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Analysis of 3D Rigid Body Motion Using the Nine Accelerometer Array System

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This paper has not been screened for accuracy nor refereed by any body of scientific peers and should not be referenced in the open literature.

ABSTRACT

The nine accelerometer array sensor package is used extensively in injury biomechanics research to obtain the rotational acceleration time histories of a rigid body. It has been shown in the past to remain computationally stable while the alternative, the six accelerometer array, becomes unstable in the presence of small inaccuracies in the individually measured accelerations. The nine accelerometer array process achieves its stability by requiring the measurement of three rotational accelerations, thus eliminating the six accelerometer array's dependency on having knowledge of the rigid body's three rotational velocities at each instant in time. The nine accelerometer array's additional three measurements also provide other important benefits: 1. Identifying whether or not any one of the nine translational acceleration measurements is inconsistent with rigid body motion, 2. If an incorrect acceleration is found, determining what the actual time history should be for that case. This paper presents the derivation of an additional set of constraint equations that a given set of nine linear accelerations must satisfy to be consistent with rigid body motion, demonstrates how an inconsistent acceleration input is discovered, and describes the process by which the true time history of the acceleration is recovered.

INTRODUCTION

The Nine Accelerometer Array Package (NAAP) uses linear accelerations to describe the angular motion of a rigid body. This sensor package typically uses nine accelerometers placed in a 3-2-2-2 configuration to track the motion of a rigid body (Padgaonkar et al., 1975).

The SIMon (Simulated Injury Monitor) finite element head model (Takhounts et. al, 2003) requires three linear and three angular velocities expressed in the global (laboratory) coordinate system as an input applied at the head center of gravity. To obtain these three linear and three angular velocities, the NAAP data is processed within the SIMon interface according to the techniques described in Padgaonkar et al. (1975) and DiMasi (1995).

However, it was noted that for some tests the model kinematics were unreasonable, e.g., rotational displacements of the head that sometimes reached 500 degrees or more. Initially, it was assumed that the transformation process described in DiMasi (1995) was inaccurate and integration error was accumulating over time. The transformation process was reworked several times but the results remained unchanged. Finally, it was realized that the problem may lay in the data itself, i.e. the nine acceleration traces may not be independent from each other and an extra set of rigid body constraint equations may exist.

This makes physical sense because there are nine measures (equations) describing the behavior of a system with six degrees of freedom (unknowns). Therefore, any three acceleration measures in NAAP are redundant and functions of the other six. This was investigated by the logic shown in Figure 1, where nine acceleration traces expressed in the head local coordinate system went through the process described in DiMasi (1995) to obtain three linear and three angular velocities expressed in the global coordinate system.

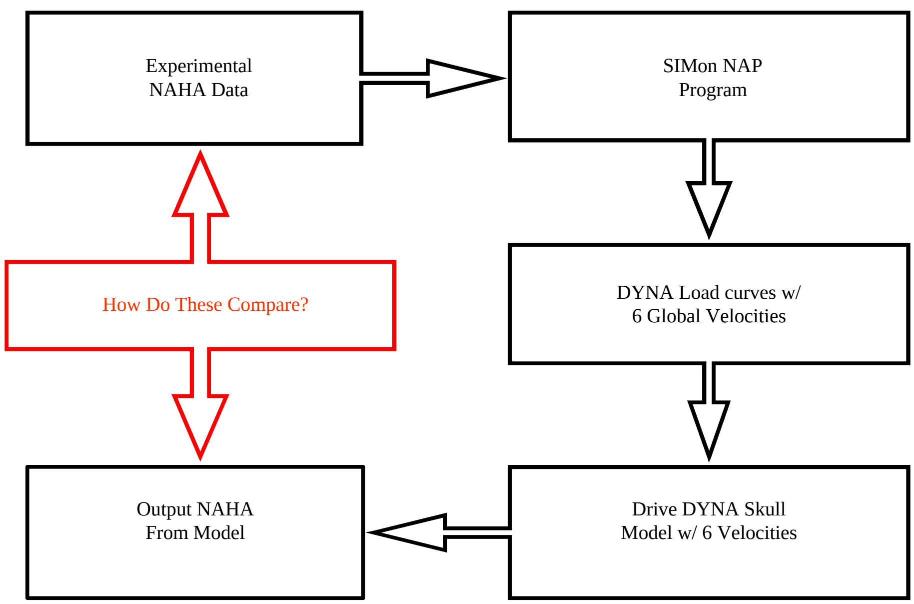

Figure 1: Diagram of the NAAP consistency check process

These six global velocities served as the input to a rigid human skull model (also provided in SIMon and seen in Figure 2), and nine accelerations in the head local coordinate system (from nodes coincident with the location of nine accelerometers) were output from the model and directly compared with the original nine acceleration traces.

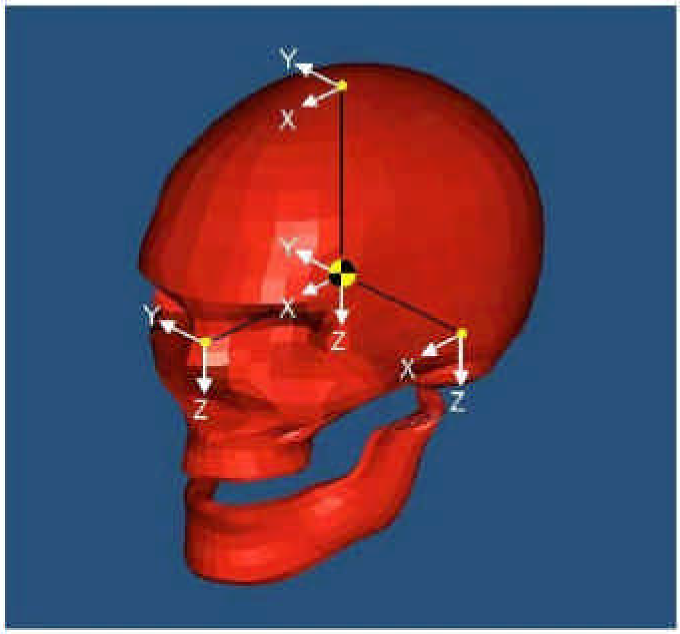

Figure 2: Rigid Human Skull Model used in SIMon FEHM, with accelerometer locations shown

All of the NCAP and side impact NAAP data used for the SIMon FEHM went through this process, and only the tests where the original and computed traces matched well were pronounced as acceptable for further analysis (passed NAAP consistency check).

This paper will examine the equations governing the motion of a 3D rigid body and the processing of NAAP data, develop a method by which nine accelerometer array package data can be checked for consistency, and discuss the process of recovering data from a corrupted channel.

