# EVALUATION OF THE SAFETY BENEFITS OF PASSIVE AND/OR ON-BOARD ACTIVE SAFETY APPLICATIONS WITH MASS ACCIDENT DATA-BASES

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## ABSTRACT

One of the main objectives of the European TRACE project (Traffic Accident Causation in Europe, January 2006 – June 2008) was the development of methodology for the evaluation of the safety benefit of existing on-board safety applications in passenger cars with the use of mass accident data-bases only.

The challenge was to evaluate passive safety applications as well as active applications and especially combinations of the two within a single investigation. In order to do so the well known concept of odds-ratio has been generalized for jointly evaluating injury mitigating effectiveness as well as accident avoiding effectiveness at once.

This paper describes statistical sound methodology that is able to evaluate the safety benefit of either a single on-board safety function or the additional gain of specific safety feature(s) (i.e. a selection of various passive safety functions and active safety functions), given that some other safety applications already are on board. In particular, the method allows for evaluation of accident avoiding effectiveness as well as injury mitigating effectiveness. Hence, it can be applied for joint evaluations of passive and on-board active safety applications.

The focus of the paper lies on the presentation of a ready-to-apply methodology, including detailed examples as well as a discussion on its advantages and its limitations.

## EFFECTIVENESS OF SINGLE SAFETY FUNCTIONS

For measuring the effectiveness of a safety function it is of critical importance to distinguish between different possible types of effects. In general there are at least four different types of safety function effects. These are:

- accident avoiding effectiveness
- injury avoiding effectiveness
- injury mitigating effectiveness
- effects of tertiary safety functions

Some safety functions aim at avoiding the accident altogether. If this is not possible, measures to prevent the involved persons from suffering injuries are taken. If this cannot be achieved either, the injury outcome for the passengers is minimized as far as possible. Afterwards, the aim is to reduce the consequences of already inflicted injuries to the largest extend possible (e.g. by automatically placing an emergency call).

A typical primary safety function aims at all of the first three types of effectiveness, whereas the effectiveness of a typical secondary safety function only consists of the injury avoiding and injury mitigating effectiveness. Tertiary safety functions aim at reducing the consequences of injuries. This paper focuses on primary and secondary safety functions and does not deal with tertiary safety functions at all.

In some sense the first three mentioned types of safety function effects are ordered hierarchically. A safety function which aims at accident avoiding typically has some measurable effect on injury avoiding and injury mitigating in cases in which the accident can not be avoided but the crash's severity can be reduced. A secondary safety function aiming at injury avoiding typically also has some effectiveness towards injury mitigating but not towards accident avoiding. Thus, a combined evaluation of different safety functions must include injury avoiding and mitigating effectiveness as well as the accident avoiding effectiveness.

However, this paper will first focus on the accident avoiding effectiveness and deal with the other types of effectiveness later.

**Relative risk – odds-ratios**

A reasonable way of measuring the effectiveness of a single safety function "SF" within a certain group of accidental situations "*A*" is to compute relative risks. For example, a relative risk easy to interpret is the ratio of the probability that a vehicle with a SF on board and active has to suffer an accident that belongs to a predetermined category *A* of accidents, and the probability of suffering an accident belonging to *A* with SF not active (cf. equation (1)).

$$\mathrm{RR} = \frac{P(\text{suffering } A \mid \text{SF active})}{P(\text{suffering } A \mid \text{SF not active})} \qquad (1)$$

This relative risk is independent of the population of interest if it is the same for both probabilities. For example, when interested in the population of all vehicles on the road within one specific year, the probabilities have to be interpreted as the "probability of suffering an accident of type *A* within the year of interest, given that SF is (not) active".

As the relative risk is the ratio of two probabilities it can take any value in the interval [0, ∞). If it equals one, the probability of suffering an accident of type *A* is independent of the safety function SF being active or not. If it is larger than one, the effectiveness of SF is negative, i.e. the safety function increases the probability of suffering an accident of category *A* when driving on a road. If the relative risk is less than one, the safety function has some positive effect, i.e. the safety function decreases the probability of suffering an accident of category *A* when driving on a road.

With simple algebra and Bayes' theorem for conditional probabilities the equivalence between this relative risk and the following odds-ratio can be shown (cf. equation (2))

$$\mathrm{RR} = \mathrm{OR} := \frac{\dfrac{P(\text{SF active} \mid A)}{P(\text{SF not active} \mid A)}}{\dfrac{P(\text{SF active} \mid N)}{P(\text{SF not active} \mid N)}} \qquad (2)$$

where *N* stands for a category of neutral accidental situations or for an internal control group of vehicle-related accidental situations. See [15] if interested in the derivation of this result. It is crucial that the relative risk of suffering an accident classified as *N* depending on SF active (P(suffering *N* | SF active)) and not (P(suffering *N* | SF not active)) respectively, must be equal or very close to one. This means that SF more or less has **no** influence on the probability of suffering an accident of neutral type *N*. For more detailed information on odds-ratios see [4], [6], [7] and [8]. It is crucial for any analysis using odds-ratios to have a reliable classification of neutral accidents *N* as the results are very sensitive to this classification!

It is important to point out the difference between accidents and vehicle-related accidental situations. There may be several vehicles involved in a single accident and the different drivers were most likely confronted with more or less different situations that led to the accident. Hence, safety functions on board of vehicles involved in one and the same accident may very well be confronted with different situations. Therefore, the effectiveness of a safety function in a specific accident highly depends on which of the involved vehicles is considered for the evaluation.

Thus, when referring to a certain type of accident, vehicle-related classification of accidents will always be in consideration.

For computing the term in equation (2) the two odds have to be estimated with the equipment-rates within the accident type of interest as shown in equation (3).

$$\frac{P(\text{SF active} \mid A)}{P(\text{SF not active} \mid A)} \approx \frac{\text{No. of cars with SF active within } A}{\text{No. of cars with SF not active within } A} \qquad (3)$$

Of course, this estimator only is adequate if the numbers of these accident counts are reasonably high. The section "Confidence intervals" deals with the accuracy of the estimated results.

With this transformation a term is derived that can easily be computed and is equivalent to the relative risk which can be interpreted, so that the effectiveness of SF within *A* can be computed as shown in equation (4).

$$eff(A) = 1 - \mathrm{OR} = 1 - \frac{\dfrac{\text{No. of cars w. SF active in } A}{\text{No. of cars w. SF not active in } A}}{\dfrac{\text{No. of cars w. SF active in } N}{\text{No. of cars w. SF not active in } N}} \qquad (4)$$

The effectiveness then describes the percentage of avoided accidents within the category *A*. To describe it more precisely:

Assume that each vehicle out of a certain fleet of vehicles is involved in a specific critical accidental situation (base unit of exposure) that in case the SF is not active would lead to accidents of type *A*. Now assume that exactly the same number of similar vehicles out of a similar fleet with the same

drivers and the exact same surrounding conditions is being confronted with the same critical situation, but this time every vehicle out of this second fleet is to be equipped with SF. Hence, this thought experiment resembles a perfect case-control-study, where each critical situation is observed twice – one time with the safety function on board, and another time without the same. Each pair represents a matched pair: Case and control. As for each pair all surrounding conditions are exactly the same except for the safety function of interest, these pairs will be referred to as a "perfect matching" in the following. Assuming that SF has some accident avoiding effectiveness, *eff*·100% of the critical situations should not have led to an accident.

