SIMULTANEOUS EVALUATION OF MULTIPLE SAFETY FUNCTIONS IN PASSENGER VEHICLES

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ABSTRACT

Up to now quite a few electronic safety functions have been developed in order to successfully increase the safety of passenger in vehicles. Among them are electronic stability control (ESC), brake assists (BAS), lane departure warning systems (LDW) and so on. There exists a short literature on the quantification of the effectiveness of such safety systems on injury outcome or crash severity. As an example the ESC in several studies impressively has been shown to be efficient in avoiding a considerable amount of loss of control or skidding accidents. Nowadays many recently registered vehicles are equipped not only with one but instead with a number of safety functions (so-called safety equipment). The present paper proposes sound statistical methodology in order to investigate the safety benefit of such composite safety equipments (in contrast to a single safety function) in passenger vehicles. It seems obvious that the effectiveness of a specific safety equipment not simply is the additive superposition of the effectiveness of the safety functions of which it consists. For example one may be interested in the additional or incremental effect of an electronic stability control when a brake assist is already on board.

As well we consider secondary safety functions which do not aim at accident avoiding but at injury avoiding or mitigating. Therefore, methodology will be presented to evaluate injury mitigating effectiveness. On this basis it is possible to deal simultaneously with any combination of primary and secondary safety functions.

The developed methodology will be demonstrated on data examples. But the main focus lies on the presentation of methodology.

EFFECTIVENESS OF SINGLE SAFETY FUNCTIONS

For measuring the effectiveness of a safety function it is of critical importance to distinguish between different possible types of effects. In general there are at least four different types of safety function effects existent. These are:

- injury accident avoiding effectiveness
- injury avoiding effectiveness
- injury mitigating effectiveness
- effects of tertiary safety functions

Some safety functions aim at avoiding the accident at all, if this still is not possible anymore, it is tried to prevent any injuries of the involved persons. If this is not possible anymore as well it is tried to reduce the injury outcome as far as possible.

A typical primary safety function aims at all of these first three types of effectiveness, whereas the effectiveness of a typical secondary safety function only consists of the types injury avoiding and injury mitigating effectiveness. In some sense the first three mentioned types of safety function effects are hierarchically ordered. A safety function which aimed at accident avoiding may have some measurable effect on injury avoiding and injury mitigating in cases in which the accident could not be avoided but for example the crash severity has been reduced. A secondary safety function aiming for injury avoiding typically will have some effectiveness on injury mitigating but not on accident avoiding. Thus it becomes clear that a combined evaluation of different safety functions must be able to evaluate injury avoiding and mitigating effectiveness as well as the accident avoiding effectiveness separately.

Afterwards, i.e. after the accident has happened and the injuries are inflicted tertiary safety functions may come into action by calling the ambulance and doing other things to reduce the consequences of the injuries. In this paper we will focus on the first three types of effectiveness.

Even though, we will focus on the accident avoiding effectiveness for the next section.

RELATIVE RISK – ODDS-RATIOS

A reasonable way of measuring the effectiveness of a single safety function "SF" within a certain group of accidental situations "*A*" is to compute relative risks. For example an easy to interpret relative risk is the ratio of the probability that a vehicle with SF on board and active has to suffer an accident that belongs to *A* and the probability of suffering an accident belonging to *A* with SF not active. (cf. Equation 1).

$$\mathrm{RR} = \frac{P(\text{suffering } A \mid \text{SF active})}{P(\text{suffering } A \mid \text{SF not active})} \tag{1}$$

As the relative risk is the ratio of two probabilities it can take any value in the interval $[0, \infty)$. If it equals one, the probability of suffering an accident of type *A* is independent of the safety function SF being active or not. If it is less than one, the safety function has some positive effect, if it is larger than one the effectiveness of SF is in the negative.

With simple algebra and Bayes law for conditional probabilities the equivalence between this relative risk and the following odds-ratio can be shown (Equation 2)

$$\mathrm{RR} = \mathrm{OR} = \frac{\dfrac{P(\text{SF active} \mid A)}{P(\text{SF not active} \mid A)}}{\dfrac{P(\text{SF active} \mid N)}{P(\text{SF not active} \mid N)}} \tag{2}$$

where *N* stands for a category of neutral accidental situations or for an internal control group of vehicle-related accidental situations. It is necessary that the relative risk of suffering an accident classified as *N* depending on SF active or not, must be equal or very close to one. This means that SF more or less has no influence on the probability of suffering an accident within the group *N*. For more detailed information on odds-ratios see Evans (1998), Kullgren et al. (1994), Hautzinger (2003), Kreiss et al. (2005).

