material properties	Piecewise Linear Plasticity
Density	2.7×10^{-6} kg/mm^3
Young modulus	70 GPa
Poisson ratio	0.3
Yield stress	0.079 GPa
Contact properties	Surface to surface contact
Static friction coefficient	0.61
Dynamic friction coefficient	0.47
Contact thickness (gap) for both slave and master side	2 mm
Impactor	Rigid
Support	Rigid

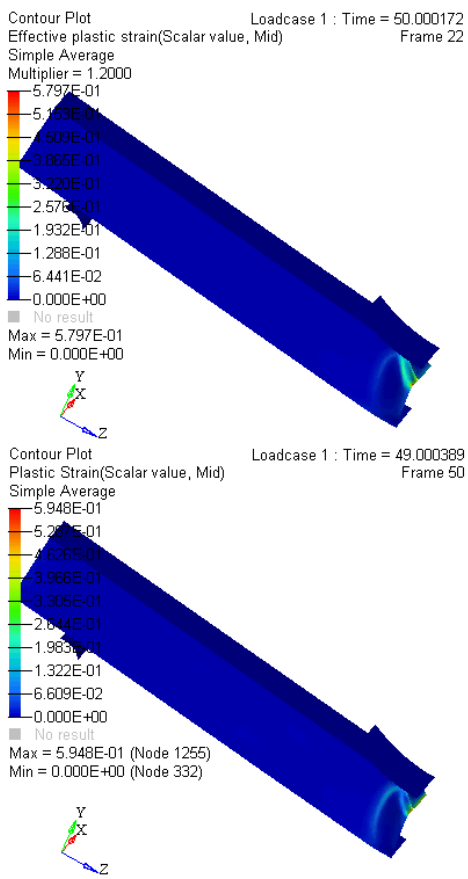


Figure 16. The VTB4 numerical result

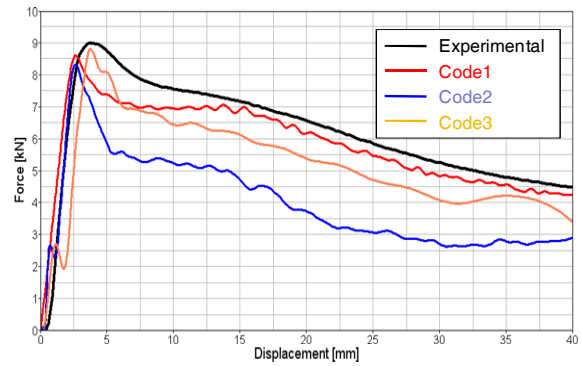


Figure 17. The VTB4 results comparison

The comparison is acceptable for all the three codes used (Figure 17). Of course, small differences cannot be avoided since different modeling (materials, elements, contacts…) are present. One of the codes gives more dissimilar results and is not really acceptable in this case.

3.5 VTB5: B-pillar impact

The VTB5 problem is summarized as follows:

- Description: bending collapse of GCM4 B-pillar
- Aim: the use of shell elements, contacts, inelastic materials
- Reference: Numerical simulations of the APROSYS GCM4 [10]

The GCM4 (Generic Car Model 4) is one of the results of the FP6 APROSYS project. Generic Car Models [10] are virtual vehicles developed to be shared between researchers without confidentiality restrictions. These models of vehicles (5 car of different sizes and a heavy truck model), provided for some typical impact scenarios, are validated against the best-in-class models on the market.
Even if this VTB seems quite simple if compared to real crash analyses, it is of interest taking into account different aspects of a complex crash analysis, and it can be useful to investigate differences between codes due to:

- the use of shell elements for sheet components
- contacts
- non elastic materials

The pillar is composed of two parts connected by means of rigid spot-welds and it is fully constrained in its extremity. A cylindrical impactor is moved with an imposed velocity of 8 m/s. The impactor is modeled as a rigid component.

An elasto-plastic material model, usually known as simplified Johnson Cook, has been used. The flow stress is given by:

$$\sigma_y = \left(A + B\varepsilon_p^N\right)\left(1 + C\ln\varepsilon^*\right)$$

Table 7.
Parameters for material model

Material Parameter (*)	Value
Density, ρ	7.8×10^{-6} kg/mm^3
Young modulus, E	200 GPa
Poisson ratio, ν	0.3
A	0.25 GPa
B	0.08 GPa
N	0.60
C	0.03