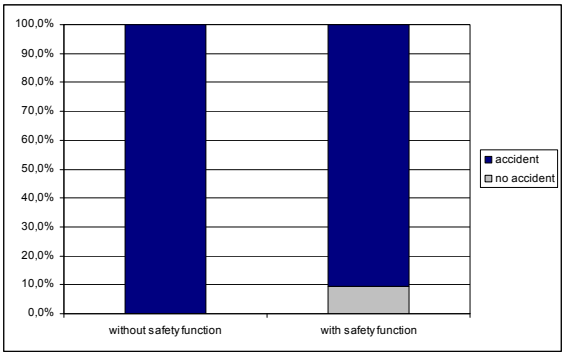


**Figure 1: Example for the interpretation of the accident avoiding effectiveness of a safety function**

In this thought experiment the accident avoiding effectiveness is about 10%, as roughly 10% less accidents within the group of equipped vehicles were observed compared to the group of non-equipped vehicles. Since the one and only difference between the two fleets is supposed to be the safety function, the effect may be postulated to be caused by the safety function's effectiveness.

Obviously, there will never be a chance of observing such an ideal situation of a perfectly matched case-control-study in the field of accident research. However, by explaining another possible way of interpreting odds-ratios, it should become clear how this problem is circumvented.

Odds-ratios compare the relative frequency of the equipment-rate within accident type *A* to the equipment-rate within type *N*. If the safety function of interest has no effect on the occurrences of accidents of type *A*, the equipment-rate within *A* should be equal to the rate within *N*. Contrary, if it has some positive effect on accident type *A*, then some accidents must have been avoided due to the safety function. Hence, these do not appear in the database. With odds-ratios it is possible to calculate the number of accidents avoided this way.

The effectiveness of a safety function is calculated by using only four different accident counts as shown in equation (4). These are the numbers of vehicles involved in accidents of type *N* or *A*, either equipped with SF or not equipped with SF. As *N* stands for a type of accidental situation not influenced by SF (neutral accidents), only one of these four counts is influenced by SF: The count of vehicles equipped with SF involved in accidents of type *A*. Therefore, any accident avoided due to the safety function has to be out of the group of SF-equipped vehicles in *A*. Hence, any change in the calculated odds-ratio may be traced back to the one group of interest. Due to that, it is possible to calculate the percentage of avoided accidents as well as the absolute number of avoided accidents within this single group of interest. This is done by looking at two different ratios only. Particularly section "evaluating injury mitigating and injury avoiding effectiveness" will make use of this way of interpreting odds-ratios.

Typically, it is not possible to identify the exact cause for these "missing" accidents – whether it is solemnly the influence of the SF or possibly due to external variables such as vehicle age, driver's experience and so on. As newer vehicles are more likely to be equipped with more safety functions than older ones, the variable vehicle-age is likely to have a confounding influence. The paper will come back to the topic of confounding variables in the section "the influence of additional external factors and logistic regression". Hence, no causal relationship between the safety function and the effectiveness can be drawn so far. For the sake of simplicity it will still be referred to as the "effectiveness of the safety function" instead of "effectiveness of the behavior of vehicles equipped with the safety function", which would be more appropriate. According to [12] all calculations made without taking external factors into account shall be referred to as "crude" calculations in this paper, e.g. crude odds-ratios and crude effectiveness.

Furthermore, it is important to point out that effectiveness always refers to accident counts instead of accidents in general. Therefore, an effectiveness of 20% for some safety function SF and some accident type *A* must be interpreted in the following way:

20% of the cases that should have been observed in group *A* are not listed in the database at hand. Thus, the probability of a vehicle equipped with SF to suffer an accident of type *A* **and to have that accident actually reported** in the database at hand is only 80% of the probability for a vehicle without SF. This plays an important role in the interpretation of the results especially when looking at injury-accident databases. Furthermore, when the probability of an accident being reported is varying among different types of accidents, the computed

effectiveness will typically be biased due to that variance.

**Overall effectiveness and misclassifications**

Besides the effectiveness for a certain group of accidental situations one might also be interested in the overall effectiveness for all accidents. There are two possible approaches to quantify the overall effectiveness of a safety function, either by extrapolation or by direct approach.

For the direct approach, *A* has to be chosen as the category of "all accidents within the database", which leads to such an overall effectiveness. This approach has the advantage that additional effects of the safety function on other than the selected sensitive accidents are taken into account as well. On the other hand more unwanted external confounding variables could be included in the overall effectiveness calculation. For example, if drivers of vehicles equipped with ESC are more likely to have a parking assistant on board as well, then the calculated overall effectiveness of ESC would include some effectiveness on parking accidents due to the correlation between ESC and parking assistants. The main disadvantage is the fact, that the category *N* of neutral accidents for this proposal will be a subset of *A*. This does not lead to any problems within the calculation itself, but calculating confidence intervals in the way described below becomes impossible.

For the other approach the effectiveness within the subgroup of accidental situations which are sensitive to the safety function has to be calculated and extrapolated to the complete set of accidents. Before discussing this approach any further, some discussion on the effects of misclassifications of accidents in a real world accident-database is in order.

There are two different possibilities for misclassifications in every single count out of the four accident counts necessary for the calculation of odds-ratios. The equipment of the vehicle of interest may be falsely classified as well as the classification of the type of accident may not be correct. It can be shown that if the system has a positive effectiveness, independent of the type of misclassification, the outcome will be an underestimation of the real effectiveness (cf. [7]). At this point the interpretation of the effectiveness is crucial. It is common to compare all accidents that are considered to be sensitive (instead of some specific accident type *A*) to the safety function to a neutral group [9]. Therefore a misclassification leads to an underestimation of the effectiveness of all sensitive cases. Typically, in analyses with accident data there will be at least some accidental situations that are not easily classifiable to be sensitive or neutral to the safety function. In many cases there even is a large group of accidents that is known to be a mixture of sensitive and neutral cases. Often it is impossible to split such a group into sensitive and neutral cases with the amount of information available. Hence, there are three possibilities to deal with such a group: Either excluding the entire group from the analysis, or including this group in the analysis and considering all these cases to be either of sensitive or neutral type. When including this group in the analysis, many of the cases will be misclassified, which results in an underestimated effectiveness for all sensitive accidents. The most convenient way to deal with such a blend of sensitive and neutral cases is to exclude them from the analysis ([7], [9]). By excluding such cases from the analysis the accuracy of the estimator for the effectiveness is higher but this estimator refers to a smaller group of sensitive cases.

Returning to the second approach, calculating the overall effectiveness via extrapolation, it is no longer of interest to have an accurate estimator for the effectiveness for sensitive accidents rather for sensitive accidents but for all accidents. The extrapolation is done as shown in equation (5).

$$eff_{overall}(\mathrm{SF}) = eff_A(\mathrm{SF}) \cdot \frac{n_0^A + \dfrac{n_1^A}{1 - eff_A(\mathrm{SF})}}{n_0^A + \dfrac{n_1^A}{1 - eff_A(\mathrm{SF})} + n_0^N + n_1^N} \qquad (5)$$

The abbreviations used in equation (5) are explained in equation (6):

$$\begin{aligned} n_1^A &:= \text{No. of accidents of type } A \text{ of cars equipped with SF} \\ n_0^A &:= \text{No. of accidents of type } A \text{ of cars not equip. with SF} \\ n_1^N &:= \text{No. of accidents of type } N \text{ of cars equip. with SF} \\ n_0^N &:= \text{No. of accidents of type } N \text{ of cars not equip. with SF} \end{aligned} \qquad (6)$$

See [15] for the derivation of this formula. This way of calculating the overall effectiveness only makes sense if definitely all accidents that are somehow sensitive to SF are included in *A*. If *A* contains some neutral accidents, then $eff_A$(SF) will be reduced accordingly. However, this leads to a larger group *A* and the resulting overall effectiveness does **not** differ from the one calculated by using a perfectly dichotomous classification of neutral and sensitive accidents at all! Therefore, a misclassification in the sense of neutral accidents being classified as sensitive does not change the calculated overall effectiveness! (cf. [15])

Hence, it is of no consequences for the calculation to include some neutral accidents into the group *A*, whereas the other types of misclassification still lead to an underestimation of the effectiveness. Therefore, if interested in an overall effectiveness it

is strongly recommended to include only these cases where the categorization is done with a sufficient precision into the neutral category. Whereas all cases without such a sufficient certainty of a correct classification should be included in group *A*.

