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Finding 3D Angular Accelerations of a Rigid Body from In-plane Accelerometers Using Optimization

V. Hasija, E. G. Takhounts, and R. H. Eppinger

ABSTRACT

This paper presents a method for determining 3D angular motion of a rigid body using linear acceleration data obtained from a set of randomly oriented accelerometers arranged in a plane. The angular accelerations are treated as design variables and are determined using an optimization methodology which minimizes the cumulative error at each time step between the measured linear accelerations and computed linear accelerations. It has been shown in the past that the six-accelerometer array becomes unstable in the presence of small inaccuracies in the measured accelerations when closed form methods are used for finding the solution. The current methodology uses an optimization technique instead of a closed form method to determine the angular accelerations and can be used with the six-accelerometer array without any instability. As a validation of the method, known angular kinematics obtained from the nine accelerometer array data are used. In-plane accelerometer subsets are used to obtain the linear acceleration data and the optimization methodology is applied to these subsets. The angular kinematics obtained with these subsets compare well with the originally known angular kinematics. Further studies are conducted with this methodology on mouthpiece accelerometers and issues like accuracy improvement and effect of cross-axis error are studied.

INTRODUCTION

Angular accelerations are determined from the linear acceleration data and are used to study rotationally induced head injuries and to drive the finite element head models. In the past, the six accelerometer array sensor package (Figure 1) and the nine-accelerometer array sensor package (Figure 2) have been used to find the angular accelerations from linear acceleration data using closed form methods.

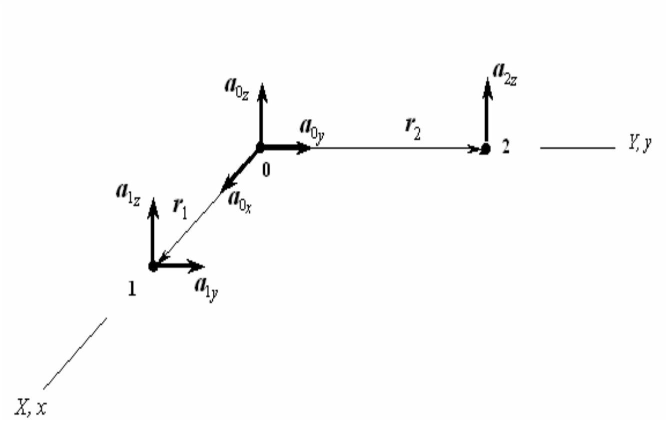

Figure 1: Six Accelerometer Array Arrangement.

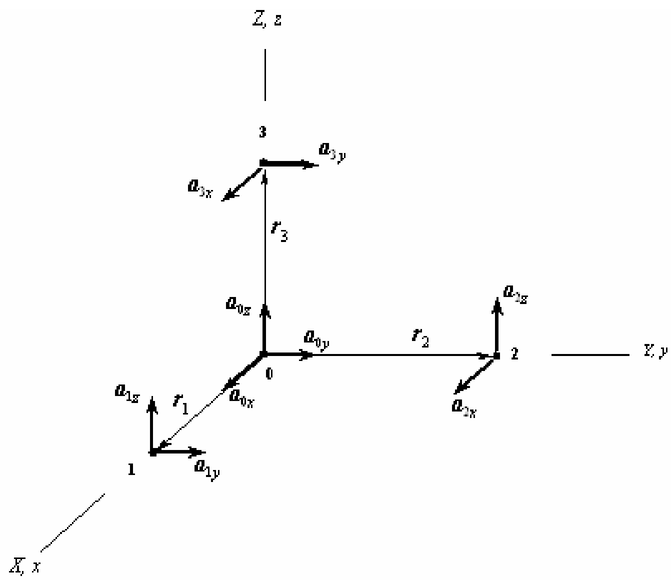

Figure 2: Nine Accelerometer Array Arrangement.

It has been shown by Padgaonkar et al. (1975) that the six-accelerometer array becomes unstable in the presence of small inaccuracies in the individually measured accelerations and also at high accelerations when closed form methods are used for finding the angular accelerations. These problems with the six-accelerometer array system were overcome by using a nine-accelerometer array system which achieves its stability by eliminating the six accelerometer array's dependency on having the knowledge of the rigid body's three rotational velocities at each instant in time.

A rigid body has only 6 degrees of freedom, and, thus only six independent measures are necessary and sufficient to completely define its 3D kinematics but the nine-accelerometer array system uses nine accelerations and is thus over-constrained. It has been shown by Takhounts et al. (2003) that these nine linear accelerations cannot be arbitrary and independent functions of time and there exists a dependency between these nine acceleration traces and error in one of the traces can lead to error propagation in other traces and thus the motion described can be very much different from the actual motion.

This paper describes a new method for finding the 3D angular accelerations of a rigid body using an optimization technique, which overcomes the disadvantages of both the six and the nine-accelerometer array system. The paper also discusses the validation of the technique and its application to mouthpiece accelerometers.

METHODS

Theoretical background – Rigid Body Motion

A brief description is given below on general rigid body motion that ends up in equations used in the optimization methodology.

For a rigid body, the 3D motion of point *B* measured by an observer located at point *A* (Figure 3) is the same as the motion of that body about a fixed point.

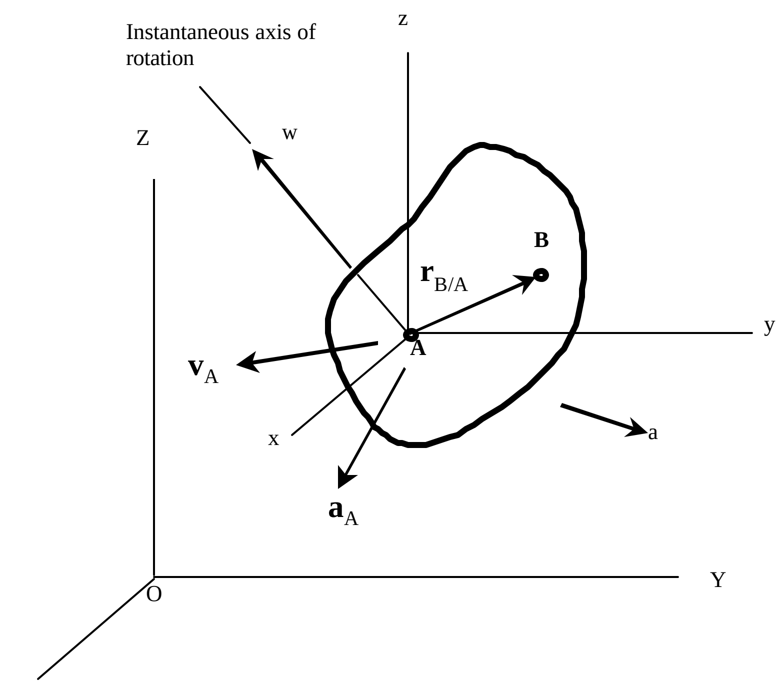

Figure 3: Rigid body subjected to a 3D general motion.