METHODS

Verification of Data Used for Model Testing

One of the well-matched NCAP tests was selected for the following analysis of the dependent (constrained) motion. For this test, one of the traces was clipped in half (Figure 3) and the rest remained unchanged.

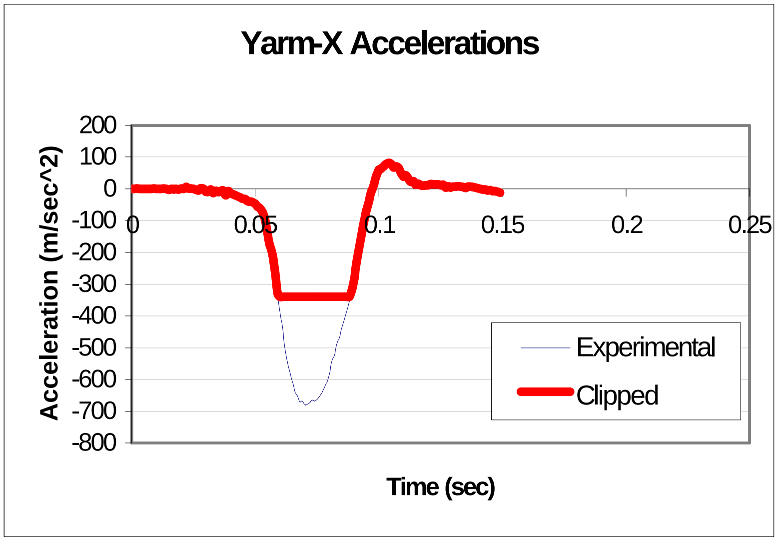

Figure 3: A clipped acceleration trace

If all nine accelerations were independent from each other, then the process of converting the accelerations to six load curves and applying these curves to the FE model should return the same nine accelerations that were input. However, the model yielded accelerations quite different from those input (Figure 4).

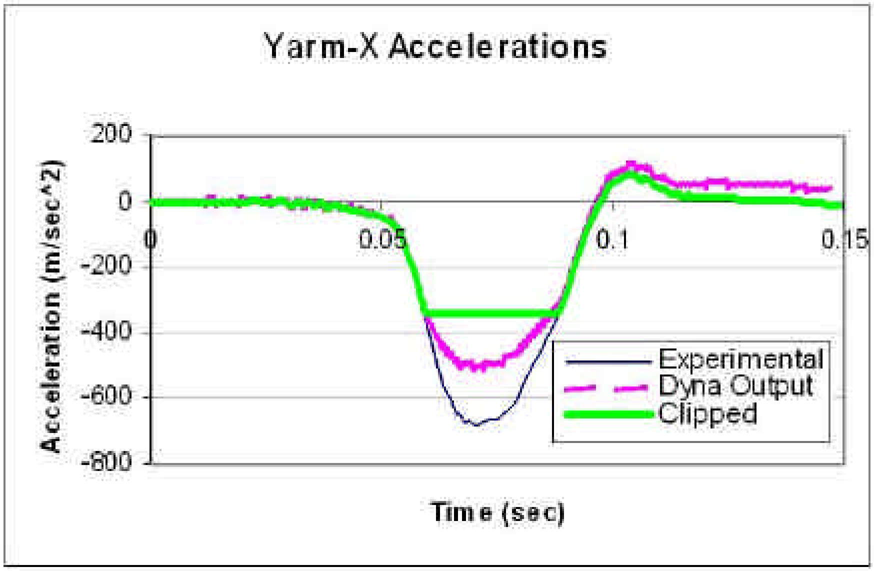

Figure 4: Results from LS-Dyna Run compared to clipped trace (input) and original experimental data

Moreover, these differences were observed not only in the clipped trace, but also in the other channels (Figure 5). This new set of nine accelerations successfully described the motion of a rigid body, but was describing a different motion than the original set.

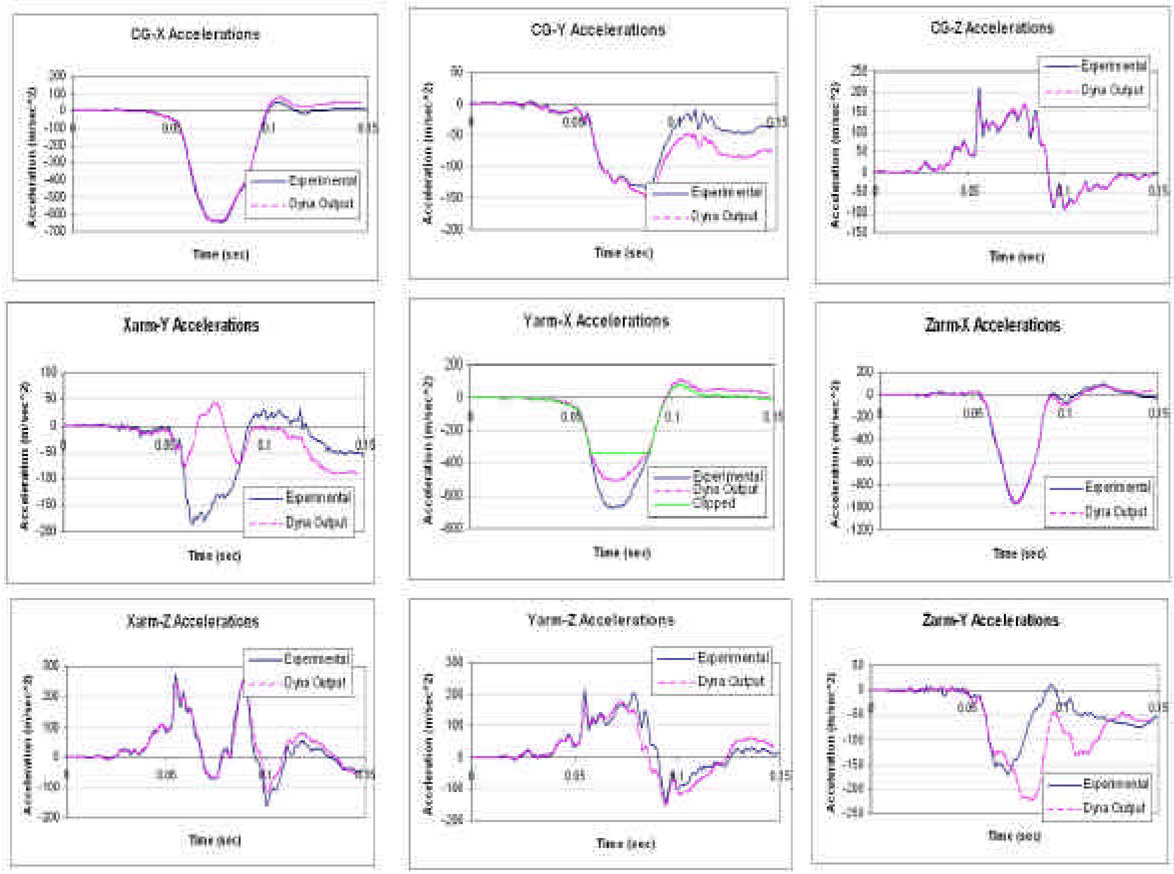

Figure 5: The acceleration trace outputted from the consistency check process compared to the inputted trace indicating that nine accelerations measured on a rigid body are not independent

This simple exercise proved a dependency exists between the nine acceleration responses and confirmed the original hypothesis of the existence of a set of additional constraint equations. A detailed theoretical analysis of measuring rigid body motion using a nine accelerometer array, derivation of the constraint equations, and methods to recover inconsistent acceleration traces are given below.