On the basis of these findings, the correct classification of a neutral type of accidental situation becomes ever more important. Although both approaches lead to comparable results, the authors recommend the usage of this extrapolation approach as in the other approach the accident counts are not independent. This is crucial as the possibility of calculating confidence intervals is eliminated.

As a last important statement of this section a word of caution is in order: It is important to point out the non-linearity of odds-ratios. When calculating the effectiveness within two disjoint groups of accidental situations, the effectiveness of the union of the two groups calculated directly will most likely differ from the one calculated by a weighted mean of each group's effectiveness. In some cases it may even occur that the directly calculated effectiveness of the union of the groups is larger (or smaller) than each of the two groups' effectiveness (cf. [2] keyword "Simpson's paradox" for more information on this behalf). Therefore, it is not advisable to calculate the overall effectiveness by dividing the data into different groups and averaging the results! In case of the overall effectiveness extrapolation approach, the 0% effectiveness of the neutral group leads to reasonable results, but for many other classifications Simpson's paradox comes into play.

**Confidence intervals**

Independently of the selection of type *A*, *N* and SF, the calculated result of the effectiveness does not take into account statistical fluctuations. For example, assume that for a certain given population of vehicles the true accident avoiding effectiveness of SF for *A* equals 20%. Then, a randomly drawn sample of accidents is reported to a database. If then the above described way of calculating the accident avoiding effectiveness is applied, it is most likely that the result will not be exactly 20.0%, due to the random drawing and therefore statistical fluctuations. Nevertheless, for a given interval it is possible to compute the probability that the true value is covered by this interval. E.g. when the probability of a certain interval for including the true value of interest is equal to 95%, then the interval is called a 95% confidence interval for that value of interest.

The requirements for calculating a confidence interval are the following:

$n^A_1$, $n^A_0$, $n^N_1$ and $n^N_0$ (cf. equation (6)) need to be pair wise stochastically independent and those accident counts need to properly follow Poisson distributions. Except for the above-mentioned first approach for calculating the overall effectiveness, these assumptions are very common in literature; cf. [10] and the literature review of [9]. If any of the two assumptions is not appropriate, a better but more time-consuming and computer-intensive method would be bootstrapping, where resampling is used to calculate a variety of different results for the effectiveness. From the variance of these different calculations, conclusions about the influence of statistical fluctuations on the effectiveness may be drawn. See [3] for more information on this topic.

When calculating the effectiveness by the means of odds-ratios, it is easily possible to calculate a confidence interval for it. If interested in a (1-α)*100% confidence interval for the effectiveness *eff(A)* with $\alpha \in (0,1)$, equation (7) shows how to compute it. See [1] page 24 for more detailed information.

$$\begin{aligned} eff_{low} &= 1 - OR \cdot \exp\left( +u_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{1}{n_1^A} + \frac{1}{n_0^A} + \frac{1}{n_1^N} + \frac{1}{n_0^N}} \right) \\ eff_{high} &= 1 - OR \cdot \exp\left( -u_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{1}{n_1^A} + \frac{1}{n_0^A} + \frac{1}{n_1^N} + \frac{1}{n_0^N}} \right) \end{aligned} \quad (7)$$

Where $u_{1-\alpha/2}$ stands for the (1-α/2)-quantile of the standard normal distribution. Please take notice of the fact that this is no symmetric confidence interval, i.e. *eff* usually will not be in the center of this interval.

If $0 \notin [eff_{low}; eff_{high}]$, the calculated effectiveness is called statistically significant with the level of significance α.

If interested in a confidence interval for the overall effectiveness described in the preceding section, the upper and lower bounds of the confidence interval for $eff_A$ need to be imputed into equation (5). The interval calculated in this way is not an exact confidence interval in the strict sense as statistical fluctuations on the percentage of neutral accidents from all accidents are not taken into account. Nevertheless, the authors recommend this approach, since as the influence of these statistical fluctuations should be negligible compared to the fluctuations the confidence interval for $eff_A$ takes into account.

In principle the presented confidence intervals or bounds are in correspondence to rates and percentages only. They are not related to absolute numbers since for example the absolute number of accidents in a certain time period is a random quantity as well. Therefore, the presented absolute

numbers should always be understood in relation to the total number of accidents (even of a specific type).

The confidence interval for the overall effectiveness is computed as shown in equation (8):

$$eff_{overall,low}(\mathrm{SF}) = eff_{A,low}(\mathrm{SF}) \cdot \frac{n_0^A + \frac{n_1^A}{1-eff_{A,low}(\mathrm{SF})}}{n_0^A + \frac{n_1^A}{1-eff_{A,low}(\mathrm{SF})} + n_0^N + n_1^N}$$

$$eff_{overall,high}(\mathrm{SF}) = eff_{A,high}(\mathrm{SF}) \cdot \frac{n_0^A + \frac{n_1^A}{1-eff_{A,high}(\mathrm{SF})}}{n_0^A + \frac{n_1^A}{1-eff_{A,high}(\mathrm{SF})} + n_0^N + n_1^N} \quad (8)$$

## EVALUATING MULTIPLE SAFETY FUNCTIONS

As a matter of course, it is of major interest to be able to evaluate a whole package of multiple safety functions as well as a single safety functions only. Odds-ratios offer a well interpretable way of comparing any two (or even more) different safety equipments. In equation (2) the odds-ratio is calculated by comparing the probabilities of suffering a certain accident depending on whether a SF is active or not. The very same approach may be applied when looking at different safety configurations instead of a single active or non-active safety function. A safety configuration is considered to be a set of various safety functions such as "any car that is equipped with anti-lock braking system, airbags and emergency brake assistant but does not contain ESC". In this paper a safety configuration is understood to be a set of safety functions always included, certain safety functions may be excluded and information about other safety functions is not of interest. In this section the effectiveness of a safety configuration SC I is compared with the effectiveness of another safety configuration SC II.

The effectiveness calculated by the use of odds-ratios then describes the additional gain of safety of SC I compared to SC II. In other words: Assume that some vehicles equipped with SC II are involved in critical accidental situations that would lead to accidents of type *A*. The effectiveness then describes how many of these accidents could have been avoided if instead of SC II the safety configuration SC I had been on board.

SC I and SC II do not have to be a single specific safety configuration but may also each describe classes of safety configurations. For example, SC II may stand for "any safety-configuration that includes the safety function SF1 but excludes SF2" and SC I could be "any safety configuration that includes SF1 as well as SF2". For the sake of an easier interpretation of the results, SC I should always include every single safety function that is included in SC II plus some additional safety function(s) that are excluded in SC II.

For SC I and SC II defined in this way, the corresponding effectiveness shown in equation (9)

$$eff(A) = 1 - \mathrm{OR} = 1 - \frac{\frac{P(\mathrm{SC\ I} \mid A)}{P(\mathrm{SC\ II} \mid A)}}{\frac{P(\mathrm{SC\ I} \mid N)}{P(\mathrm{SC\ II} \mid N)}} \quad (9)$$

describes the additional gain of SF2 within accident type *A*, given that SF1 is already existent. If interested in safety configurations which contain information on more than two safety functions, the interpretation of the results is analogous:

The effectiveness then describes the additional gain of all these safety functions that are included in SC I but excluded in SC II, given that all the safety functions that are included in both, SC I and SC II are already present.