This relative motion occurs about the instantaneous center of rotation and is defined by:

$$\boldsymbol{v}_{B/A} = w \times \boldsymbol{r}_{B/A} \qquad (1)$$

and $$\boldsymbol{a}_{B/A} = a \times \boldsymbol{r}_{B/A} + w \times (w \times \boldsymbol{r}_{B/A}), \qquad (2)$$

where $\boldsymbol{v}_{B/A}$ and $\boldsymbol{a}_{B/A}$ are the relative velocity and acceleration of point *B* with respect to point *A*. For translating axes, the relative motions are related to absolute motions by $\boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{v}_{B/A}$ and $\boldsymbol{a}_B = \boldsymbol{a}_A + \boldsymbol{a}_{B/A}$, and the absolute velocity and acceleration of point *B* are determined from the following equations:

$$\boldsymbol{v}_B = \boldsymbol{v}_A + w \times \boldsymbol{r}_{B/A} \qquad (3)$$

$$\boldsymbol{a}_B = \boldsymbol{a}_A + a \times \boldsymbol{r}_{B/A} + w \times (w \times \boldsymbol{r}_{B/A}) \qquad (4)$$

If the vectors are defined as: $\boldsymbol{a}_A = [a_{Ax}, a_{Ay}, a_{Az}]^T$, $\boldsymbol{a}_B = [a_{Bx}, a_{By}, a_{Bz}]^T$, $a = [a_x, a_y, a_z]^T$, $w = [w_x, w_y, w_z]^T$, and $\boldsymbol{r}_{B/A} = [r_x, r_y, r_z]^T$, then equation 4 can be rewritten in the component form as:

$$\begin{bmatrix} a_{Bx} \\ a_{By} \\ a_{Bz} \end{bmatrix} = \begin{bmatrix} a_{Ax} \\ a_{Ay} \\ a_{Az} \end{bmatrix} + \begin{bmatrix} a_y r_z - a_z r_y \\ a_z r_x - a_x r_z \\ a_x r_y - a_y r_x \end{bmatrix} + \begin{bmatrix} w_x(w_y r_y + w_z r_z) - r_x(w_y^2 + w_z^2) \\ w_y(w_z r_z + w_x r_x) - r_y(w_z^2 + w_x^2) \\ w_z(w_x r_x + w_y r_y) - r_z(w_x^2 + w_y^2) \end{bmatrix} \qquad (5)$$

This general equation 5 is used for setting up equations in the optimization methodology.

Optimization methodology-General Approach

This new methodology uses an optimization technique rather than closed form methods to find the 3D angular motion of a rigid body from linear accelerations data.

The linear acceleration data can be obtained from any set of accelerometers placed at random locations in the rigid body and arranged in a plane as shown in Figure 4.

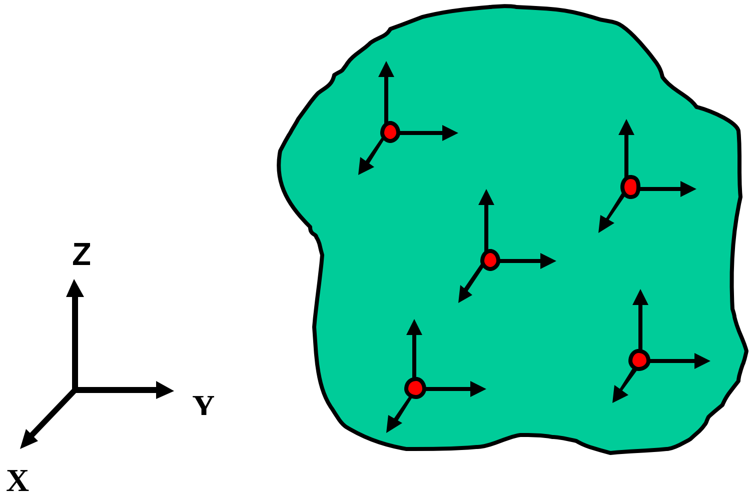

Figure 4: Rigid body with accelerometers at random locations.

Once the measured linear accelerations are obtained, equations are set up for these linear accelerations in terms of angular accelerations using the general equation 5. These equations can be set up along any path as long as the distance between the different accelerometer locations are known. One of the possibilities is shown in Figure 5.

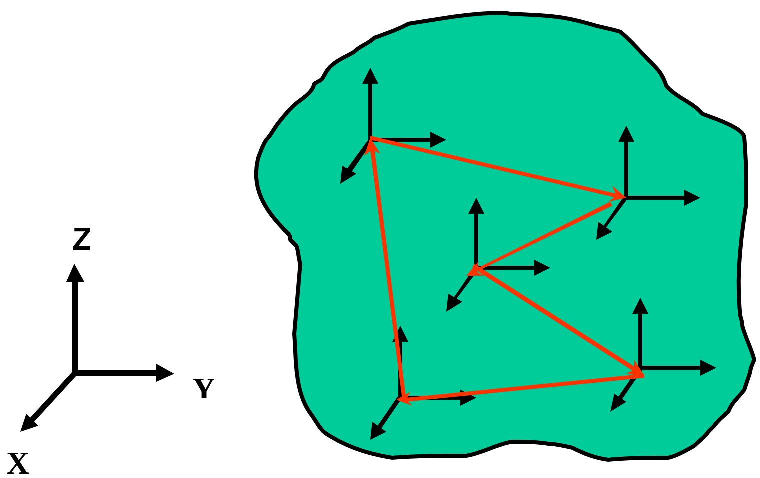

Figure 5: Shows a path that can be used for equation set up.

After the equations are set up and the measured linear accelerations are known, an optimization process is carried out using the angular accelerations (a_x, a_y, a_z) as design variables so as to minimize the cumulative error, given by equation 6, at each time step between the measured linear accelerations and the corresponding linear accelerations computed using the derived set of equations.

The objective function (error function) minimized at each time step is defined as:

$$\sqrt{\sum_{i=1}^{n}\left[(a_{ix}^{m}-a_{ix}^{c})^2+(a_{iy}^{m}-a_{iy}^{c})^2+(a_{iz}^{m}-a_{iz}^{c})^2\right]} \tag{6}$$

where,

n =number of points from which the acceleration data is obtained
m=measured data
c= computed data

This methodology helps determine 3D angular accelerations that best satisfy the linear acceleration time histories at all the given points. Since this is an optimization technique, the more the number of points the better is the estimate of the angular accelerations. This methodology has the advantage of being stable at high accelerations and also the solution is not affected much due to small inaccuracies in the measured data. Thus reliable 3D angular kinematics can be obtained using this methodology from in-plane accelerometer data (e.g. six accelerometer array, mouthpiece accelerometer) without stringent accuracy requirements.

