MATHEMATICAL ANALYSIS OF BRAKING IN REAR-END CRASH SCENARIOS

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ABSTRACT

Recently, a test-based assessment method for forward-looking crash avoidance and mitigation systems (pre-crash systems) was developed within the EU project ASSESS. In this method, injury outcome is correlated with the change of velocity (Delta-V) during the crash and the effect of the pre-crash system on this quantity is investigated. Delta-V may be related to relative impact speed (i.e., the speed difference between the vehicles at the moment of collision) by certain assumptions. It is therefore a central question in the ASSESS assessment method to determine the relative impact speed in all rear-end test scenarios in the test protocol. This is most challenging in the scenario when the vehicle under test (striking vehicle) is approaching a braking lead vehicle.

In this paper, the corresponding question is addressed in a much more general setting. In particular, instead of the fixed values used in the test protocol for the ASSESS method, all possible values for the initial speeds and decelerations of the vehicles and the initial headway of the lead vehicle are considered in the important case when the two vehicles have equal initial speeds and decelerations. The question addressed here is the following: given arbitrary values of all these parameters, is it possible to determine the speed difference between the two vehicles at the time of collision from the impact speed of the vehicle under test using mathematically rigorous methods?

The results show that under simplifying assumptions, the relative impact speed may be expressed as a function of the impact speed of the vehicle under test. The set of valid test results is characterized for all possible parameter values together with the requirements on how early braking needs to be performed in order to avoid the crash.

Possible generalizations of the method (including the feasibility of removing some of the theoretical assumptions) are also discussed. The results may be of use in the design of future test protocols for radar-based braking systems as well as in the development of such systems.

INTRODUCTION

Forward-looking crash avoidance and mitigation systems (pre-crash systems) constitute an important family of active safety systems. The safety effects of such systems – forward collision warning (FCW), pre-crash brake assist (PBA) and autonomous pre-crash braking (PB) in particular – in real-world rear-end crashes have been assessed in e.g. [3,4,5]. The results showed tangible benefits in terms of a reduced number of injuries in both vehicles involved in a rear-end crash.

It is a common feature of pre-crash systems that radar technologies of high accuracy are used to scan the road in front of the equipped vehicle to detect any object or vehicle that poses a potential safety risk. This way the system can intervene if necessary by issuing warning, providing brake enforcement or performing autonomous emergency braking. Therefore, a proper understanding of vehicle dynamics in rear-end scenarios is essential for the development and testing of pre-crash systems. The mathematical analysis described in this paper will give an exact description of the corresponding motions under certain simplifying assumptions.

Recently, a test-based assessment method for pre-crash systems was developed within the EU project ASSESS, see [1,2]. In this method, the injury-reducing capacity of a pre-crash system is assessed based on test results in three rear-end scenarios in which the vehicle under test (which is the striking vehicle) is approaching a slower lead vehicle, a stationary lead vehicle and a braking lead vehicle, respectively. In the current paper, the same classification of rear-end scenarios is considered and the braking lead vehicle scenario will be analysed in detail.

It is convenient for the implementation of the ASSESS method to determine the relative impact speed (i.e., the speed difference between the vehicles at the moment of collision) from the impact speed of the subject vehicle in all rear-end test scenarios in the test protocol. This is easily done in the scenarios when the lead vehicle is stationary or travelling at a constant speed but it is much more challenging when the lead vehicle is braking. With the help of the mathematical analysis of the braking lead vehicle scenario described here, it will be possible to find a solution to this problem for a multitude of parameter values.

In this paper the following basic model of the braking lead vehicle scenario is analyzed. A target vehicle (TV) and a subject vehicle (SV) move along a straight line in a given direction, with TV having an initial headway of $h > 0$. TV has initial speed $v_T > 0$ and is braking at deceleration $b_T > 0$ from time 0 on as long as it has positive velocity. SV, having initial speed $v_S > 0$, is driving at constant speed v_S until a time $T_b \geq 0$ after which it is braking at deceleration $b_S > 0$ as long as it has positive velocity. In both cases the deceleration associated with braking is immediately reached (i.e. no ramp-up period is taken into account).