At this point again the neutral accident type *N* is crucial. This type of accident must not be influenced by **any** of the safety functions that distinguish SC I from SC II.

Analogous to evaluating a single safety function, the confidence intervals may be computed by applying equation (7).

The overall effectiveness calculation (cf. equation (5)) is still possible. However, the fact that the computed value refers to the group of all vehicles equipped either with SC I or SC II has to be taken into consideration. All other safety configurations are excluded and therefore the different overall effectiveness calculations are not always comparable as shown in the following.

The herewith described algorithm is applied to a data example in [15] in detail.

## EVALUATING INJURY MITIGATING AND INJURY AVOIDING EFFECTIVENESS

So far odds-ratios have only been used for evaluating the accident avoiding effectiveness, but as pointed out in the beginning of the paper the other types of effectiveness (e.g. injury avoiding and injury mitigating) are of major interest as well. Typically these types of effectiveness can be quantified on the basis of in-depth accident studies and simulations based on accident-reconstructions. But as these in-depth studies are not always

applicable, this paper intends to propose a general approach that as far as possible is independent on the type of safety function or configuration of interest, such an approach shall be presented in the following.

As seen in the previous sections, odds-ratios are able to evaluate the accident avoiding effectiveness of some safety configurations within a certain type of accidental situation called *A*. To evaluate the effectiveness of a safety configuration on different severity levels of injuries, *A* has to be split up into *n* different subgroups, enumerated according to an increasing severity of the accident. Thus *A1* may stand for all accidents within category *A* with material damage only, *A2* may stand for all accidents within category *A* with slightly injured passengers only, up to *An* which may stand for accidents of category *A* with fatally injured passengers. As the described classification of the accidents is vehicle-related, only the occupants of the vehicle of interest are relevant for the classification $Ax, x=1,\ldots,n,$ and not for example the most severely injured person involved in an accident. For such a classification both approaches of either looking at the injury status of the driver only, or looking at the maximum injury severity of any occupant of the vehicle is feasible. As injury mitigation stands for a reduction of the severity, it may be expressed by a shifting of cases from higher groups to lower groups due to the safety configuration of interest.

**Explanation with the help of a thought experiment**

For the sake of an easier understanding this subclassification shall now be introduced to the thought experiment above: Two almost identical fleets of vehicles are to be involved in critical situations. The vehicles of the first fleet are not equipped with the safety configuration of interest, the vehicles out of the second fleet are all to be equipped with the safety configuration. All other variables that may have an influence on the accident outcome such as driver behavior are supposed to be exactly the same for the vehicles of both fleets. Hence, this thought experiment again resembles a perfect case-control-study, where each critical situation is observed twice – one time with the safety configuration on board and another time without the same. Each pair represents a matched pair: Case and control. Assuming that the safety configuration indeed has some effect on the injury severity of the accident-involved persons, this should be observable. The distribution of the accidents through the different severity types *A1* to *An* within the fleet of equipped vehicles should differ from the distribution of the non-equipped fleet. For the sake of consistency the event "no accident" is to be named *A0*.

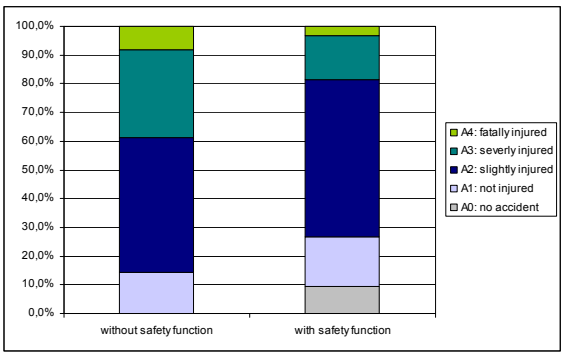


**Figure 2: Example for a different injury severity distribution**

This example is similar to the example in Figure 1, just the group "accidents" has been subclassified according to the above introduced scheme of accident severity. Similar to the accident avoidance effectiveness which may be observed by the decreasing amount of accidents, the injury mitigating effectiveness expresses itself in shifts within the group "accidents" from one group of injury severity to another one. In this example the amount of vehicles with fatally injured occupants has decreased in the group of SF-equipped vehicles compared to the non-equipped ones. Unfortunately it is not possible to tell, what has happened with these "missing" fatalities. Did these accidents not happen anymore (accident avoidance) or was it an accident still but with a lesser severity (injury avoidance or injury mitigation)?

At this point one major assumption has to be made:

It is assumed that the safety configuration of interest has to have a positive effectiveness. This means that every accident that is prevented from being of severity $x$ and instead of severity $y$, it **must** be assumed that $y \leq x$ will always hold! In other words for every matched pair of cases and controls the severity of the case (that is a vehicle equipped with this specific safety configuration) must be of the same or of a lower level than the severity of the control (non-equipped vehicle)!

Obviously this assumption sometimes may not be true in single specific accidents, such as if a vehicle falls into water where drowning may become more likely for a belted person. Nevertheless, for the majority of the cases the assumption is not unrealistic and as the assumption is crucial for all the follow-ups, the few exceptions shall be ignored.

Due to this assumption it may be taken for granted that all accidents out of the second fleet, classified to a certain severity level has its matching partner within the same or a higher group of accident-severity. For example all fatal accidents in the group of equipped vehicles have their matching non-equipped counterpart in the group of fatal accidents.

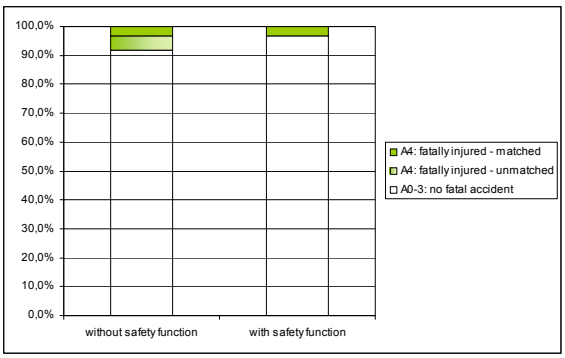


**Figure 3: Example for different amounts of fatal accidents**

A short recall: In the thought experiment every critical situation is observed twice with identical surrounding conditions except for the safety function. Each of these pairs is called a matched pair. In the context of Figure 1 "matched" stands for matched within the same severity level and if the matching partner of a specific critical situation is not within the same group it is denoted as being "unmatched".

All non-equipped vehicles out of the group *A4* that are not matched have been avoided to stay in the group of fatal accidents due to the safety configuration. Thus, the injury mitigating effectiveness for the group of fatal accidents is somehow expressed by an accident avoiding effectiveness of this subgroup. If the above assumption does not hold, it is not possible to tell anything about the amount of unmatched vehicles within the non-equipped group *A4*, as it may be the case that some of the equipped fatal accidents may have their matching partner in a non-fatal group. That is why the assumption is crucial for the following.

The same argument may be applied to any group of accident where it is for sure that all vehicles in this group out of the equipped fleet have their matching counterpart in the same injury severity group within the non-equipped fleet. Hence, in the equipped fleet no shifts from other groups to the group of interest are allowed. Due to the above assumption this will always hold if this group of interest is what shall be referred to as a "topmost group", which may be described as a group of **at least** a certain severity. Thus if the group with a certain severity $x$, that is group $Ax$ is to be investigated, it is necessary to look at this very group and all more severe groups which may be defined as $Ax^{+} := Ax \cup A(x+1) \cup \ldots \cup A(n-1) \cup An$. Only then it is for sure that theoretically all cases of equipped vehicles in this group $Ax^{+}$ have their control of non-equipped vehicles in the group $Ax^{+}$ as well.