Validation

For the purpose of validation, known angular kinematics obtained from the nine-accelerometer array data of one of the NCAP tests are used. The peak linear acceleration for this case is around 64 g's. Two in-plane accelerometer subsets (X-CG-Y) and (X-Y-Z) as shown in Figure 6 are used for the validation study.

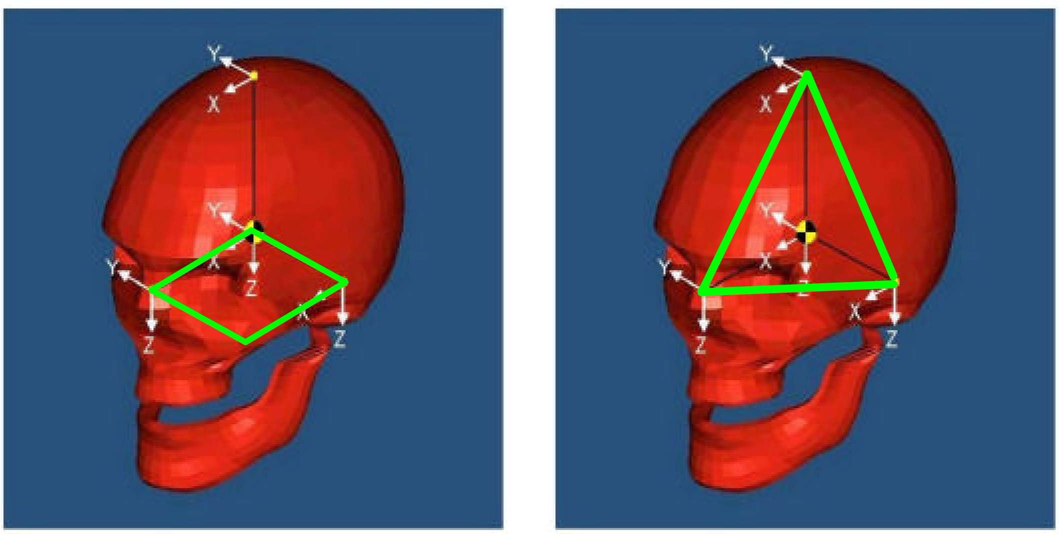

Figure 6: Rigid body Head Model with X-CG-Y and X-Y-Z In-plane subsets.

The linear acceleration data is obtained from these subsets using the rigid body head model, which is driven using the known angular kinematics. For these subsets, a set of equations is derived for the linear accelerations in terms of angular accelerations using equation 5. Equations 7 and 8 show the set up for the X-CG-Y subset and equations 9-11 show the set up for the X-Y-Z subset.

$$\begin{bmatrix} a_{1x} \\ a_{1y} \\ a_{1z} \end{bmatrix} = \begin{bmatrix} a_{0x} \\ a_{0y} \\ a_{0z} \end{bmatrix} + \begin{bmatrix} 0 \\ a_z r_{xz} \\ -a_y r_x \end{bmatrix} + \begin{bmatrix} -r_x\left(w_y^2+w_z^2\right) \\ w_y\left(w_x r_x\right) \\ w_z\left(w_x r_x\right) \end{bmatrix} \tag{7}$$

$$\begin{bmatrix} a_{2x} \\ a_{2y} \\ a_{2z} \end{bmatrix} = \begin{bmatrix} a_{0x} \\ a_{0y} \\ a_{0z} \end{bmatrix} + \begin{bmatrix} -a_z r_y \\ 0 \\ a_x r_y \end{bmatrix} + \begin{bmatrix} w_x\left(w_y r_y\right) \\ -r_y\left(w_z^2+w_x^2\right) \\ w_z\left(w_y r_y\right) \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} a_{1x} \\ a_{1y} \\ a_{1z} \end{bmatrix} = \begin{bmatrix} a_{2x} \\ a_{2y} \\ a_{2z} \end{bmatrix} + \begin{bmatrix} -\alpha_z r_y \\ \alpha_z r_x \\ \alpha_x r_y - \alpha_y r_x \end{bmatrix} + \begin{bmatrix} w_x (w_y r_y) - r_x (w_y^2 + w_z^2) \\ w_y (w_x r_x) - r_y (w_z^2 + w_x^2) \\ w_z (w_x r_x + w_y r_y) \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} a_{2x} \\ a_{2y} \\ a_{2z} \end{bmatrix} = \begin{bmatrix} a_{1x} \\ a_{1y} \\ a_{1z} \end{bmatrix} + \begin{bmatrix} -\alpha_z r_y \\ \alpha_z r_x \\ \alpha_x r_y - \alpha_y r_x \end{bmatrix} + \begin{bmatrix} w_x (w_y r_y) - r_x (w_y^2 + w_z^2) \\ w_y (w_x r_x) - r_y (w_z^2 + w_x^2) \\ w_z (w_x r_x + w_y r_y) \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} a_{3x} \\ a_{3y} \\ a_{3z} \end{bmatrix} = \begin{bmatrix} a_{1x} \\ a_{1y} \\ a_{1z} \end{bmatrix} + \begin{bmatrix} \alpha_y r_z \\ \alpha_z r_x - \alpha_x r_z \\ -\alpha_y r_x \end{bmatrix} + \begin{bmatrix} w_x (w_z r_z) - r_x (w_y^2 + w_z^2) \\ w_y (w_z r_z + w_x r_x) \\ w_z (w_x r_x) - r_z (w_x^2 + w_y^2) \end{bmatrix} \tag{11}$$

For the X-CG-Y subset, the Z-arm acceleration data is ignored and for the X-Y-Z subset the CG acceleration data is ignored. After the equations are set up, the optimization process is carried to find the 3D angular accelerations. The results are shown in Figure 7 and Figure 8 and it can be seen that a very good match is obtained between the original (known) and the optimized angular accelerations for both in-plane subsets.