The rest of this paper is concerned with the case of $v_S = v_T = v_0$ and $b_S = b_T = b$, i.e., when the two vehicles have equal initial speeds and decelerations. The importance of this special case is due to the fact that it represents the real-life situation when both vehicles drive at the speed limit of the road that they are driving on and their emergency brakings result in similar decelerations.

METHODS

All results in this paper are obtained via mathematically rigorous methods. The main step in the analysis is the reduction of the problem to the case of $T_b = 0$ by determining the velocities of the vehicles and the distance between them at time T_b, and substituting the corresponding parameters in the solutions for the case $T_b = 0$. However, the description of the detailed computations involved is out of the scope of this paper and will therefore be published separately.

RESULTS

First, it is important to note that there is an upper bound for the value of T_b, namely the time when SV reaches TV without braking, since the possibility of braking after the two vehicles have crashed is excluded in the current analysis. The resulting upper bounds are the following:

$$T_b \leq \sqrt{2h/b} \qquad (1)$$

if $h \leq v_0^2/(2b)$ (see curve C in Figure 1), and

$$T_b \leq h/v_0 + v_0/(2b) \qquad (2)$$

if $h > v_0^2/(2b)$ (see curve D in Figure 1). Note that the expressions on the right hand sides of (1) and (2) take the same value (namely v_0/b) in $h = v_0^2/(2b)$, hence their concatenation determines a region above these curves (marked as region IV in Figure 1). This parameter region is disregarded in the current analysis.

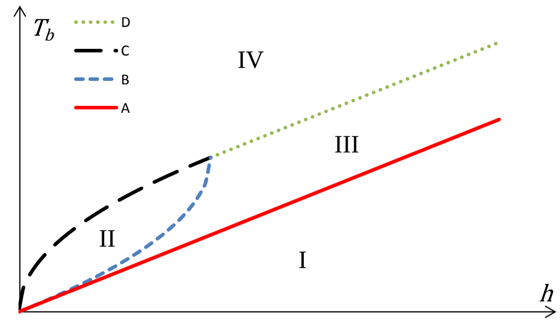

Figure 1. Illustration of the functions and regions that are relevant to the analysis in the case of $v_0 = 1$ and $b = 1$.

It is of special interest to determine which values of T_b allow the avoidance of the crash. The corresponding parameter region in Figure 1 is region I (with curve A as a non-inclusive upper bound), which is determined by the equation

$$T_b < h/v_0 \qquad (3).$$

In other words, the crash is avoided if and only if (3) holds.

A crash may occur before or after the target vehicle has stopped. It turns out that a crash while both vehicles are in motion may occur only for the following values: $h < v_0^2/(2b)$ and

$$\left(v_0 - \sqrt{v_0^2 - 2bh}\right)/b < T_b \leq \sqrt{2h/b} \qquad (4).$$

In Figure 1, the corresponding region is II. It is important to note that $\left(v_0 - \sqrt{v_0^2 - 2bh}\right)/b$ and $\sqrt{2h/b}$ indeed take the same values for $h = 0$ as well as for $h = v_0^2/(2b)$ while for $0 < h < v_0^2/(2b)$, it holds that

$$h/v_0 < \left(v_0 - \sqrt{v_0^2 - 2bh}\right)/b < \sqrt{2h/b} \qquad (5).$$

This shows that II is indeed a well-defined parameter region which is situated between h/v_0 and $\sqrt{2h/b}$ as shown in Figure 1.

Finally, for $h < v_0^2/(2b)$ and

$$h/v_0 \leq \mathrm{T_b} \leq (v_0 - \sqrt{\mathrm{v}_0^2 - 2\mathrm{bh}})/\mathrm{b} \qquad (6)$$

or for $h \geq \mathrm{v}_0^2/(2b)$ and

$$h/v_0 \leq \mathrm{T_b} \leq h/v_0 + v_0/(2b) \qquad (7)$$

it holds that a crash occurs when the target vehicle has already stopped. The corresponding parameter region in Figure 1 is region III.