Theoretical Background for Nine Accelerometer Array Measurements

For a rigid body, the 3D motion of point *B* measured by an observer located at point *A* (Figure 6) is the same as the motion of that body about a fixed point.

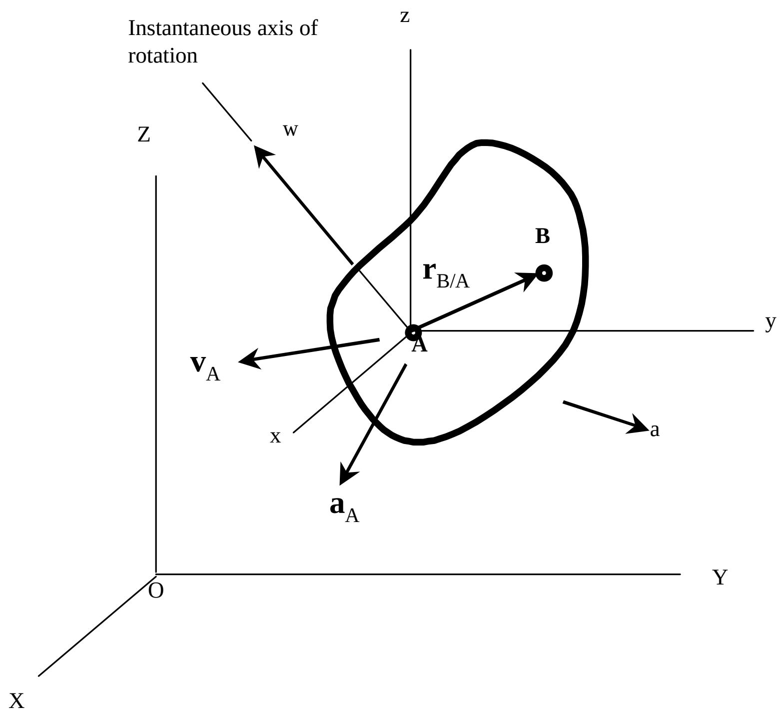

Figure 6: Rigid body subjected to a 3D general motion

This relative motion occurs about the instantaneous center of rotation and is defined by:

$$\boldsymbol{v}_{B/A} = \omega \times \boldsymbol{r}_{B/A} \tag{1}$$

and

$$\boldsymbol{a}_{B/A} = \alpha \times \boldsymbol{r}_{B/A} + \omega \times (\omega \times \boldsymbol{r}_{B/A}), \tag{2}$$

where $\boldsymbol{v}_{B/A}$ and $\boldsymbol{a}_{B/A}$ are the relative velocity and acceleration of point *B* with respect to point *A*. For translating axes, the relative motions are related to absolute motions by $\boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{v}_{B/A}$ and $\boldsymbol{a}_B = \boldsymbol{a}_A + \boldsymbol{a}_{B/A}$, and the absolute velocity and acceleration of point *B* are determined from the following equations:

$$\boldsymbol{v}_B = \boldsymbol{v}_A + \omega \times \boldsymbol{r}_{B/A} \tag{3}$$

$$\boldsymbol{a}_B = \boldsymbol{a}_A + \alpha \times \boldsymbol{r}_{B/A} + \omega \times (\omega \times \boldsymbol{r}_{B/A}). \tag{4}$$

If the vectors are defined as: $\boldsymbol{a}_A = [a_{Ax}, a_{Ay}, a_{Az}]^T$, $\boldsymbol{a}_B = [a_{Bx}, a_{By}, a_{Bz}]^T$, $\alpha = [\alpha_x, \alpha_y, \alpha_z]^T$, $\omega = [\omega_x, \omega_y, \omega_z]^T$, and $\boldsymbol{r}_{B/A} = [r_x, r_y, r_z]^T$, then equation 4 can be rewritten in the component form:

$$\begin{bmatrix} a_{Bx} \\ a_{By} \\ a_{Bz} \end{bmatrix} = \begin{bmatrix} a_{Ax} \\ a_{Ay} \\ a_{Az} \end{bmatrix} + \begin{bmatrix} \alpha_y r_z - \alpha_z r_y \\ \alpha_z r_x - \alpha_x r_z \\ \alpha_x r_y - \alpha_y r_x \end{bmatrix} + \begin{bmatrix} \omega_x\left(\omega_y r_y + \omega_z r_z\right) - r_x\left(\omega_y^2 + \omega_z^2\right) \\ \omega_y\left(\omega_z r_z + \omega_x r_x\right) - r_y\left(\omega_z^2 + \omega_x^2\right) \\ \omega_z\left(\omega_x r_x + \omega_y r_y\right) - r_z\left(\omega_x^2 + \omega_y^2\right) \end{bmatrix} \tag{5}$$

Consider now a nine accelerometer array system, arranged within a rigid body in the manner shown in Figure 7.