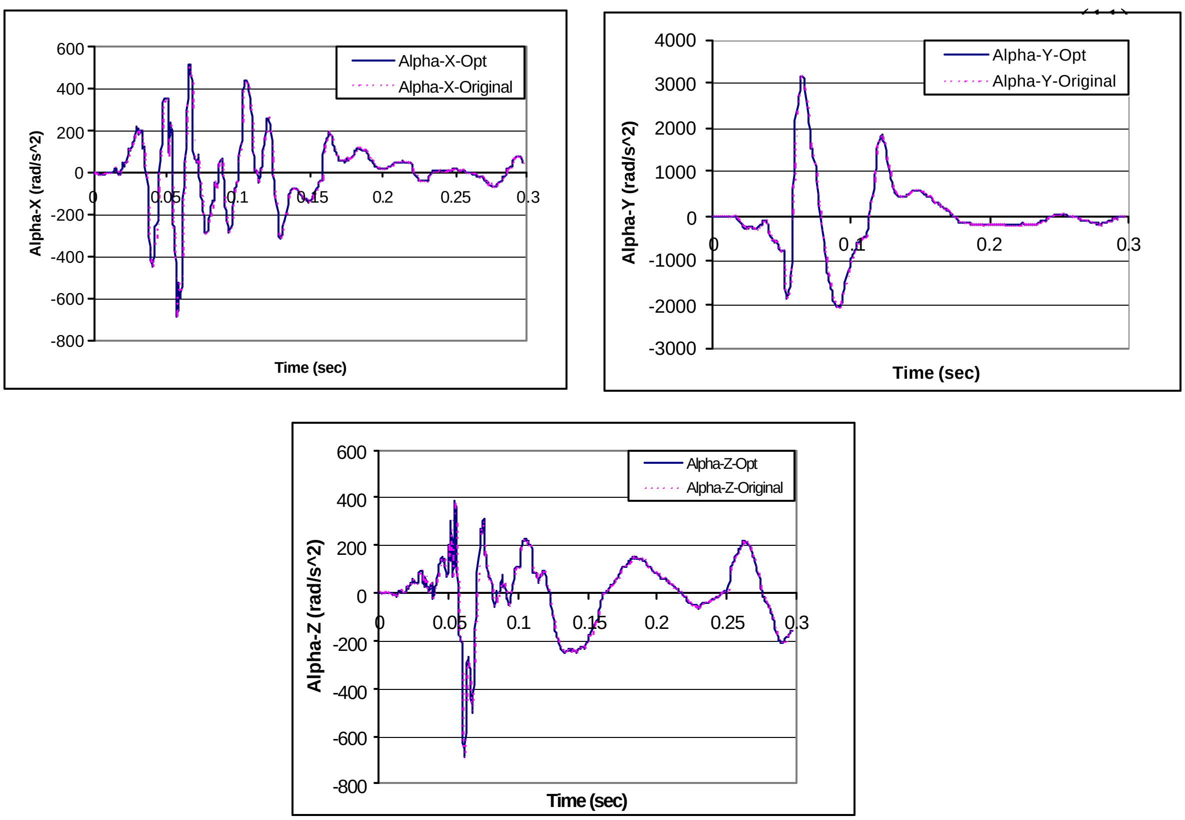

Figure 7: Plots showing the comparison between the original and the optimized angular accelerations with the X-CG-Y subset.

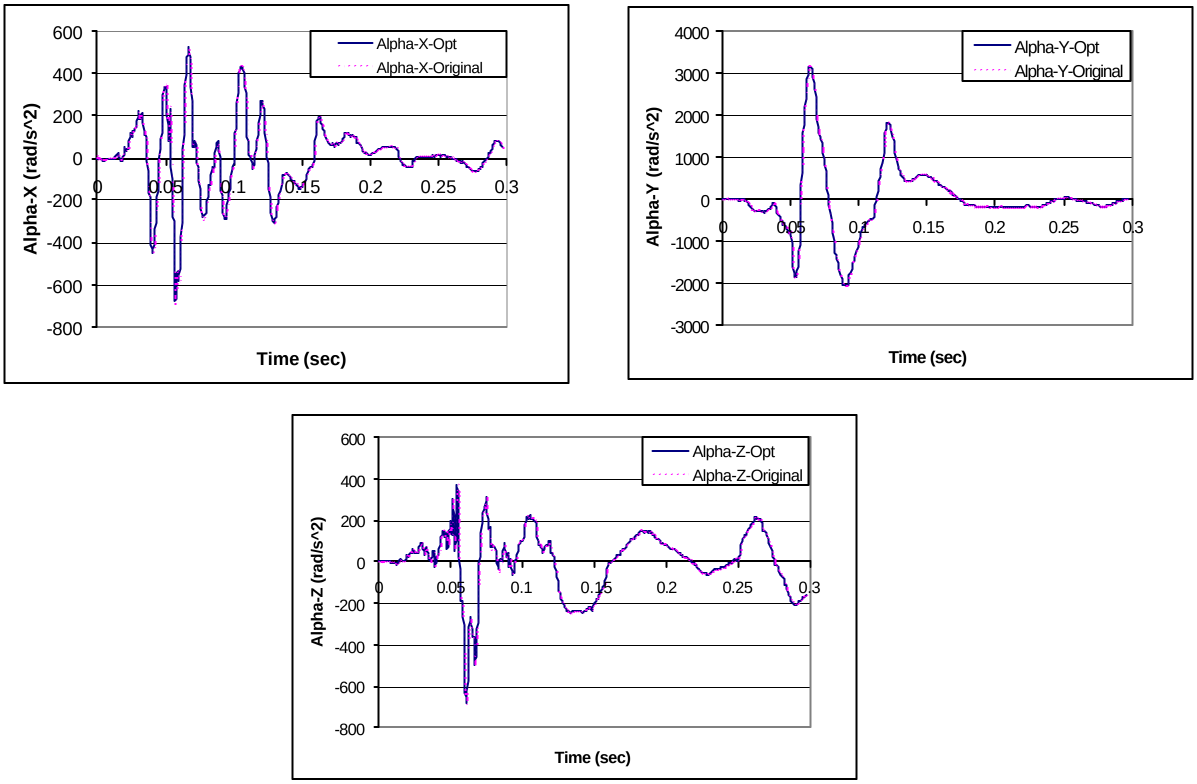

Figure 8: Plots showing the comparison between the original and the optimized angular accelerations with the X-Y-Z subset.

It is known that the six-accelerometer array becomes unstable in the presence of small inaccuracies in the measured data when closed form methods are used for finding the solution. In order to check how the errors would affect the solution, the optimization methodology is applied to a data set where random errors ($\pm 2\%$) are introduced in the non-CG acceleration traces according to equation 12. The X-CG-Y subset is used for this study.

$$
\begin{aligned}
& a(t)_{new} = (1 - Error)a(t)_{org} \\
& -0.02 \leq Error \leq 0.02
\end{aligned}
\tag{12}
$$

Figure 9 shows the variation of error with time. The peak linear acceleration with error is 64.42 g's, which is slightly higher than the original data without error (64.02 g's).