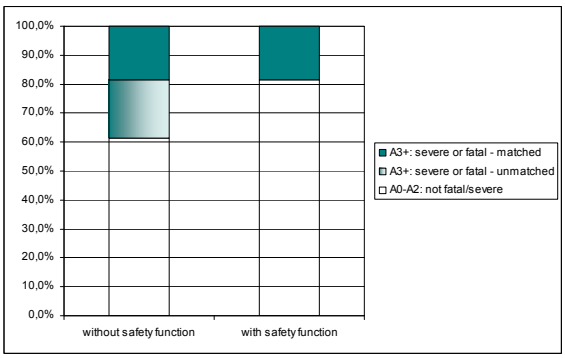


**Figure 4: Example for different amounts of severe or fatal accidents**

For every topmost group the injury mitigation effectiveness may be calculated in this way and it describes the amount of non-equipped vehicles (or controls) that are downshifted from the topmost group to some other groups. It stays unclear to which group these controls are shifted, i.e. to which group the matching case-vehicle belongs.

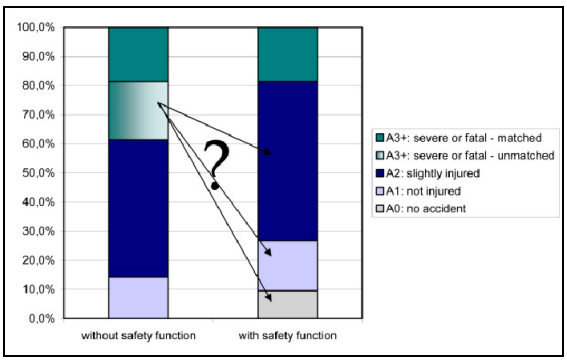


**Figure 5: Example for unknown amount of downshift to group of interest: A2**

When now looking at a not topmost group, for example *A2*, it is obviously impossible to tell how many of the $A3^{+}$-unmatched controls are matched to a case in *A2*. One extreme would be that **all** of these are matched to a case in *A2*, which would describe the situation that due to the safety configuration, the outcome of all these accidents has been mitigated to slight injuries only. This is expressed in Figure 6, where all the $A3^{+}$-unmatched controls are matched to a case in *A2*. For calculating the injury mitigation effectiveness within group *A2* it is again necessary to identify the amount of unmatched controls, that is non-equipped vehicles, in group *A2*. Because of the assumed non-negative effectiveness, it becomes clear that all leftover cases in *A2* are matched to controls of the same severity (dark blue areas in Figure 6). With this knowledge the exact amount of controls that are not matched to cases of the same severity level may be obtained, which is as well shown in Figure 6.

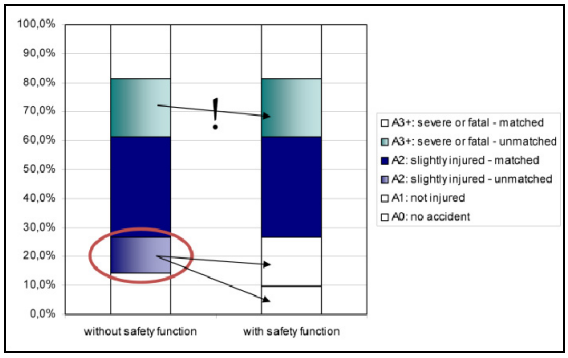


**Figure 6: Upper bound for effectiveness in group *A2***

It has to be kept in mind that due to the assumption, that every time the safety configuration prevented an accident from being severe or fatal, the outcome then has necessarily to be classified as belonging to group *A2*. If this is not true and at least some injury mitigation leads to a less severe classification, then the amount of matchings from case group *A2* to control group *A2* increases and therefore the amount of unmatched controls in group *A2* decreases. As the injury mitigation effectiveness decreases with a decreasing amount of unmatched controls, the above described extreme leads to an upper bound for the effectiveness!

The other extreme would be to assume, that all controls of the group $A3^+$ that are not matched to cases in $A3^+$ as well are matched to a less severe group than *A2*. Thus, as no cases in *A2* are matched to a higher group, all the observed cases in *A2* need to have their matching counterpart in the control group *A2*. By then looking at the size of both groups *A2* and seeing which is bigger and therefore leaves some cases or controls unmatched within the same severity level, a lower bound for the injury mitigating effectives may be calculated. In this example one observes more cases of severity level *A2* than controls of the same severity level. This would lead to a negative effectiveness for the group *A2* which contradicts the general assumption of a non-negative effectiveness. On the other hand this extreme scenario, where every time the safety configuration prevents an accident from being fatal or severe, the outcome must be no injury or no accident seems rather unlikely. Therefore a negative lower bound of the effectiveness does not necessarily contradict the non-negativity-assumption.

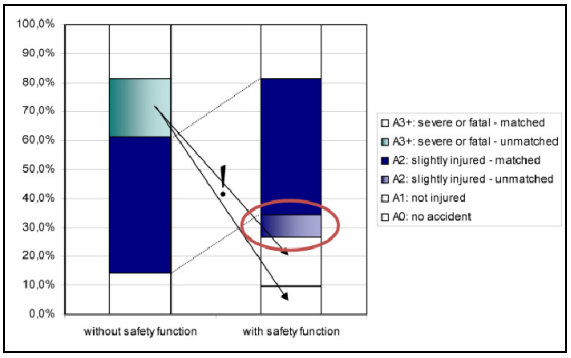


**Figure 7: Lower bound for effectiveness in group *A2***

A negative lower bound for the effectiveness has to be interpreted almost similar to a lower bound equal to zero with the only difference that it indicates a high effectiveness in the higher groups and in some cases possibly a low effectiveness in the group of interest.

It is important to point out, that the interval given by the lower and upper bound do have nothing in common with a confidence interval. This interval just includes all possible values for the effectiveness for the group *A2* that are conformable with the data at hand. Confidence intervals need to be computed separately for each of the two boundaries!

Of course all the above explanations are applicable to any of the different severity groups *A1* to *An* as well as especially $A1^+$ to $A(n\text{-}1)^+$. When then having a closer look and the classification of *A0* and *A1* is similar to the example above, the effectiveness of group $A1^+$ describes the amount of cases that are shifted to group *A0* due to the safety configuration. In other words the safety configuration has changed an event from being an accident of a certain severity to become an event "no accident". Thus the effectiveness of group $A1^+$, denoted by $eff(A1^+)$, is equivalent to the accident avoiding effectiveness. Accordingly $eff(A2^+)$ is a combination of the injury avoiding effectiveness and the accident avoiding effectiveness.

Analogous to the section "effectiveness of single safety functions" it is important to point out the impossibility of observing such a perfectly matched case-control study in the real world similar to the above thought experiment. Nevertheless, if there is a neutral type of accidental situation according to the requirements for such mentioned above, this group may be used as the control group and some other group of interest then will be the case group. These two groups of course then are not case wise matched as it is assumed in the thought experiment, but these matchings were only introduced for the sake of an easier understanding, odds-ratios do not require such an assumption.

**Resulting formulas for injury mitigation analysis**

When implying the above described theory to the formulas for odds-ratios, the effectiveness for a topmost group is calculated as shown in equation (10), where SC I stands for the safety configuration of interest that is about to be compared to the other safety configuration SC II. *N* stands for the neutral type of accidental situation.

$$eff\left(Ax^{+}\right) = 1 - \mathrm{OR} = 1 - \frac{\dfrac{P\left(\mathrm{SC\ I} \mid Ax \cap A(x+1) \cap \ldots \cap An\right)}{P\left(\mathrm{SC\ II} \mid Ax \cap A(x+1) \cap \ldots \cap An\right)}}{\dfrac{P\left(\mathrm{SC\ I} \mid N\right)}{P\left(\mathrm{SC\ II} \mid N\right)}} \quad (10)$$

The probability-rates such as P(SC I | *N*)/P(SC II | *N*) may be understood as the equipment-rates within the groups of interest as shown in equation (3).