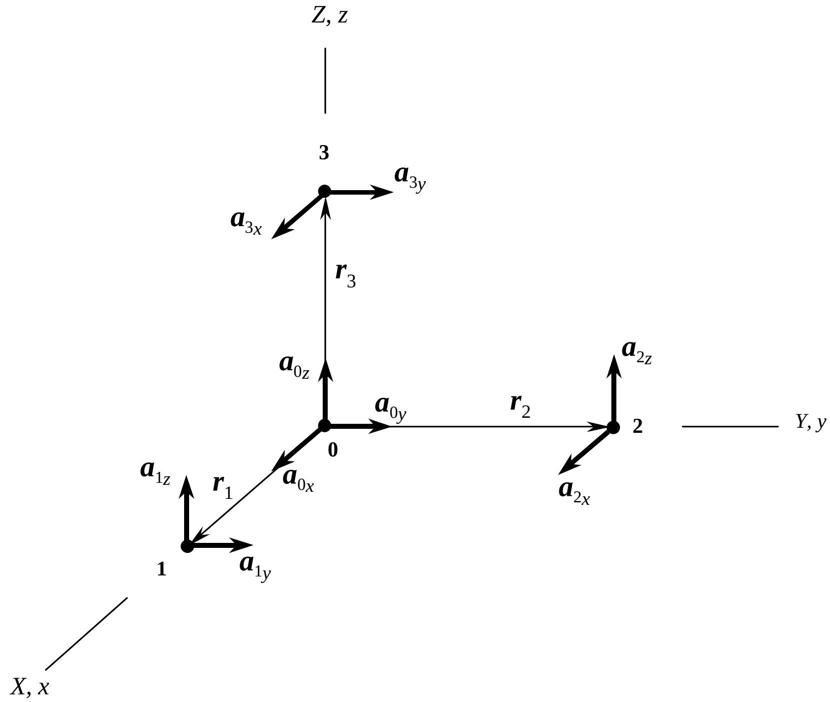

Figure 7: Nine accelerometer array arrangement.

Then, the accelerations of points 1, 2, and 3 with respect to point 0 can be expressed using equations 5. For points 1 and 0 substitute 1 for B, 0 for A, $\boldsymbol{r}_{B/A} = \boldsymbol{r}_1 = [r_x, 0, 0]^T$, where r_x is the distance between points 1 and 0:

$$\begin{bmatrix} - \\ a_{1y} \\ a_{1z} \end{bmatrix} = \begin{bmatrix} - \\ a_{0y} \\ a_{0z} \end{bmatrix} + \begin{bmatrix} - \\ \alpha_z r_x \\ -\alpha_y r_x \end{bmatrix} + \begin{bmatrix} - \\ \omega_y \omega_x r_x \\ \omega_z \omega_x r_x \end{bmatrix}. \qquad (6)$$

Similarly, for points 2 and 0, substitute 2 for B, 0 for A, $\boldsymbol{r}_{B/A} = \boldsymbol{r}_2 = [0, r_y, 0]^T$, where r_y is the distance between points 2 and 0:

$$\begin{bmatrix} a_{2x} \\ - \\ a_{2z} \end{bmatrix} = \begin{bmatrix} a_{0x} \\ - \\ a_{0z} \end{bmatrix} + \begin{bmatrix} -\alpha_z r_y \\ - \\ \alpha_x r_y \end{bmatrix} + \begin{bmatrix} \omega_x \omega_y r_y \\ - \\ \omega_z \omega_y r_y \end{bmatrix}. \qquad (7)$$

For points 3 and 0, substitute 3 for B, 0 for A, $\boldsymbol{r}_{B/A} = \boldsymbol{r}_3 = [0, 0, r_z]^T$, where r_z is the distance between points 3 and 0:

$$\begin{bmatrix} a_{3x} \\ a_{3y} \\ - \end{bmatrix} = \begin{bmatrix} a_{0x} \\ a_{0y} \\ - \end{bmatrix} + \begin{bmatrix} \alpha_y r_z \\ -\alpha_x r_z \\ - \end{bmatrix} + \begin{bmatrix} \omega_x \omega_z r_z \\ \omega_y \omega_z r_z \\ - \end{bmatrix}. \qquad (8)$$

From equations 7 and 8, α_x is found to be:

$$\begin{cases} \alpha_x = \dfrac{a_{2z} - a_{0z}}{r_y} - \omega_y \omega_z \\ \alpha_x = -\dfrac{a_{3y} - a_{0y}}{r_z} + \omega_y \omega_z \end{cases} \tag{9}$$

Similarly, α_y is found from equations 6 and 8:

$$\begin{cases} \alpha_y = -\dfrac{a_{1z} - a_{0z}}{r_x} + \omega_z \omega_x \\ \alpha_y = \dfrac{a_{3x} - a_{0x}}{r_z} - \omega_z \omega_x \end{cases} \tag{10}$$

and α_z is found from equations 6 and 7:

$$\begin{cases} \alpha_z = \dfrac{a_{1y} - a_{0y}}{r_x} - \omega_x \omega_y \\ \alpha_z = -\dfrac{a_{2x} - a_{0x}}{r_y} + \omega_x \omega_y \end{cases} \tag{11}$$

By adding the paired equations 9, 10, and 11, the three angular accelerations α_x, α_y, and α_z can be expressed as functions of combinations of the nine linear accelerations and accelerometer arm lengths, i.e.:

$$\begin{cases} \alpha_x = \dfrac{a_{2z} - a_{0z}}{2r_y} - \dfrac{a_{3y} - a_{0y}}{2r_z} \\ \alpha_y = \dfrac{a_{3x} - a_{0x}}{2r_z} - \dfrac{a_{1z} - a_{0z}}{2r_x} \\ \alpha_z = \dfrac{a_{1y} - a_{0y}}{2r_x} - \dfrac{a_{2x} - a_{0x}}{2r_y} \end{cases} \tag{12}$$

Equations 12 are given in Padgaonkar et al. (1975), and currently serve as the basis for derivation of angular motion of a rigid body from a set of nine linear accelerations. Linear accelerations, as shown in equations 12, are usually functions of time. Therefore, the angular velocities of the rigid body could be obtained by simply integrating equations 12 with respect to time:

$$\begin{cases} \omega_x(t) = \int_0^t \alpha_x(t)\,dt \\ \omega_y(t) = \int_0^t \alpha_y(t)\,dt \\ \omega_z(t) = \int_0^t \alpha_z(t)\,dt \end{cases} \tag{13}$$

However, by subtracting the paired equations 9, 10, and 11 from each other, another set of equations for linking the angular motion of the rigid body can be found:

$$\begin{cases} \omega_y \omega_z = \dfrac{a_{2z} - a_{0z}}{2r_y} + \dfrac{a_{3y} - a_{0y}}{2r_z} \\ \omega_z \omega_x = \dfrac{a_{3x} - a_{0x}}{2r_z} + \dfrac{a_{1z} - a_{0z}}{2r_x} \\ \omega_x \omega_y = \dfrac{a_{1y} - a_{0y}}{2r_x} + \dfrac{a_{2x} - a_{0x}}{2r_y} \end{cases} \tag{14}$$

Since a rigid body in space has six independent degrees of freedom and nine measurements were recorded, it is reasonable that three additional equations linking the measurements to the kinematic rigid body descriptors would exist. This also implies, since both sets of equations (12 and 14) require nine inputs to derive six unknowns, that the nine inputs cannot be independent of each other. The following example illustrates this point.