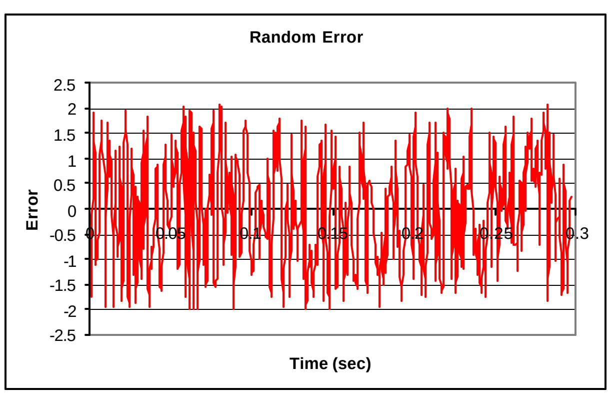

Figure 9: Plot showing the error variation with time.

The optimization is carried out with this erroneous data set to get the 3D angular accelerations. The results are shown in Figure 10. It is seen that even with errors introduced in the measured data and at high accelerations, a very good comparison is obtained between the original (known) and the optimized angular accelerations and the solution does not become unstable.

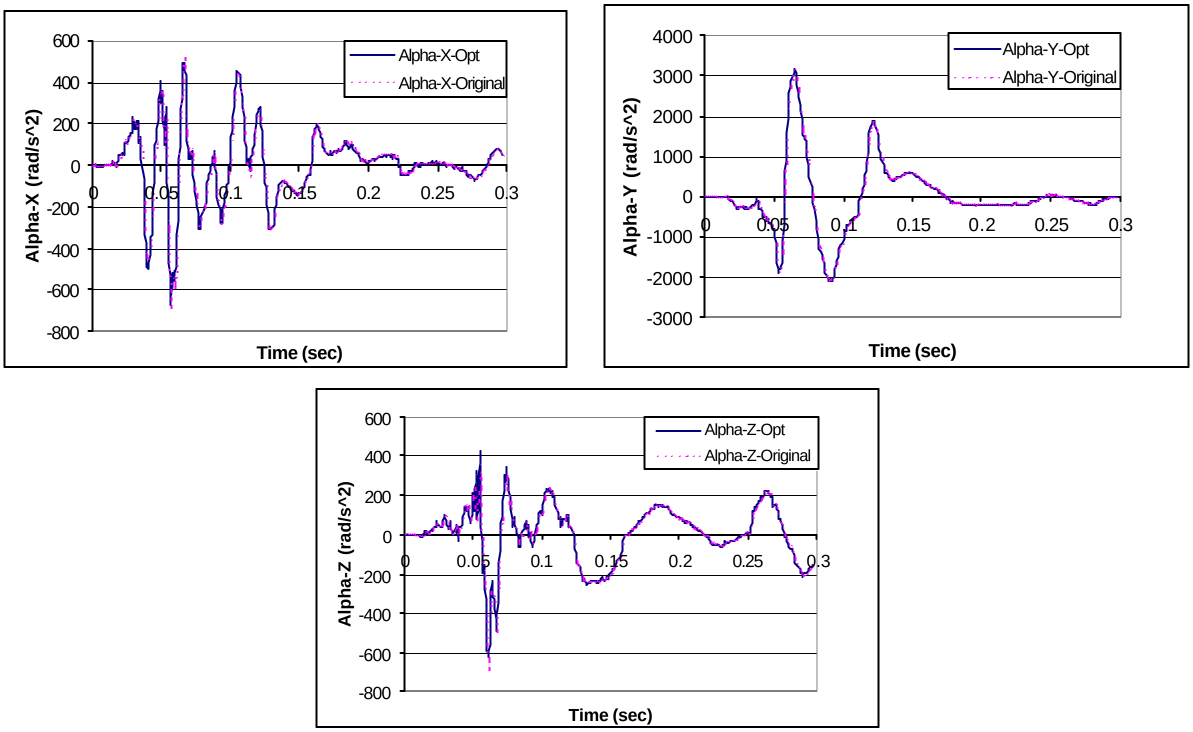

Figure 10: Plots showing the comparison between the original and the optimized angular accelerations with erroneous data.

Application to Mouthpiece Accelerometer

A mouthpiece accelerometer as shown in Figure 11 is set up in the rigid body head model. A constraint of 1.25 inches on the length of the side of the triangular plate is maintained to make sure that the size of the plate is small and can easily be fitted in small confined spaces.

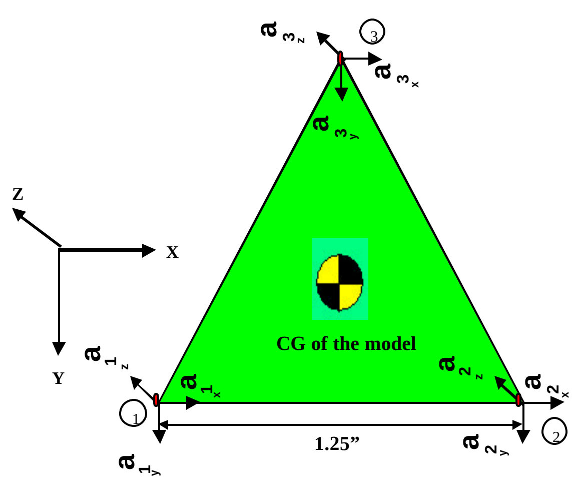

Figure 11: Set up of Mouthpiece Accelerometer.

This rigid body head model is then driven using the known angular kinematics and the linear acceleration data is obtained in the three orthogonal directions (Z-direction is out-of-plane of the paper) from points 1, 2 and 3. Equations are set up for the mouthpiece accelerometer and are given by equations 13-15.

$$\begin{bmatrix} a_{1x} \\ a_{1y} \\ a_{1z} \end{bmatrix} = \begin{bmatrix} a_{2x} \\ a_{2y} \\ a_{2z} \end{bmatrix} + \begin{bmatrix} 0 \\ a_z r_x \\ -a_y r_x \end{bmatrix} + \begin{bmatrix} -r_x\left(w_y^2 + w_z^2\right) \\ w_y\left(w_x r_x\right) \\ w_z\left(w_x r_x\right) \end{bmatrix} \tag{13}$$

$$\begin{bmatrix} a_{2x} \\ a_{2y} \\ a_{2z} \end{bmatrix} = \begin{bmatrix} a_{1x} \\ a_{1y} \\ a_{1z} \end{bmatrix} + \begin{bmatrix} 0 \\ a_z r_x \\ -a_y r_x \end{bmatrix} + \begin{bmatrix} -r_x\left(w_y^2 + w_z^2\right) \\ w_y\left(w_x r_x\right) \\ w_z\left(w_x r_x\right) \end{bmatrix} \tag{14}$$

$$\begin{bmatrix} a_{3x} \\ a_{3y} \\ a_{3z} \end{bmatrix} = \begin{bmatrix} a_{1x} \\ a_{1y} \\ a_{1z} \end{bmatrix} + \begin{bmatrix} -a_z r_y \\ a_z r_x \\ a_x r_y - a_y r_x \end{bmatrix} + \begin{bmatrix} w_x\left(w_y r_y\right) - r_x\left(w_y^2 + w_z^2\right) \\ w_y\left(w_x r_x\right) - r_y\left(w_z^2 + w_x^2\right) \\ w_z\left(w_x r_x + w_y r_y\right) \end{bmatrix} \tag{15}$$

The optimization methodology is applied to the data obtained from the mouthpiece accelerometer and the angular accelerations are obtained. A good match is obtained between the original (known) and the optimized angular accelerations as shown in Figure 12.