If there is a sufficient amount of accidents involved in accidents of type *N* in the database at hand, the group *N* may be divided into subgroups according to the subclassification of *A*. But as the difference between SC I and SC II is considered to have no influence on crash occurrence and injury severity for the group *N*, the equipment-rate should not differ between the different severity levels. Hence, there should be no need to do so. In some cases it even will occur that the equipment rate within different severity levels within *N* do differ (either due to statistical fluctuations or to substantial differences) but then it seems to be questionable to take these differences into account for calculating the different types of effectiveness.

The confidence interval for this effectiveness may be computed by applying equations (7) which shall be repeated in equations (11).

$$\begin{aligned} eff_{low} &= 1 - OR \cdot \exp\left( +u_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{1}{n_1^A} + \frac{1}{n_0^A} + \frac{1}{n_1^N} + \frac{1}{n_0^N}} \right) \\ eff_{high} &= 1 - OR \cdot \exp\left( -u_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{1}{n_1^A} + \frac{1}{n_0^A} + \frac{1}{n_1^N} + \frac{1}{n_0^N}} \right) \end{aligned} \quad (11)$$

Again, $u_{1-\alpha/2}$ stands for the (1-α/2)-quantile of the standard normal distribution and $n^A{}_1$, $n^A{}_0$, $n^N{}_1$ and $n^N{}_0$ are the four different absolute numbers used in the calculation for the odds-ratio OR (cf. [15]).

As this paper now comes back to the earlier discussed lower and upper bounds it is important to point out the difference between these upper and lower bounds and confidence intervals. The upper and lower bounds only account for different possible amounts of injury mitigations from more severe groups to the group of interest, whereas confidence intervals account for statistical fluctuations!

As argued above where Figure 7 is described, the lower bound for the effectiveness is calculated by not taking any possible injury mitigation (or downshifts) from more severe accident categories to the group of interest into account. Therefore this effectiveness is calculated by the crude odds-ratio of this group as shown in equation (12).

$$\begin{aligned} eff_{\min}\left(Ax\right) &= 1 - \frac{\dfrac{\text{No. of vehicles with SC I in } Ax}{\text{No. of vehicles with SC II in } Ax}}{\dfrac{\text{No. of vehicles with SC I in } N}{\text{No. of vehicles with SC II in } N}} \\ &= 1 - \frac{n_1^x}{n_2^x} \Big/ \frac{n_1^N}{n_2^N} \end{aligned} \quad (12)$$

For calculating the confidence interval for this lower bound, only a one-sided interval is of use, as the maximal possible value for the lower bound does not provide any useful information. Therefore it must be calculated with using the (1-α)-quantile of the standard normal distribution instead of the (1-α/2)-quantile and equation (13) describes the (1-α) lower confidence bound for the minimal effectiveness.

$$eff_{\min,low}\left(Ax\right) = 1 - \frac{\dfrac{n_1^x}{n_2^x}}{\dfrac{n_1^N}{n_2^N}} \cdot \exp\left( +u_{1-\alpha} \cdot \sqrt{\frac{1}{n_1^x} + \frac{1}{n_2^x} + \frac{1}{n_1^N} + \frac{1}{n_2^N}} \right) \quad (13)$$

The calculation of the upper bound is a bit more complicated, so only the resulting formulas are presented here. The derivation of these results may be found in [15]. The crucial point for this upper bound for a certain group *Ax* is a proper estimation of the downshifts from higher groups to this group *Ax* as described with Figure 6. Therefore many different variables needed for the formula are defined, as well as some abbreviations comparable to equation (6) are introduced in equations (14).

$$\begin{aligned} n_1^x &:= \text{No. of vehicles with SC I in } Ax \\ n_2^x &:= \text{No. of vehicles with SC II in } Ax \\ z &:= x+1 \\ n_1^{z+} &:= \text{No. of vehicles with SC I in } Az^{+} \\ n_2^{z+} &:= \text{No. of vehicles with SC II in } Az^{+} \\ n_1^N &:= \text{No. of vehicles with SC I in } N \\ n_2^N &:= \text{No. of vehicles with SC II in } N \end{aligned} \quad (14)$$

Then the upper bound for the effectiveness within group *Ax* may be calculated as explained in [15]. In a first step, the maximum amount of downshifts into the group of interest ist computed (cf. equation (15)).

$$\#shifts = \frac{n_2^{z+} \cdot n_1^{\mathrm{N}}}{n_2^{\mathrm{N}}} - n_1^{z+} \tag{15}$$

Then, for the sake of consistency this value needs to be truncated to the interval [0, $n_1^x$] as any other value would lead to negative accident counts (cf. equation (16)).

$$\begin{aligned} &\text{IF } \#shifts \le 0 \quad \text{THEN } \#shifts := 0 \\ &\text{IF } \#shifts > n_1^x \quad \text{THEN } \#shifts := n_1^x \end{aligned} \tag{16}$$

Finally, the effectiveness is computed by subtracting the amount of shifts from the accident count of interest and computing the effectiveness in the standard way with the use of odds-ratios.

$$eff_{\max}(Ax) = 1 - \frac{\left(n_1^x - \#shifts\right) \cdot n_2^N}{n_2^x \cdot n_1^N} \tag{17}$$

If interested in a direct calculation of the effectiveness without explicitely computing the amount of shifts, equation (18) may be used.

$$\begin{aligned} &\text{IF } \left( \frac{n_2^{z+} \cdot n_1^{\mathrm{N}}}{n_2^{\mathrm{N}}} + n_1^{z+} \ge n_1^x \right) \\ &\quad \text{THEN } eff_{\max}(Ax) = 100\% \\ &\text{ELSE IF } \left( n_1^{z+} - \frac{n_2^{z+} \cdot n_1^{\mathrm{N}}}{n_2^{\mathrm{N}}} \le 0 \right) \\ &\quad \text{THEN } eff_{\max}(Ax) = eff_{\min}(Ax) \\ &\text{ELSE} \\ &eff_{\max}(Ax) = \frac{n_1^N \cdot \left(n_2^x + n_2^{z+}\right) - n_2^N \cdot \left(n_1^x + n_1^{z+}\right)}{n_2^x \cdot n_1^N} \end{aligned} \tag{18}$$

When interested in an upper confidence bound for the maximal effectiveness, the proceeding is analogously. First the maximum number of shifts needs to be calculated, this time including confidence intervals (cf. equation (19)).

$$\begin{aligned} &\#shifts_\alpha \\ &= n_1^{z+} \cdot \left( \frac{1}{1 - eff_{high}\left(Az^+\right)} - \frac{1 - eff_{high}\left(Az^+\right)}{1 - eff\left(Az^+\right)} \right) \end{aligned} \tag{19}$$

Please take special note of the fact that for the calculation of $eff_{high}(Az^+)$ the **one-sided** upper confidence bound instead of the two-sided one was used!