Consider a simple example of pure rotation of a rigid body where:

$$a_{0x}(t) = a_{0y}(t) = a_{0z}(t) = 0, \tag{15}$$

and the remaining six accelerations are identical functions of time:

$$a_{1y}(t) = a_{1z}(t) = a_{2x}(t) = a_{2z}(t) = a_{3x}(t) = a_{3y}(t) = t. \tag{16}$$

Then, equations (12) become:

$$\begin{cases} \alpha_x(t) = \dfrac{t}{2r_y} - \dfrac{t}{2r_z} \\ \alpha_y(t) = \dfrac{t}{2r_z} - \dfrac{t}{2r_x} \\ \alpha_z(t) = \dfrac{t}{2r_x} - \dfrac{t}{2r_y} \end{cases} \tag{17}$$

Integrating these equations (17) with respect to time gives the following angular velocities:

$$\begin{cases} \omega_x(t) = \dfrac{t^2}{4r_y} - \dfrac{t^2}{4r_z} \\ \omega_y(t) = \dfrac{t^2}{4r_z} - \dfrac{t^2}{4r_x} \\ \omega_z(t) = \dfrac{t^2}{4r_x} - \dfrac{t^2}{4r_y} \end{cases} \tag{18}$$

Using equations 18, the products of the angular velocities are found to be:

$$\begin{cases} w_y(t)w_z(t) = -\dfrac{t^4(r_x - r_y)(r_x - r_z)}{16r_x^2 r_y r_z} \\ w_z(t)w_x(t) = \dfrac{t^4(r_x - r_y)(r_y - r_z)}{16r_x r_y^2 r_z} \\ w_x(t)w_y(t) = -\dfrac{t^4(r_x - r_z)(r_y - r_z)}{16r_x r_y r_z^2} \end{cases} \tag{19}$$

However, substituting equations 15 and 16 into 14 directly yields another relationship for the products of the angular velocities:

$$\begin{cases} w_y(t)w_z(t) = \dfrac{t}{2r_y} + \dfrac{t}{2r_z} \\ w_z(t)w_x(t) = \dfrac{t}{2r_z} + \dfrac{t}{2r_x} \\ w_x(t)w_y(t) = \dfrac{t}{2r_x} + \dfrac{t}{2r_y} \end{cases} \tag{20}$$

By comparing equations 19 and 20 and noticing that they are not equal, one can conclude that the conditions 12, 13, and 14 cannot be satisfied simultaneously if nine linear accelerations are chosen in the form of 15 and 16. In other words, in order to satisfy equations 12, 13, and 14, the nine linear accelerations cannot be arbitrary and independent functions of time. This statement is true because a rigid body has only 6 degrees of freedom, and, thus, only six independent measures are necessary and sufficient to completely define its 3D kinematics.

Given this conclusion, the next step taken was to see if it was possible to "recover " an erroneous acceleration trace. Suppose it is known that one of the non-CG accelerations $a_{1y}(t)$, $a_{1z}(t)$, $a_{2x}(t)$, $a_{2z}(t)$, $a_{3x}(t)$, or $a_{3y}(t)$ is not measured properly, but not known which one is incorrect. Assume first, that the CG accelerations $a_{0x}(t)$, $a_{0y}(t)$, and $a_{0z}(t)$ are measured properly. The question then becomes to find the improperly measured acceleration trace and correct it, so that it will be consistent with the rigid body constraints.

From the first set of equations (14) the acceleration trace $a_{2z}(t)$ can be expressed as:

$$a_{2z}(t) = 2r_y\left(w_y(t)w_z(t) - \frac{a_{3y}(t) - a_{0y}(t)}{2r_z}\right) + a_{0z}(t) \tag{21}$$

Using equations 13 for $w_y(t)$ and $w_z(t)$ and equations 12 for $a_y(t)$ and $a_z(t)$, and substituting them into the equation above for $a_{2z}(t)$ (21), we get:

$$
\begin{aligned}
a_{2z}(t) &= 2r_y \left\{ \int_t a_y(t)dt \int_t a_z(t)dt - \frac{a_{3y}(t) - a_{0y}(t)}{2r_z} \right\} + a_{0z}(t) \\
&= 2r_y \left\{ \int_t \left[\frac{a_{3x}(t) - a_{0x}(t)}{2r_z} - \frac{a_{1z}(t) - a_{0z}(t)}{2r_x} \right] dt \int_t \left[\frac{a_{1y}(t) - a_{0y}(t)}{2r_x} - \frac{a_{2x}(t) - a_{0x}(t)}{2r_y} \right] dt \right. \\
&\quad \left. - \frac{a_{3y}(t) - a_{0y}(t)}{2r_z} \right\} + a_{0z}(t)
\end{aligned} \tag{22}
$$

In a functional form the above equation can be rewritten:

$$a_{2z}(t) = f\left[a_{0x}(t), a_{0y}(t), a_{0z}(t), a_{1y}(t), a_{1z}(t), a_{2x}(t), a_{3x}(t), a_{3y}(t), r_x, r_y, r_z\right]. \tag{22a}$$

It should be observed that the acceleration trace $a_{2z}(t)$ is a function of all the other eight acceleration traces in the array and the distances r_x, r_y, and r_z. This means that if acceleration trace $a_{2z}(t)$ was measured incorrectly, but all the rest traces were correct, the correct $a_{2z}(t)$ can be calculated using equation 22.

Similarly, for the rest of non-CG accelerations the relationship will be:

$$
\begin{aligned}
a_{1y}(t) &= 2r_x \left\{ \int_t a_x(t)dt \int_t a_y(t)dt - \frac{a_{2x}(t) - a_{0x}(t)}{2r_y} \right\} + a_{0y}(t) \\
&= 2r_x \left\{ \int_t \left[\frac{a_{2z}(t) - a_{0z}(t)}{2r_y} - \frac{a_{3y}(t) - a_{0y}(t)}{2r_z} \right] dt \int_t \left[\frac{a_{3x}(t) - a_{0x}(t)}{2r_z} - \frac{a_{1z}(t) - a_{0z}(t)}{2r_x} \right] dt \right. \\
&\quad \left. - \frac{a_{2x}(t) - a_{0x}(t)}{2r_y} \right\} + a_{0y}(t)
\end{aligned} \tag{23}
$$

$$
\begin{aligned}
a_{1z}(t) &= 2r_x \left\{ \int_t a_z(t)dt \int_t a_x(t)dt - \frac{a_{3x}(t) - a_{0x}(t)}{2r_z} \right\} + a_{0z}(t) \\
&= 2r_x \left\{ \int_t \left[\frac{a_{1y}(t) - a_{0y}(t)}{2r_x} - \frac{a_{2x}(t) - a_{0x}(t)}{2r_y} \right] dt \int_t \left[\frac{a_{2z}(t) - a_{0z}(t)}{2r_y} - \frac{a_{3y}(t) - a_{0y}(t)}{2r_z} \right] dt \right. \\
&\quad \left. - \frac{a_{3x}(t) - a_{0x}(t)}{2r_z} \right\} + a_{0z}(t)
\end{aligned} \tag{24}
$$