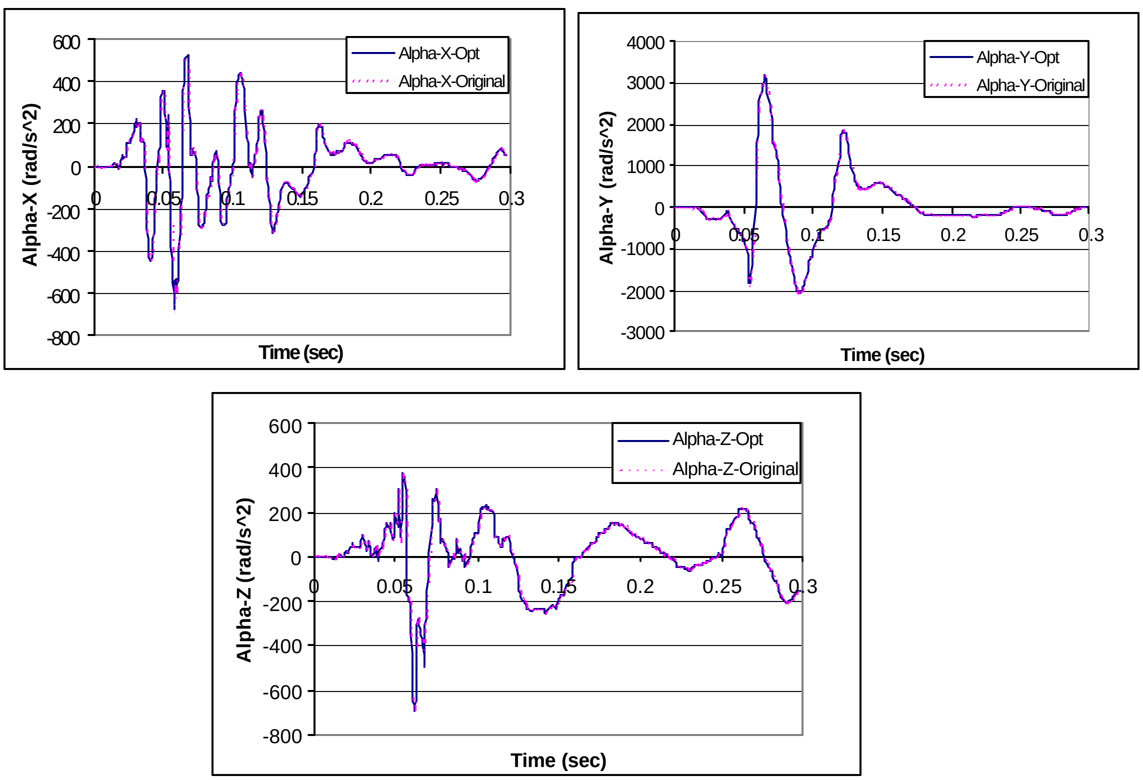

Figure 12: Plots showing the comparison between the original and the optimized angular accelerations for mouthpiece accelerometer.

In order to investigate the effect of cross-axis sensitivity on the optimized angular accelerations, a 3% error is introduced in each linear acceleration trace obtained from the mouthpiece accelerometer according to equation 16:

$$a_{ix} = a_{ix} + 0.03a_{iy} + 0.03a_{iz}$$
$$a_{iy} = a_{iy} + 0.03a_{ix} + 0.03a_{iz} \qquad i = 1 \text{ to } 3 \qquad (16)$$
$$a_{iz} = a_{iz} + 0.03a_{ix} + 0.03a_{iy}$$

This takes into account the cross-axis error from the other two directions at that instant of time. With this data and the equation set up given by equations 13-15, the optimization is carried out to find the 3D angular kinematics. Figure 13 shows the results. Though there are some differences between the optimized and the original Alpha-Z trace, the overall match is good for Alpha-X, Alpha-Y, and Alpha-Z, and the solution does not become unstable with 3% cross-axis error.