In the parameter regions where the two vehicles crash, the speeds of the vehicles at impact are computed as follows. In region III, the impact speed of the subject vehicle is

$$\mathrm{v_S^{impact}} = \sqrt{2\mathrm{b}}\sqrt{v_0\mathrm{T_b} - \mathrm{h}} \qquad (8).$$

Since the target vehicle is standing still at the time of impact in this parameter region, the relative impact speed $R = \mathrm{v_S^{impact}} - \mathrm{v_T^{impact}}$ equals $R = \mathrm{v_S^{impact}}$ whose value is determined in equation (8). In region II, the subject vehicle has an impact speed of

$$\mathrm{v_S^{impact}} = (\mathrm{bT_b^2} + 2\mathrm{v_0T_b} - 2\mathrm{h})/(2\mathrm{T_b}) \qquad (9)$$

while the target vehicle has impact speed

$$\mathrm{v_T^{impact}} = \mathrm{v_S^{impact}} - \mathrm{bT_b} \qquad (10)$$

hence the relative impact speed in region II is $R = \mathrm{bT_b}$.

It may be noted that the right hand sides of both (8) and (9) are strictly increasing and continuous functions of $\mathrm{T_b}$ (in the relevant intervals of h), and the limit of (9) as $\mathrm{T_b}$ tends to $(v_0 - \sqrt{\mathrm{v}_0^2 - 2\mathrm{bh}})/\mathrm{b}$ from above equals the value of (8) at braking time $T_b = (v_0 - \sqrt{\mathrm{v}_0^2 - 2\mathrm{bh}})/\mathrm{b}$ (with the value in question being $v_0 - \sqrt{\mathrm{v}_0^2 - 2\mathrm{bh}}$). This implies that for all initial parameters $\mathrm{v_0}$, b and h, $\mathrm{v_S^{impact}}$ is a strictly increasing and continuous function of $\mathrm{T_b}$ in the regions where a crash occurs. This function has a minimal value of 0 (which is taken at $\mathrm{T_b} = h/v_0$) and a maximal value of v_0 (at $\mathrm{T_b} = \sqrt{2\mathrm{h}/\mathrm{b}}$ if $h \leq \mathrm{v}_0^2/(2b)$ and at $\mathrm{T_b} = h/v_0 + v_0/(2b)$ if $h > v_0^2/(2b)$). According to Bolzano's theorem about continuous functions on closed intervals, all values between the minimum and maximum are taken at respective values of $\mathrm{T_b}$. In other words, any impact speed between 0 and v_0 is a possible impact speed for the subject vehicle.

It is now possible for each $0 \leq \mathrm{v_S^{test}} \leq \mathrm{v_0}$ to determine the corresponding value of $\mathrm{T_b}$ (i.e. the value which results in $\mathrm{v_S^{impact}} = \mathrm{v_S^{test}}$) and the relative impact speed as follows. If $h \geq v_0^2/(2b)$, then

$$\mathrm{T_b} = ((\mathrm{v_S^{test}})^2 + 2bh)/(2bv_0) \qquad (11),$$

and the relative impact speed R is determined by

$$R = \mathrm{v_S^{test}} \qquad (12).$$

If $h < v_0^2/(2b)$ and $0 \leq \mathrm{v_S^{test}} \leq v_0 - \sqrt{\mathrm{v}_0^2 - 2\mathrm{bh}}$, then $\mathrm{T_b}$ and the relative impact speed are again given by the equations (11) and (12), respectively. On the other hand, if it holds that $h < v_0^2/(2b)$ and $v_0 - \sqrt{\mathrm{v}_0^2 - 2\mathrm{bh}} \leq \mathrm{v_S^{test}} \leq v_0$, then the subject vehicle started braking at time

$$\mathrm{T_b} = \frac{\mathrm{v_S^{test}} - v_0 + \sqrt{(\mathrm{v_S^{test}} - v_0)^2 + 2\mathrm{bh}}}{\boldsymbol{b}} \qquad (13),$$

and the relative impact speed is

$$R = \mathrm{v_S^{test}} - v_0 + \sqrt{(\mathrm{v_S^{test}} - v_0)^2 + 2\mathrm{bh}} \qquad (14).$$

These results show that the relative impact speed may indeed be expressed as a function of the impact speed of the striking vehicle and the initial parameters.