Again, for the sake of consistency this value needs to be truncated to the interval [0, $n_1^x$] as any other value would lead to negative accident counts (cf. equation (16)).

$$\begin{aligned} &\text{IF } \#shifts_\alpha \le 0 \quad \text{THEN } \#shifts_\alpha := 0 \\ &\text{IF } \#shifts_\alpha > n_1^x \quad \text{THEN } \#shifts_\alpha := n_1^x \end{aligned} \tag{20}$$

This amount of shifts has to be subtracted from the accident count of interest for the computation of the upper bound of the effectiveness as shown in equation (21).

$$\begin{aligned} &eff_{\max,high}(Ax) = 1 - \frac{\left(n_1^x - \#shifts_\alpha\right) \cdot n_2^N}{n_2^x \cdot n_1^N} \\ &\cdot \exp\left( -u_{1-\alpha} \cdot \sqrt{\frac{1}{\left(n_1^x - \#shifts_\alpha\right)} + \frac{1}{n_2^x} + \frac{1}{n_1^N} + \frac{1}{n_2^N}} \right) \end{aligned} \tag{21}$$

The resulting value will most certainly be equal or larger than the actual upper confidence bound. The left over uncertainty and the exact derivation of these formulas is explained in detail in [15].

As a matter of course, the overall effectiveness introduced in the section "overall effectiveness and misclassifications" is of interest for the different groups of injury severity. The formula given in equation (5) may be transformed accordingly to any group $Ax$ or $Ax^+$, subgroup of $A$ with an injury severity $x$ or $x^+$ respectively as shown in equation (22). Please pay special attention to the necessity of splitting the accident type $N$ according to the injury severity splitting of accident type $A$. This is necessary for the calculation of the overall effectiveness **within** a certain injury severity level and does not contradict the recommendation from above not to do such a splitting for the calculation of $eff_{Ax}$ or $eff_{Ax+}$. The abbreviations used are according to equation (14) except for the substitution of $N$ with $Nx$ due to the subclassification of $N$. According to the definition of group $Ax^+$ the group $Nx^+$ stands for "accidents of type $N$ classified with a severity of at least $x$".

$$\begin{aligned} &eff_{overall}^{severity:\,x}(\mathrm{SF}) = eff_{Ax}(\mathrm{SF}) \\ &\cdot \frac{n_2^x + \dfrac{n_1^x}{1 - eff_{Ax}(\mathrm{SF})}}{n_2^x + \dfrac{n_1^x}{1 - eff_{Ax}(\mathrm{SF})} + n_2^{\mathrm{Nx}} + n_1^{\mathrm{Nx}}} \\ &eff_{overall}^{severity:\,x+}(\mathrm{SF}) = eff_{Ax+}(\mathrm{SF}) \\ &\cdot \frac{n_2^{x+} + \dfrac{n_1^{x+}}{1 - eff_{Ax}(\mathrm{SF})}}{n_2^{x+} + \dfrac{n_1^{x+}}{1 - eff_{Ax}(\mathrm{SF})} + n_2^{\mathrm{Nx+}} + n_1^{\mathrm{Nx+}}} \end{aligned} \tag{22}$$

For calculating the overall effectiveness of an upper bound for the effectiveness of a not topmost injury-group, the shifts described in equation (15) and (19) respectively need to be taken into consideration which leads to equation (23):

$$eff^{severity:\,x}_{\max,overall}(\mathrm{SF}) = eff_{\max,Ax}(\mathrm{SF}) \cdot \frac{n_2^x + \dfrac{n_1^x - \#\mathrm{shifts}}{1 - eff_{\max,Ax}(\mathrm{SF})}}{n_2^x + \dfrac{n_1^x - \#\mathrm{shifts}}{1 - eff_{\max,Ax}(\mathrm{SF})} + n_2^{\mathrm{Nx}} + n_1^{\mathrm{Nx}}} \tag{23}$$

$$eff^{severity:\,x}_{\max,high,overall}(\mathrm{SF}) = eff_{\max,high,Ax}(\mathrm{SF}) \cdot \frac{n_2^x + \dfrac{n_1^x - \#\mathrm{shifts}_\alpha}{1 - eff_{\max,Ax}(\mathrm{SF})}}{n_2^x + \dfrac{n_1^x - \#\mathrm{shifts}_\alpha}{1 - eff_{\max,high,Ax}(\mathrm{SF})} + n_2^{\mathrm{Nx}} + n_1^{\mathrm{Nx}}}$$

If interested in the overall effectiveness of a group $Ax^+$ instead of $Ax$ the formulas are almost identically and therefore will not be repeated. Again it shall be stressed, that the interval given by the extrapolation of confidence boundaries for the effectiveness within a certain accident group to an interval for the overall effectiveness is not any longer an exact (1-α) confidence interval as statistical fluctuations on the percentage of neutral accidents from all accidents are not taken into account.

The herewith described algorithm is applied to a data example in [15] in detail.

## THE INFLUENCE OF ADDITIONAL EXTERNAL FACTORS AND LOGISTIC REGRESSION

In drawing conclusions from a statistical analysis one always has to be careful. A causal relationship between two variables always leads to some kind of statistical dependence between these two quantities. The opposite assertion that an existing statistical dependence between two quantities leads to a causal relationship between the corresponding variables in general is not true. The easiest example one may think of is as follows. Assume that one variable Z has a causal relationship to the variables X and Y which are of interest to the investigator. If one considers or observes the variables X and Y only, then they typically will show up some kind of dependence. But the true story is that both variables depend on the third one Z. In the context of this paper this could mean that if the driver populations of vehicles equipped and not-equipped with a specific safety equipment are completely different or even disjoint. Then the observed effectiveness of this safety equipment may be entirely due to the difference in the driver population. One easily can think of other examples which in some and even in relevant cases may lead to a significant misinterpretation of the results. In pure statistical theory one therefore usually assumes that the test conditions of the two experiments are completely equal except for the variable of interest as in the thought experiments in the preceding section. Having such an ideal situation at hand, all observed differences in accident outcome between equipped and non-equipped vehicles are due to the safety configuration for sure. But the above mentioned theoretical assumption is far from being realistic when investigating real world accident data. In reality the equipment of vehicles not only differs in specific safety functions and the driver population rarely is the same for different vehicles. Therefore methodology is needed to deal with this situation.

One simple idea is to create different categories of accidents in which all relevant external variables like driver's age and gender, size of the vehicle, weather conditions at the accident spot, accidental situation etc are as similar as possible. Within every group of such categorized accidents one may compute an odds-ratio as described above. The variation of the odds-ratio over the different categories easily may be interpreted as a quantification of the influence of the accident characteristics within a single category. This approach perfectly works if one has sufficient accident data at hand and not too many external variables in mind. If only one of these two hypotheses is not true one ends up with very few cases in each category which leads to non reliable statistical quantities within each category. Even if only five external variables are considered, for which each of them may take five different values at least hundred thousand and more accidents are needed in order to obtain reliable and interpretable results. Thus, even for a rather low number of external variables the so-called curse of dimensionality arises.

Another possibility in order to quantify the influence of external variables to the accident outcome is given by the statistical concept of logistic regression. A detailed explanation of the concept of logistic regression models may be found in any textbook of categorical data (cf. for example [1]). Before starting the explanation, a brief word of caution is in order. Logistic regression is not able to circumvent the above mentioned curse of dimensionality. Logistic regression is a statistical tool which is able to deal with a moderate and sometimes even high number of external variables by the price of assuming that the influence of the external variables is to some extend easily structured. From a principle point of view logistic regression assumes that the influence of the external variables to a slightly transformed output quantity is just as simple as a linear influence.