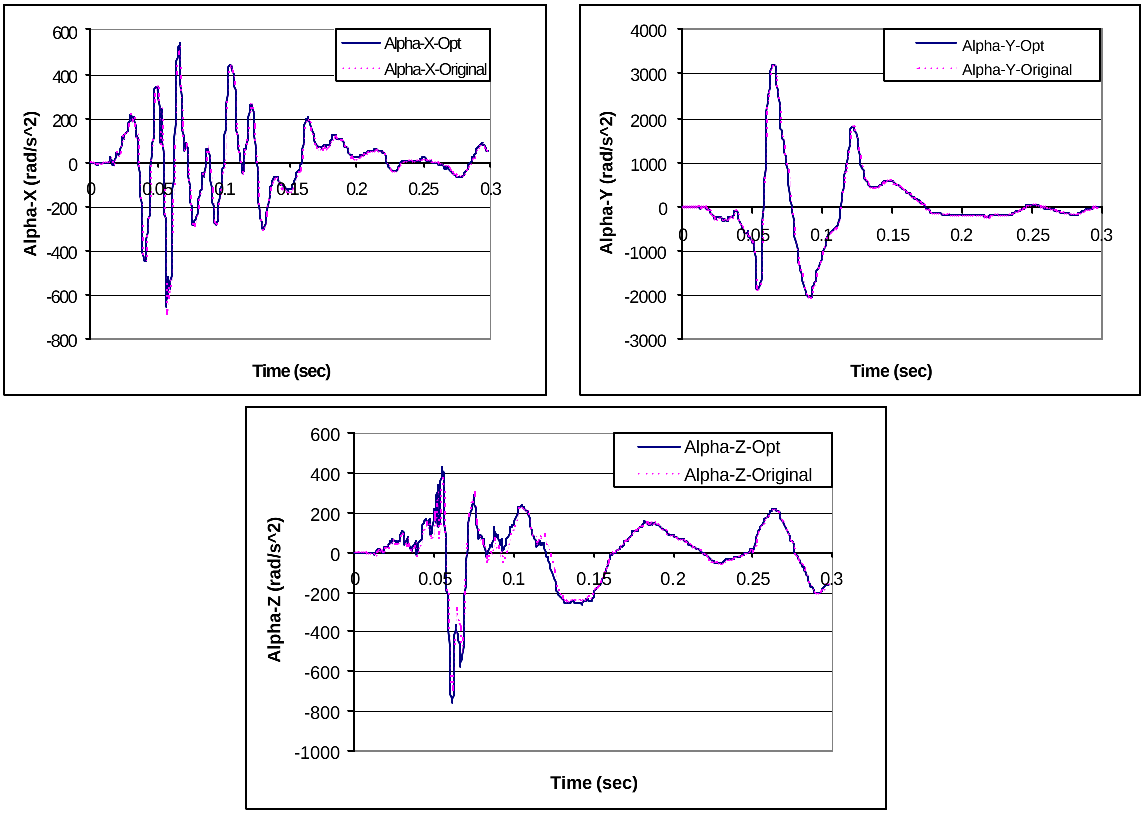

Figure 13: Plots showing the comparison between the original and the optimized angular accelerations for mouthpiece accelerometer with 3% cross-axis sensitivity.

Further studies are carried out with the mouthpiece accelerometer data to study how the results improve due to additional channels. For this study, the 3% cross axis error data is used. Optimization is carried out using 6 and all the 9 channels of the mouthpiece accelerometer. The six and the nine channels used for optimization are shown in Figure 14.

It is important to note that the six channels which are used for optimization should be chosen in such a way that when the equations are formed all three angular accelerations are used in the equation set up. For the equation set up given by equations 13-15 for the mouthpiece accelerometer, it is imperative to choose in all three points out-of-plane acceleration orientations to properly calculate three angular accelerations, and the other three acceleration orientations can be chosen arbitrarily which in this case are the y-accelerations.

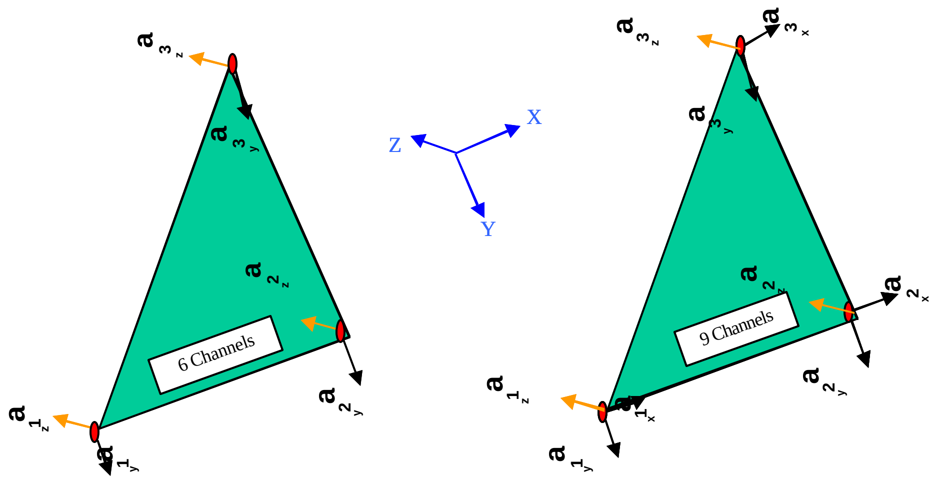

Figure 14: Six and Nine channels used for optimization.

The results with six and nine channel optimization are shown in Figure 15 and Figure 16. Table 1 shows the accuracy improvement between 6 and 9 channel optimization by comparing the cumulative error in Alpha-X, Alpha-Y and Alpha-Z. The improvement in the accuracy is observed because the erroneous data is used for optimization.

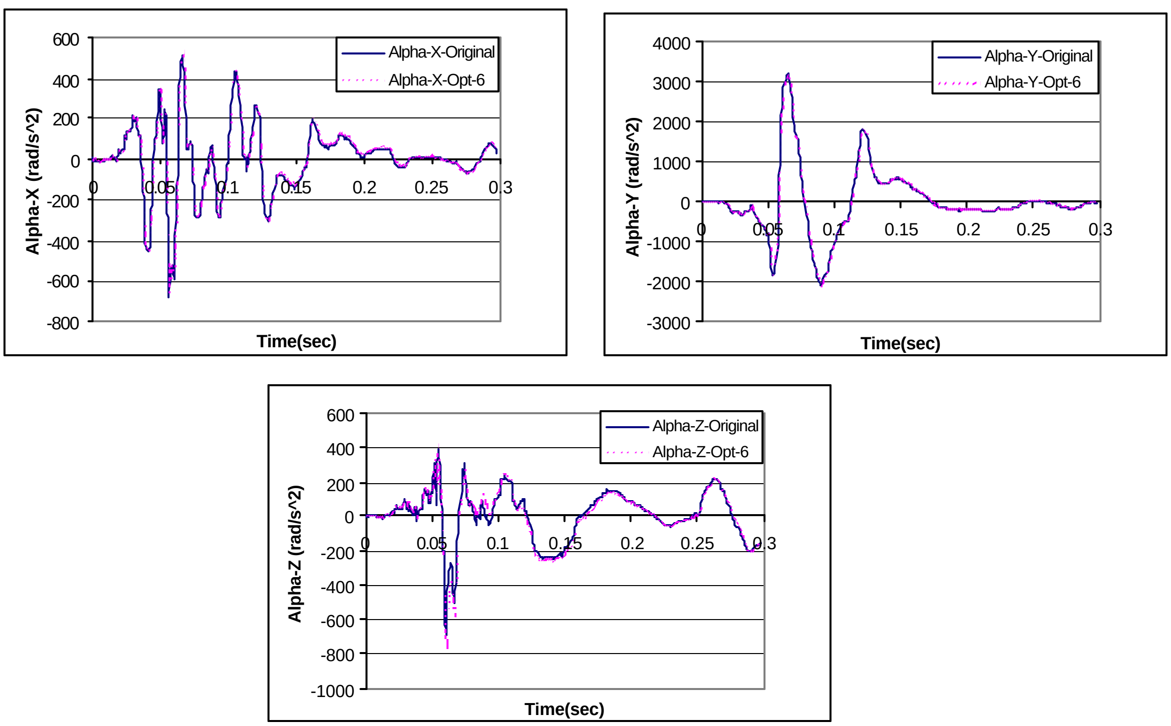

Figure 15: Plots showing the comparison between the original and the optimized angular accelerations with 6-channel optimization.