It is important to point out the difference between accidents and vehicle-related accidental situations. There may be several vehicles involved in a single accident and the different drivers were most probably confronted with different situations that led to the accident. So safety functions on board of vehicles involved in one and the same accident also are confronted with different situations. Therefore the effectiveness of a safety function on a specific accident highly depends on which of the involved vehicles is considered for the evaluation.

Thus, from now on when referring to a certain type of accident we are always talking about a vehicle-related classification of accidents.

For computing the term in equation (2) the two odds have to be estimated with the equipment-rates within the accident type of interest as it is shown in (Equation 3).

$$\frac{P(\text{SF active} \mid A)}{P(\text{SF not active} \mid A)} \approx \frac{\text{No. of cars with SF active within } A}{\text{No. of cars with SF not active within } A} \tag{3}$$

With this transformation we end up with a term that easily can be computed and is equivalent to the relative risk that easily may be interpreted, so that the effectiveness of SF within *A* can be computed as (Equation 4)

$$\mathit{eff} = 1 - \mathrm{OR} \tag{4}$$

The effectiveness then describes the percentage of avoidable accidents within the category *A*. To describe it more precisely:

Given that some vehicles are involved in critical accidental situations that in case SF is not active would lead to accidents of type *A*, then in $\mathit{eff} \cdot 100\%$ of the cases the accident could be avoided if SF would have been active.

Most safety functions do not have an influence on every accidental situation. In order to quantify the overall effectiveness of a safety function there are two possible approaches. Either the effectiveness within the subgroup of accidental situations which are sensitive to the safety function of interest has to be calculated and extrapolated to the complete set of accidents. Choosing *A* to be the category of "all accidents within the data-base" also leads to such an overall effectiveness. This second approach has the advantage that additional effects of the safety function on other than the selected sensitive accidents are not ignored. On the other hand it may be possible to include unwanted external variables in the overall effectiveness calculation. So for example if drivers of vehicles equipped with ESC typically have a parking assistant on board as well, then the calculated overall effectiveness of ESC would include some effectiveness on parking accidents due to the correlation between ESC and parking assistants.

However the category N of neutral accidents for the second proposal will then be a subset of A. This does not lead to any problems within the calculation.

Classically type A is meant to be the type of accident which is influenced most by SF. But as for some studies a general effectiveness of a certain safety function is of greater interest we would like to point out this possibility of calculating the overall effectiveness of some safety function SF. Even though one has to keep in mind, that this effectiveness will be way less significant than some effectiveness concerning only a certain single accident type.

DATA EXAMPLE (1)

For a better understanding a short data example is presented in this section. Let us assume that our aim is to evaluate the overall effectiveness of each of two safety functions SF1 and SF2. So in both cases the accident type of interest A is any accident within a fictional data-base, where the information of the equipment with the safety function of interest is at hand.

As it is explained in the section above the first step is to characterize a type of neutral accidental situations for each safety function. The group of neutral accidental situations concerning safety function SF1 is labelled N_1 and the other one accordingly N_2.

Within our fictive data-base we find for 4632 vehicles the information whether or not they are equipped with SF1 and for 3354 vehicles whether or not they are equipped with SF2. For the calculation of the odds-ratio we simply need to count the number of cases within the following 2x2-contingency-table.

Table 1
Cross-tabular for calculating the odds-ratio

all vehicles where SF1 equipment is known		type of accident	
		N_1	A
SF1	Yes	56	641
	No	328	3991
Sum		384	4632

With this data the overall effectiveness of SF1 may be calculated with the formulas from equation (2), (3) and (4) which are combined in (Equation 5):

$$eff(A) = 1 - \frac{\dfrac{\text{No. of cars with SF1 within } A}{\text{No. of cars without SF1 within } A}}{\dfrac{\text{No. of cars with SF1 within } N}{\text{No. of cars without SF1 within } N}} \quad (5)$$

So we obtain the effectiveness of SF1 within A, that is the overall effectiveness by (Equation 6)

$$eff(A) = 1 - \frac{641}{3991} \bigg/ \frac{56}{328} \approx 5.93\% \quad (6)$$

Remember at this point, that we calculated the effectiveness of SF1 to any given accident and not only within the types of accidents, where SF1 is supposed to have the largest effectiveness which typically is much higher than this calculated 6%.