$$
\begin{aligned}
a_{2x}(t) &= 2r_y \left\{ \int_t a_x(t)dt \int_t a_y(t)dt - \frac{a_{1y}(t) - a_{0y}(t)}{2r_x} \right\} + a_{0x}(t) \\
&= 2r_y \left\{ \int_t \left[\frac{a_{2z}(t) - a_{0z}(t)}{2r_y} - \frac{a_{3y}(t) - a_{0y}(t)}{2r_z} \right] dt \int_t \left[\frac{a_{3x}(t) - a_{0x}(t)}{2r_z} - \frac{a_{1z}(t) - a_{0z}(t)}{2r_x} \right] dt \right. \\
&\quad \left. - \frac{a_{1y}(t) - a_{0y}(t)}{2r_x} \right\} + a_{0x}(t)
\end{aligned} \tag{25}
$$

$$\begin{aligned}
a_{3x}(t) &= 2r_z\left\{\int_t \alpha_z(t)dt\int_t \alpha_x(t)dt - \frac{a_{1z}(t)-a_{0z}(t)}{2r_x}\right\} + a_{0x}(t)\\
&= 2r_z\left\{\int_t\left[\frac{a_{1y}(t)-a_{0y}(t)}{2r_x} - \frac{a_{2x}(t)-a_{0x}(t)}{2r_y}\right]dt\int_t\left[\frac{a_{2z}(t)-a_{0z}(t)}{2r_y} - \frac{a_{3y}(t)-a_{0y}(t)}{2r_z}\right]dt\right.\\
&\quad\left. - \frac{a_{1z}(t)-a_{0z}(t)}{2r_x}\right\} + a_{0x}(t)
\end{aligned}\tag{26}$$

$$\begin{aligned}
a_{3y}(t) &= 2r_z\left\{\int_t \alpha_y(t)dt\int_t \alpha_z(t)dt - \frac{a_{2z}(t)-a_{0z}(t)}{2r_y}\right\} + a_{0y}(t)\\
&= 2r_z\left\{\int_t\left[\frac{a_{3x}(t)-a_{0x}(t)}{2r_z} - \frac{a_{1z}(t)-a_{0z}(t)}{2r_x}\right]dt\int_t\left[\frac{a_{1y}(t)-a_{0y}(t)}{2r_x} - \frac{a_{2x}(t)-a_{0x}(t)}{2r_y}\right]dt\right.\\
&\quad\left. - \frac{a_{2z}(t)-a_{0z}(t)}{2r_y}\right\} + a_{0y}(t)
\end{aligned}\tag{27}$$

Each one of the six computed acceleration traces (22 through 27) in the nine accelerometer array can be compared with the corresponding originally measured acceleration trace. If one of the computed traces is not coincident with that originally measured, then the originally measured acceleration trace contains an error and should be replaced with the computed one. If more than one of the computed traces is not coincident with the corresponding originally measured traces, then the error could be in one of the CG accelerations [$a_{0x}(t)$, $a_{0y}(t)$, or $a_{0z}(t)$], or may be due to more than one of the acceleration traces (or arm lengths) being measured improperly.

To check for an incorrect trace, first we will derive additional equations for the three (CG) acceleration traces $a_{0x}(t)$, $a_{0y}(t)$, and $a_{0z}(t)$. From the second equation of set 14 and by using relations 12 and 13, $a_{0x}(t)$ can be expressed as:

$$\begin{aligned}
a_{0x}(t) &= -2r_z\left\{\int_t \alpha_z(t)dt\int_t \alpha_x(t)dt - \frac{a_{1z}(t)-a_{0z}(t)}{2r_x}\right\} + a_{3x}(t)\\
&= -2r_z\left\{\int_t\left[\frac{a_{1y}(t)-a_{0y}(t)}{2r_x} - \frac{a_{2x}(t)-a_{0x}(t)}{2r_y}\right]dt\int_t\left[\frac{a_{2z}(t)-a_{0z}(t)}{2r_y} - \frac{a_{3y}(t)-a_{0y}(t)}{2r_z}\right]dt\right.\\
&\quad\left. - \frac{a_{1z}(t)-a_{0z}(t)}{2r_x}\right\} + a_{3x}(t)
\end{aligned}\tag{28a}$$

Similarly, from the third equation of set 14:

$$\begin{aligned}
a_{0x}(t) &= -2r_y\left\{\int_t \alpha_x(t)dt\int_t \alpha_y(t)dt - \frac{a_{1y}(t)-a_{0y}(t)}{2r_x}\right\} + a_{2x}(t)\\
&= -2r_y\left\{\int_t\left[\frac{a_{2z}(t)-a_{0z}(t)}{2r_y} - \frac{a_{3y}(t)-a_{0y}(t)}{2r_z}\right]dt\int_t\left[\frac{a_{3x}(t)-a_{0x}(t)}{2r_z} - \frac{a_{1z}(t)-a_{0z}(t)}{2r_x}\right]dt\right.\\
&\quad\left. - \frac{a_{1y}(t)-a_{0y}(t)}{2r_x}\right\} + a_{2x}(t)
\end{aligned}\tag{28b}$$

If equations 28a and 28b yield the same result, then all nine traces are self-consistent and no further investigation is required. However, if these equations don't yield the same result, then we must try to express the acceleration trace $a_{0x}(t)$ as a function of the remaining eight acceleration traces. One of the possible ways to accomplish this is to subtract equation 28b from 28a and solve the newly obtained equation for the velocity $\int_t a_{0x}(t)dt$:

$$\int_t a_{0x}(t)dt = \frac{r_y r_z}{r_y^2+r_z^2}\left\{-2r_z\int_t\left[\frac{a_{1y}(t)-a_{0y}(t)}{2r_x}-\frac{a_{2x}(t)}{2r_y}\right]dt+2r_y\int_t\left[\frac{a_{3x}(t)}{2r_z}-\frac{a_{1z}(t)-a_{0z}(t)}{2r_x}\right]dt\right.$$
$$\left.+\frac{1}{\int_t\left[\frac{a_{2z}(t)-a_{0z}(t)}{2r_y}-\frac{a_{3y}(t)-a_{0y}(t)}{2r_z}\right]dt}\left[-2r_y\frac{a_{1y}(t)-a_{0y}(t)}{2r_x}+2r_z\frac{a_{1z}(t)-a_{0z}(t)}{2r_x}-a_{2x}(t)+a_{3x}(t)\right]\right\} \quad (29)$$

The equation above allows for the calculation of the acceleration $a_{0x}(t)$ when the other eight accelerations in the nine accelerometer array are known.