For describing the essentials of logistic modeling some external variables $x_1, x_2, \ldots, x_d$ are introduced. These variables could take values 0 or 1, in case of gender as an example, or could take numbers (like the age of the driver of the vehicle) and so on. One or more of the variables denotes the coding whether a specific safety function in the vehicle is on or off. Then logistic modeling for the

probability $P(A|x_1, x_2, \ldots, x_d)$ of having an accident of type $A$ given that the external variables take the specific values $x_1, x_2, \ldots, x_d$ reads as follows (cf. equation (24)).

$$P(A|x_1,\ldots,x_d) = \frac{\exp(\beta_0 + \beta_1 x_1 + \ldots + \beta_d x_d)}{1+\exp(\beta_0 + \beta_1 x_1 + \ldots + \beta_d x_d)} \quad (24)$$

For the so-called odds this means

$$\begin{aligned} \text{logit } P(A|x_1,\ldots,x_d) &= \ln \frac{P(A|x_1,\ldots,x_d)}{1-P(A|x_1,\ldots,x_d)} \quad (25) \\ &= \beta_0 + \beta_1 x_1 + \ldots + \beta_d x_d \end{aligned}$$

which just indicates the above mentioned linearity assumption of logistic modeling. When the accident-database at hand is split into two groups – accidents of type $A$ and accidents of type $N$ which is neutral to the safety function of interest – statistical routine software is able to estimate the parameters $\beta_1, \beta_2, \ldots, \beta_d$. With the use of equation (26) which is shown to be valid in [15], the relative risk introduced in the section "relative risk – odds-ratios" may be computed.

$$\begin{aligned} &\text{for } x_1 = \begin{cases} 0, & \text{safety configuration: SC II} \\ 1, & \text{safety configuration: SC I} \end{cases} \\ &\frac{P\left(A \mid x_1 = 1 \,\&\, x_k \in \Omega_k \text{ for } k \in \{2,\ldots,d\}\right)}{P\left(A \mid x_1 = 0 \,\&\, x_k \in \Omega_k \text{ for } k \in \{2,\ldots,d\}\right)} = \exp\left(\beta_1\right) \end{aligned} \quad (26)$$

In equation (26) $\Omega_k$ stands for the set of all possible values $x_k$ can take. Hence, for any given specific combination of values for the different external factors $x_k$ 's, the relative risk of suffering an accident of type $A$ with SC I compared to SC II is calculable. It even is independent on the specific values of $x_2, \ldots, x_k$ which means that no matter the specific situation of external factors, the relative risk always stays the same. This may be interpreted as the relative risk shown in equation (1) where all the confounding influences of the variables $x_2, \ldots, x_k$ have been canceled out!

It shall be stressed that the classification and choosing of the variables $x_2, \ldots, x_k$ may have an enormous influence on the estimation of $\beta_1$.

First, the type of variable is of interest: When a variable has more than two values possible to take, the logistic model always assumes that the influence of that variable continuously increases or decreases. E.g. if looking at the influence of the driver's age on the probability to be involved in a skidding accident the model either assumes that the older the driver, the higher (or lower) the probability of being involved in a skidding accident will be. It is not possible to model a high risk for young drivers, a low one for middle-aged and again a high one for old drivers when using a logistic regression. Therefore the authors strongly recommend using binary variables only. This would not be a loss of generality as every variable describing a single external factor $x_k$ with $h$ different possible values may be expressed by binary variables only. In order to do so, one of the possible outcomes needs to be set as a reference group. Then this single factor may be coded as ($h$-1) different variables $x_{k1}, \ldots, x_{k(h-1)}$ each to be coded as 0, if the value $x_k$ equals the reference group. If the value equals one of the other ($h$-1) possible outcomes, e.g. the $m$-st possible value, then $x_{km}$ is set to 1 and all the others to 0. Thus, any categorized variable with $h$ different possible values may be coded as ($h$-1) different binary variables. (Many statistical software-packages do this automatically when applying a categorical logistic regression.)

When dealing with continuous variables, these may be categorized as well without losing much information due to the limited number of cases within a database and a limited interpretability – is there much information to gain by distinguishing between a driver who is 37 years and 360 days old and a 38 years old driver?

Second, including one more variable or excluding a single one from the model may have a big influence on the estimation of $\beta_1$ as well. Accordingly, the inclusion or exclusion of certain interactions of multiple of the different variables may have a big influence. Thus, for the logistic regression it is of crucial importance to choose the "right" variables and interactions of variables to be included into the model only. There are multiple ways of identifying the so-called "goodness of fit" of a model which may be used to select the model most appropriate for the data at hand. See [1] for more information.

Another important remark is that it is crucial that $\beta_1$ is interpretable in such a good way due to the specific choice of neutral accident types. Please see [15] on what requirements need to be fulfilled in order to be able to interpret $\beta_k$ with $k \neq 1$.

It shall be stressed, that when applying a logistic regression typically a confidence interval for the parameter $\beta_1$ is given. Imputing these in equation (26) directly leads to confidence intervals for the effectiveness of interest.

Finally a logistic regression model is only able to deal with data sets that have complete information on all variables included in the model. As a logistic regression model typically includes plenty of different confounder variables this typically leads to a considerable reduction of the amount of data available to run the regression on. To circumvent this problem two approaches are promising:

Either a missing data imputation algorithm [5] should be run on the data-base, or the attribute

"missing" should be used as another category of the corresponding categorical confounding variable.

## GENERAL REMARKS

The methodology presented in this paper is applicable for any accident-database with a sufficient amount of information available to identify the crucial group of neutral accidents. Even though to obtain representative results it is crucial to have a database which is representative for the population of interest. Many databases are not representative as for example most of the times the injury outcome of any accident has an influence on the probability of a specific accident to be recorded in the database. This problem of representativeness may be dealt with by using an appropriate weighting algorithm. See [13] for more details.

Another important remark deals with missing values in the database. Often many entries within a database are missing. Many studies assume these values to be missing at random and therefore simply exclude these cases from further analyses. However if this nonexistence of information is not entirely at random this proceeding results in a possible bias. Therefore it is recommended to take missing value imputation methods into consideration as they are described in [5]. Especially when calculating the effectiveness with a logistic regression to cancel out the influence of different confounder variables this is crucial. That is because a logistic regression requires complete information on all variables of all cases that are to be involved in the analysis. Typically this is only true for a comparatively small number of cases within a database.

## CONCLUSIONS

So far the findings of the preceding sections are more or less stand-alone results. As a matter of course, it is of great interest to combine these findings in order to be able to do an injury mitigation analysis for multiple safety functions including external factors. Therefore, the results are summarized in the following and it is explained how to combine the various methods.

First, the paper describes how odds-ratios may be interpreted, how to use these values in order to calculate the effectiveness of a safety function, how to extrapolate the results to an overall effectiveness and how to compute confidence intervals.

Afterwards, these findings are generalized in order to evaluate more than a single safety function. This is done by evaluating different combinations of the safety functions. Hence, if interested in an evaluation of more than a single safety function, multiple analyses have to be made. These multiple evaluations distinguish from each other in a different focus on the safety configuration of interest.

Section "evaluating injury mitigating and injury avoiding effectiveness" again describes a generalization of the use of odds-ratios and again multiple evaluations have to be made in order to quantify the injury mitigation effectiveness of a safety function. This time the multiple evaluations necessary differ in the injury severity of interest. Thus, it is no problem to combine the different generalization methods of odds-ratios: Each evaluation necessary for evaluating multiple safety functions at once may be split up into the multiple injury mitigation evaluations.

It is important to point out, that for all calculations presented so far, only aggregated data is necessary. Only with introducing a logistic regression in order to deal with confounding variables it becomes necessary to have more detailed case by case information!

Section "the influence of additional external factors and logistic regression" suggests a logistic regression instead of calculating the crude odds-ratios. By doing so, external factors may be taken into account in order to reduce interfering influences. Therefore, each calculation necessary should be done with the use of logistic regression. This is easily possible for almost all evaluations described above. Only if so-called "shifts" need to be taken into account, the logistic regression may not be applied directly. Nevertheless it is possible to apply a logistic regression in these cases as well. See [15] for a detailed description.

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