* Units in table are referred to a mm/ms/kg/kN set

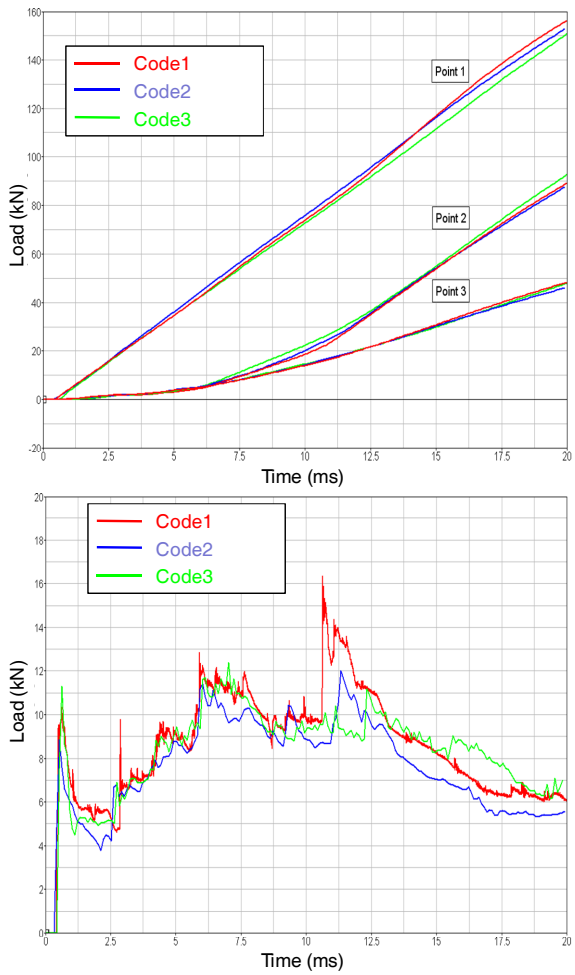


Figure 18. The VTB4 numerical result

Values for the parameters are summarized in Table 7.

Final deformed shapes, contact forces and node displacements have been taken as references.

In Figure 18 a comparison of the displacement of some reference points between the three examined codes is reported, together with a comparison of the global load exchanged with the supports.

The results (phase 2 analyses) are quite encouraging despite the difficulty related to the complexity of the problem (Figure 19).

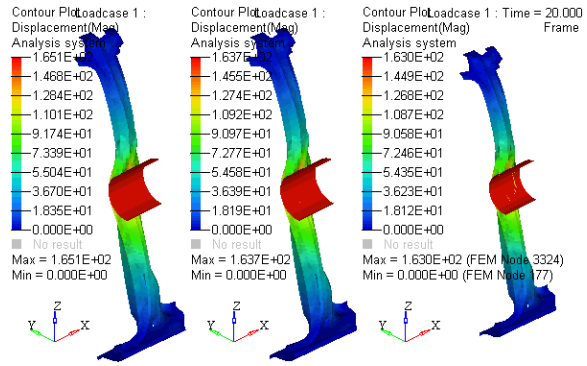


Figure 19. B-pillar results comparison between three different codes

4. DISCUSSION

The examples shown demonstrate the capability of different software codes to replicate real crash type scenarios accurately.

The accuracy of results obtained has shown to be improved if material data has been obtained prior to performing the analysis. The implications for full scale virtual testing mean that smaller component tests may be critical in building more accurate models. These smaller component tests may be a simple material test or a sub assembly of the structure which can be tested in isolation.

Although it is recognized that there are differences in the material models of different codes, in general, they demonstrate the capability to reproduce real life scenarios.

When performing a blind simulation the need to provide as much information as possible is essential. This has been demonstrated mainly in the knee impactor scenario. Material and boundary conditions must be accurately stated.

When a physical test was repeated for the knee impactor, a cloud of data was obtained. In a similar manner, there is also a scatter of results when using different software codes. For future VT legislation it is important to characterize the scatter from repeat tests and understand the implications when performing simulations.

5. CONCLUSIONS

A first step towards electronic certification of numerical codes for crash analysis and of the operators working on impact simulations has been made. This step is very important to fully exploit the potentiality of virtual testing and make it acceptable as a tool for certification.
To do this a series of simple case studies, called Virtual Testing benchmarks (VTB) has been defined.
The examples presented in this work are representative of situations mainly related to head and leg impact situations (EC regulations 2003/102/EC). Each different case tries to analyze an important aspect of the model and considers different topics arising in each situation (element type, material modeling, etc.). References are also given, from experimental, analytical, or numerical results. The simulations allow to certificate by comparison either the operators (phase 1) and the codes (phase 2). In most cases some codes were checked and fully validated. An important further step is the definition of metrics to evaluate and accept or reject the obtained results. It is also to be noted that we have not presented here the complete 3R method (rating, reliability and robustness) which was devised during the APROSYS-SP7 project which provide a full assessment methodology for a virtual testing scenario to be implemented (a paper is in preparations).
This is a first proposal to define VTBs, to be included in a full or partial virtual testing regulation investigation. More cases are being defined by the authors and the results which include the widest range of simulation capabilities will be published regularly, depending on the accident scenario investigated. Up to now VTBs can be used for internal self certification, but some quantitative assessment system is necessary to use virtual testing in regulations.

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