Similarly for the acceleration trace $a_{0y}(t)$, we get from the first equation of set 14:

$$a_{0y}(t) = -2r_z\left\{\int_t \alpha_y(t)dt\int_t \alpha_z(t)dt-\frac{a_{2z}(t)-a_{0z}(t)}{2r_y}\right\}+a_{3y}(t)$$
$$= -2r_z\left\{\int_t\left[\frac{a_{3x}(t)-a_{0x}(t)}{2r_z}-\frac{a_{1z}(t)-a_{0z}(t)}{2r_x}\right]dt\int_t\left[\frac{a_{1y}(t)-a_{0y}(t)}{2r_x}-\frac{a_{2x}(t)-a_{0x}(t)}{2r_y}\right]dt\right.$$
$$\left.-\frac{a_{2z}(t)-a_{0z}(t)}{2r_y}\right\}+a_{3y}(t)$$

(30a)

And, by following the previous method, from the third equation of set 14, we have:

$$a_{0y}(t) = -2r_x\left\{\int_t \alpha_x(t)dt\int_t \alpha_y(t)dt-\frac{a_{2x}(t)-a_{0x}(t)}{2r_y}\right\}+a_{1y}(t)$$
$$= -2r_x\left\{\int_t\left[\frac{a_{2z}(t)-a_{0z}(t)}{2r_y}-\frac{a_{3y}(t)-a_{0y}(t)}{2r_z}\right]dt\int_t\left[\frac{a_{3x}(t)-a_{0x}(t)}{2r_z}-\frac{a_{1z}(t)-a_{0z}(t)}{2r_x}\right]dt\right.$$
$$\left.-\frac{a_{2x}(t)-a_{0x}(t)}{2r_y}\right\}+a_{1y}(t)$$

(30b)

By subtracting 30b from 30a, we get:

$$\int_t a_{0y}(t)dt = \frac{r_x r_z}{r_x^2 + r_z^2}\left\{ -2r_x \int_t \left[\frac{a_{2z}(t) - a_{0z}(t)}{2r_y} - \frac{a_{3y}(t)}{2r_z}\right]dt + 2r_z \int_t \left[\frac{a_{1y}(t)}{2r_x} - \frac{a_{2x}(t) - a_{0x}(t)}{2r_y}\right]dt \right.$$
$$\left. + \frac{1}{\int_t \left[\frac{a_{3x}(t) - a_{0x}(t)}{2r_z} - \frac{a_{1z}(t) - a_{0z}(t)}{2r_x}\right]dt}\left[2r_x \frac{a_{2x}(t) - a_{0x}(t)}{2r_y} - 2r_z \frac{a_{2z}(t) - a_{0z}(t)}{2r_y} + a_{1y}(t) - a_{3y}(t)\right]\right\} \quad (31)$$

Again, for the acceleration trace $a_{0z}(t)$, from the first equation of set 14 we have:

$$a_{0z}(t) = -2r_y \left\{\int_t a_y(t)dt \int_t a_z(t)dt - \frac{a_{3y}(t) - a_{0y}(t)}{2r_z}\right\} + a_{2z}(t)$$
$$= -2r_y \left\{\int_t \left[\frac{a_{3x}(t) - a_{0x}(t)}{2r_z} - \frac{a_{1z}(t) - a_{0z}(t)}{2r_x}\right]dt \int_t \left[\frac{a_{1y}(t) - a_{0y}(t)}{2r_x} - \frac{a_{2x}(t) - a_{0x}(t)}{2r_y}\right]dt - \frac{a_{3y}(t) - a_{0y}(t)}{2r_z}\right\} + a_{2z}(t) \quad (32a)$$

and from the second equation of set 14 we have:

$$a_{0z}(t) = -2r_x \left\{\int_t a_z(t)dt \int_t a_x(t)dt - \frac{a_{3x}(t) - a_{0x}(t)}{2r_z}\right\} + a_{1z}(t)$$
$$= -2r_x \left\{\int_t \left[\frac{a_{1y}(t) - a_{0y}(t)}{2r_x} - \frac{a_{2x}(t) - a_{0x}(t)}{2r_y}\right]dt \int_t \left[\frac{a_{2z}(t) - a_{0z}(t)}{2r_y} - \frac{a_{3y}(t) - a_{0y}(t)}{2r_z}\right]dt - \frac{a_{3x}(t) - a_{0x}(t)}{2r_z}\right\} + a_{1z}(t) \quad (32b)$$

By subtracting 32b from 32a, we get:

$$\int_t a_{0z}(t)dt = \frac{r_x r_y}{r_x^2 + r_y^2}\left\{ -2r_y \int_t \left[\frac{a_{3x}(t) - a_{0x}(t)}{2r_z} - \frac{a_{1z}(t)}{2r_x}\right]dt + 2r_x \int_t \left[\frac{a_{2z}(t)}{2r_y} - \frac{a_{3y}(t) - a_{0y}(t)}{2r_z}\right]dt \right.$$
$$\left. + \frac{1}{\int_t \left[\frac{a_{1y}(t) - a_{0y}(t)}{2r_x} - \frac{a_{2x}(t) - a_{0x}(t)}{2r_y}\right]dt}\left[2r_y \frac{a_{3y}(t) - a_{0y}(t)}{2r_z} - 2r_x \frac{a_{3x}(t) - a_{0x}(t)}{2r_z} + a_{2z}(t) - a_{1z}(t)\right]\right\} \quad (33)$$

Therefore, the consistency check process to "recover" a trace can be summarized as follows:

1. Measure nine translational accelerations
2. Compute a_x, a_y, a_z using Padgaonkar equations
3. Compute $w_x w_y$, $w_x w_z$, $w_y w_z$ using rigid body constraint equations
4. Compute each translational acceleration as a function of the remaining 8 accelerations and radius vectors
5. Compare computed and original 9 translational accelerations and find a cumulative error for each trace
6. Substitute the trace with greatest cumulative error with the computed trace
7. Return to step 2, and repeat if needed.

RESULTS

Using a test containing the same clipped trace for the YarmX acceleration as seen in Figure 3, the NAAP consistency check described above was performed and an attempt was made to recover the incorrect trace. In a single iteration, the solution converged to yield the correct trace for the YarmX acceleration, as seen in Figure 8.