DISCUSSION

The results in the previous section address all relevant aspects of rear-end scenarios with a braking lead vehicle that were considered in the Introduction. Since the results are derived using mathematical computations, they yield definite answers to the underlying questions under the theoretical assumptions that describe the dynamics of the two vehicles. Therefore, it is important to discuss the relevance of these assumptions to real-world rear-end crashes and the possibility of weakening some of the assumptions.

The first limitation of this study is that only the special case of equal initial speeds and decelerations is considered. It would also be important to consider $\mathrm{v_S} = \mathrm{v_T} = \mathrm{v_0}$ and $\mathrm{b_S} < b_\mathrm{T}$ since this case determines whether the crash can be avoided by moderate braking or rather emergency braking is necessary. Even $\mathrm{v_S} = \mathrm{v_T} = \mathrm{v_0}$ and $\mathrm{b_S} > b_\mathrm{T}$ is relevant due to the fact that the vehicle under test is typically a new vehicle and hence may be able to achieve greater deceleration by emergency braking than an average vehicle on the road does.

Furthermore, $v_S \neq v_T$ should also be analysed at various deceleration values. These more general cases will be addressed in future work.

However, even the most general case with fully arbitrary values of the initial speeds and decelerations of the vehicles is rather idealistic. In fact, each one of the assumptions of the underlying model could be criticized for incomplete practical relevance. For example, there will always be some delay in real-world vehicles after the application of the brakes until the deceleration associated to the braking is attained. Therefore, the model could be made more realistic by considering a ramp-up period; for instance, it may be assumed that the deceleration increases from 0 to the desired level linearly with a certain slope.

It is also reasonable to allow the subject vehicle to increase its deceleration at a time point after T_b. This is especially relevant for those values of T_b that are too large for avoiding the crash. Another generalization of the current model is to allow the target vehicle to stop braking at a certain time or even allow re-acceleration. Note that both of these changes would enable collision avoidance at larger values of T_b than those in the present study.

Nevertheless, the current simple model is fully solvable. Although this model (as well as its proposed generalizations) may easily be simulated, having exact solutions has numerous advanages; for example, the formulas derived in the previous section instantly provide solutions that are perfectly accurate and do not suffer from rounding errors. Although the simulations are very fast themselves, using exact solutions can save substantial time in those applications that would otherwise require multiple simulations. Also, exact solutions may be used in other studies; see e.g. [4] where mathematical formulas were applied in a creative way to evaluate potential outcomes of rear-end crashes had the involved vehicles been equipped with certain pre-crash systems. Therefore, depending on the desired application, the theoretical assumptions in this study may be a reasonable compromise in order to ensure solvability. Finally, solutions of the current simple model may pave the way for finding exact solutions in more advanced models.

CONCLUSIONS

In this paper, a mathematical analysis of rear-end scenarios with a braking lead vehicle under simplifying theoretical assumptions is presented. In particular, it is shown that the relative impact speed may be expressed as a function of the impact speed of the striking vehicle, and it is determined how early the striking vehicle needs to start braking in order to avoid the crash. The results are obtained by mathematically rigorous methods hence their only limitations are the underlying theoretical assumptions regarding the dynamics of the vehicles. However, these assumptions capture only a fraction of the complexity of real-world rear-end scenarios; therefore, the results presented here may be used as a first approximation, as well as a stepping stone for the analysis of more advanced models.

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