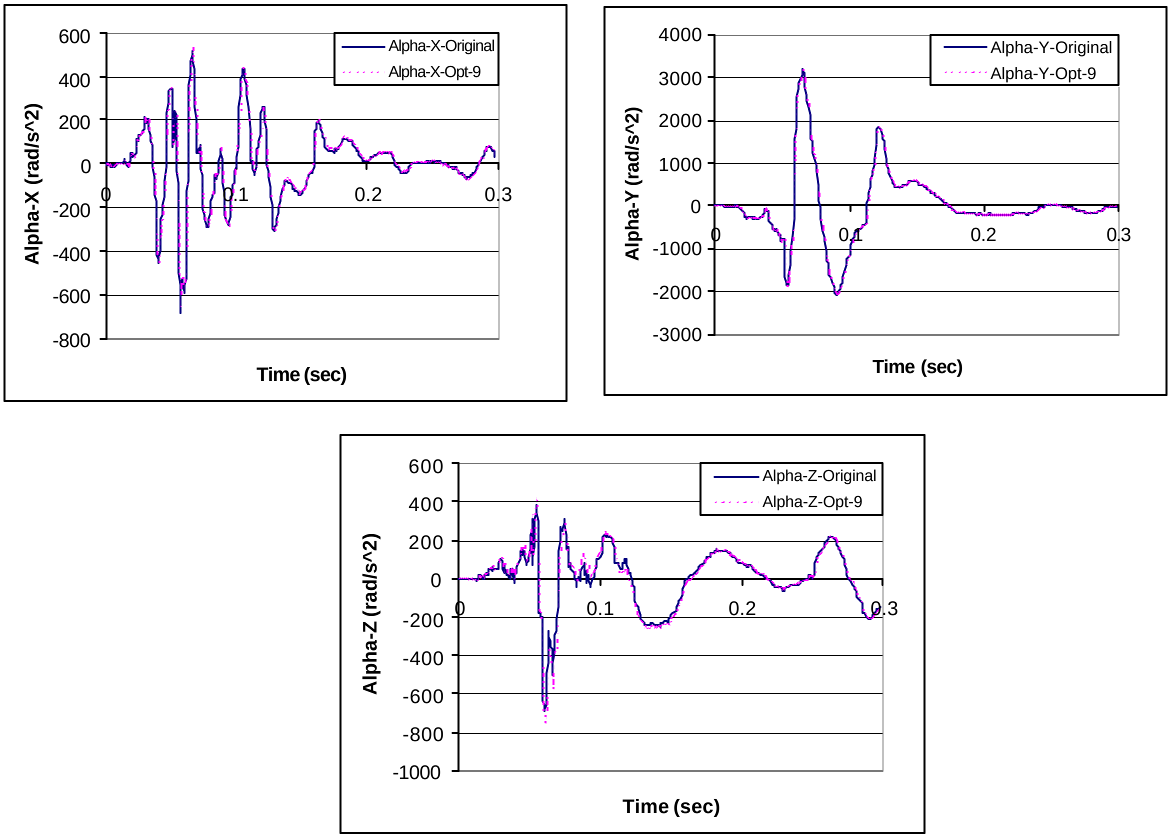

Figure 16: Plots showing the comparison between the original and the optimized angular accelerations with 9-channel optimization.

Table 1. Accuracy comparison between 6 and 9-channel optimization.

	Cumulative Error		
Number of Channels	Alpha-X	Alpha-Y	Alpha-Z
6	124.138	102.88	444.816
9	100.67	95.549	407.237

CONCLUSIONS

A new optimization methodology for determining 3D angular kinematics of a rigid body from the linear acceleration data has been developed and validated. This methodology has been shown to be stable even at high accelerations and also in the presence of small inaccuracies in the measured data. As a result, this technique can be used with the six-accelerometer array data without incurring the risk of instability. Since this methodology can be used with the six-accelerometer array system and six independent measures are sufficient to completely define the 3D kinematics of a rigid body, the over-constrained nine-accelerometer array system can be avoided.

In-plane accelerometers (e.g. mouthpiece accelerometers) can be easily fitted in small confined spaces as compared to a nine-accelerometer array package, which poses attachment difficulties in small spaces. Since this methodology has been shown to give good results with in-plane accelerometer data, it can be used for determining 3D angular accelerations from mouthpiece accelerometers to study 3D head kinematics of football players, boxers, hockey players, small dummy heads, etc.

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PADGAONKAR, A.J., KRIEGER, K.W., AND KING, A.I. (1975). Measurement of angular acceleration of a rigid body using linear accelerometers. Journal of Applied Mechanics, pp. 552-556.

TAKHOUNTS, E., EPPINGER, R., TANNOUS, R., CAMPBELL, J.Q., POWER, E., SHOOK, L., HASIJA, V. (2003). Analysis of 3D Rigid Body Motion using the Nine Accelerometer Array System. Injury Biomechanics Research, Proceedings of the Thirty-First International Workshop, pp. 59-75.

DISCUSSION

PAPER: **Finding 3D Angular Accelerations of a Rigid Body from In-Plane Accelerometers Using Optimization**

PRESENTER: ***Vikas Hasija, GESAC Inc. and National Highway Traffic Safety Administration***

QUESTION: *Guy Nusholtz, DaimlerChrysler*

I'll do the first part of that. This is more a comment than a question. There's a couple of other errors that you want to include: miscalibration of the accelerometers, and you also want to include Brownian type of error.

ANSWER: Brownian?

Q: You got random, but Brownian—In other words, accelerometers drift over time and you have a change in the dc value. The way it's typically modeled is by Brownian. Okay? If you don't understand it, I'll explain it to you after.

A: Okay.

Q: And the next, the actual question is: Did you try and do a comparison between this process and some of the other processes that are already available to see whether it's an improvement, such as Wayne State's method or SGA?

A: Oh no. We didn't do any comparisons. I showed, like, earlier work was done by Padgaonkar, which showed that 6 accelerometer arrays become unstable at high accelerations and there are some inaccuracies. I assure that this method is stable and it works at high acceleration, as well as in the presence of inaccuracies in the measured data.

Q: But things like the Wayne State 9 accelerometers: They're just versions of that, which you could easily compare it to. And, the other thing, which you might want to show, is what happens when you have 12 accelerometers versus, say, 9 accelerometers and then compare it to the current 9 accelerometers.

A: Okay.

Q: Information. One more just small comment is: You have some additional information inside of your response that you could use, probably, to improve the result. Okay. Thank you.

A: Thanks.