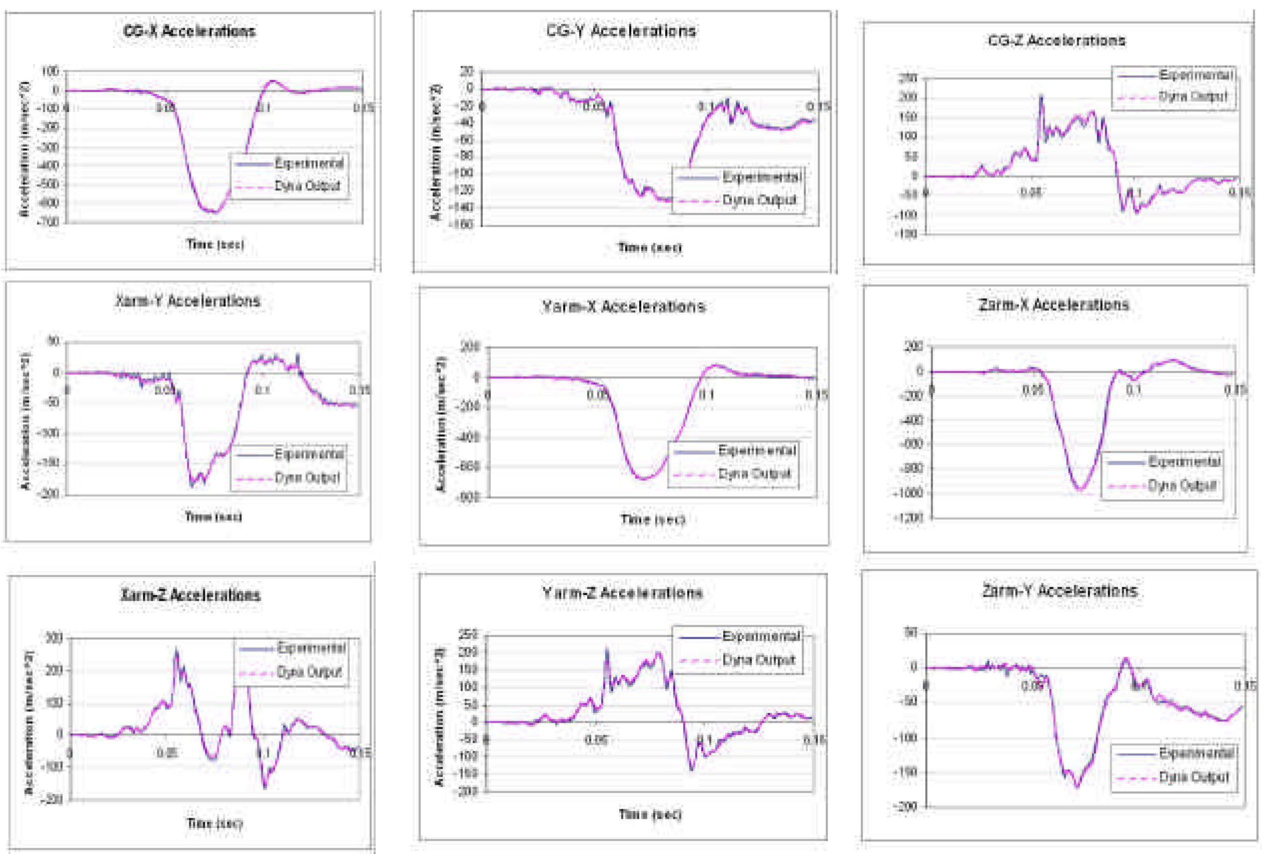

Figure 8: Consistency check results for data set with clipped trace after one iteration

CONCLUSIONS

The process of determining the consistency of the experimental NAAH data has proven to be an invaluable research tool for several reasons. First, it has helped to identify proper acceleration traces and put to rest speculations, such as "sharp, non-physical head acceleration spikes" causing high HIC values. For example, test 4246-01 had a HIC15 of 15,151, a peak angular acceleration equal to 136,910 rad/sec^2, and a peak angular velocity equal to 41.8 rad/s. (Takhounts et al., 2003) These peak values may seem unreasonably high, too noisy, and even non-physical. The consistency check process for this case, however, returned acceleration traces quite similar to the experimentally measured ones, thus confirming the validity of the measured data.

Second, since the process goes through several routines of numerical integration and differentiation, and still returns consistent traces over the whole integration time interval, it shows that a speculation such as "error accumulation due to numerical integration" may have been overused in the past. For example, when an inconsistent set of accelerations was created by clipping one of the traces (Figure 3) in the consistent set and run through the process, some of the returned traces (Figure 5) appeared to have an "error accumulation" problem. This example showed that, although traces may appear to have an error accumulation problem upon first inspection, the nature of the actual problem might be quite different.

Third, the process identifies and "recovers" an inconsistent trace, and thus may salvage an expensive experimental investment. Finally, it allows for a visual assessment of the 3D kinematics of a rigid body that can be compared to the videos of the event.

The work presented here shows the importance of examining the nine accelerometer array package data for consistency before it is processed and used as the basis for other work. The redundancy of the nine accelerometer array sensor package must also be considered when describing the system mathematically. In this case, it has been shown that an additional set of constraint equations exist that must also be satisfied to define the motion of a rigid body. Currently, this process can only be used to recover a trace from a data set where only a single acceleration trace has been corrupted. Future work on this topic will include investigating the possibility of recovering more than one inconsistent trace from the same data set.

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DISCUSSION

PAPER: **Analysis of 3D Rigid Body Motion Using the Nine Accelerometer Array System**

PRESENTER: ***Erik Takhounts, National Highway Traffic Safety Administration***

QUESTION: *Guy Nusholtz, Daimler/Chrysler*
Theoretically, you should be able to correct for up to three accelerations depending on the amount of error that you have in your system. The problem that I see when I do the correction is when you start doing, like, two or three accelerations, then you start to propagate them faster. And it even becomes worse when you start having errors in all 9 of your accelerations, then sometimes your correction schemes kind of backfire. Have you looked at anything beyond the single acceleration with an error?

ANSWER: Yes, I'll start with a simple case – when one trace has an error. Here, I could actually obtain a closed form solution. If I mess up two traces, then I have to derive some kind of an optimization scheme that all nine traces will have to go through. And if three traces will have errors simultaneously, then the routine becomes even more complicated. But, I believe as you said, we have 6 degrees of freedom for a rigid body, and consequently we only need six independent measures.

Q: Theoretically.

A: Theoretically. So, I believe that we can correct up to three traces.

Q: Three. And typically when you get your remaining six, then if there's any error in those other six, boy do they propagate fast. When you go through it, you'll probably run into that.

A: Okay. I'll share my knowledge with you.

Q: Thank you! That would be nice.