Similarly we calculate the effectiveness of SF2 by counting the number of cases from the following 2x2-table.

Table 2
Cross-tabular for calculating the odds-ratio

all vehicles where SF2 equipment is known		type of accident	
		N_2	A
SF2	Yes	30	300
	no	279	3054
sum		309	3354

Using a similar formula as in equation (5) we obtain the overall effectiveness of SF2 in (Equation 7):

$$eff(A) = 1 - \frac{300}{3054} \bigg/ \frac{30}{279} \approx 8.64\% \quad (7)$$

So far we did not take into account any external variables that may have an influence, such as driver's age, surrounding conditions etc. See subsection "logistic regression and […]" for more information on that.

EVALUATING MULTIPLE SAFETY FUNCTIONS

Of course it is of major interest to be not only able to evaluate a single safety function but as well a whole package of multiple safety functions. Odds-ratios offer a well interpretable way of comparing any two (or even more) different safety equipments. In the above formula (Equation 2) the odds-ratio is calculated by somehow comparing the probabilities of suffering a certain accident given a safety function SF is active or not active. The very same approach may be applied if not looking at a single active or not active safety function but instead at some safety configurations. A safety configuration is considered to be a set of different safety functions such as "any car that is equipped with anti-lock braking system, airbags and emergency brake assistant but does not contain ESC". So in our understanding a safety configuration means that certain safety functions are always included, certain safety functions may be excluded and no information on other safety functions are of interest. Let us assume that we intend to compare the effectiveness of some safety configuration SC I with the effectiveness of some safety configuration SC II.

The effectiveness calculated via the odds-ratio then describes the additional gain of safety of SC I compared to equipment SC II. Described in the words as above: Given that some vehicles equipped with SC I are involved in critical accidental situations that would lead to accidents of type *A*, then the question is, how many of these accidents could have been avoided if instead of SC I the safety configuration SC II would have been on board.

Of course SC I and SC II do not have to be a single specific safety configuration but as well may each describe classes of safety configurations. For example SC II may stand for "any safety-configuration that includes the safety function SF1 but excludes SF2" and SC I could be "any safety configuration that includes SF1 as well as SF2". For the sake of an easier interpretation of the results SC I should always include every single safety function that is included in SC II plus some additional safety function(s).

Let us come back to the example above. The corresponding effectiveness (Equation 8)

$$eff = 1 - \mathrm{OR} = 1 - \frac{\dfrac{P(\mathrm{SC\ I} \mid A)}{P(\mathrm{SC\ II} \mid A)}}{\dfrac{P(\mathrm{SC\ I} \mid N)}{P(\mathrm{SC\ II} \mid N)}} \tag{8}$$

then describes the additional gain of SF2 within accident type *A*, given that SF1 is already existent.

Crucial at this point is the neutral accident type *N*. This type of accident has to be independent on **every** safety function that distinguishes SC I from SC II on its own!

For more detailed information on what we call multi-dimensional odds-ratios see Kreiss et al. (2006). Especially we want to stress the fact that it is not possible in general to calculate confidence intervals for odds-ratios of multiple safety functions.

DATA EXAMPLE (2)

Staying at the evaluation of SF1 and SF2 within our fictive data sample, we now want to investigate the interactions of the two safety functions. Recall that the effectiveness of SF1 was 5.93% and the effectiveness of SF2 was 8.64%. When analysing the interactions of the two safety functions we again need a group of neutral types of accidental situations. As we already identified some neutral types of accidents for each safety function, e.g. we obtain one possible group of neutral types of accidents concerning both safety functions by using the intersection of N_1 and N_2. Let us assume we obtain the following 2x2-table.

Table 3
Cross-tabular for calculating the odds-ratio

all vehicles within data-base, where SF1 and SF2 equipment is known		type of accident	
		$N_1 \cap N_2$	A
SF1 & SF2	Both	18	176
	None	239	2612
Sum		257	2788

This leads to the effectiveness of having both SF1 and SF2 instead of none of these as it is calculated in (Equation 9):

$$eff(A) = 1 - \frac{176}{2612} \Big/ \frac{18}{239} \approx 10.53\% \tag{9}$$

So the effectiveness of both safety functions is less than the sum of the two single safety function's effectiveness. Obviously there exists some interaction of the two safety functions.

Now we want to find out more details about these interactions. Table (4) helps us to evaluate the effectiveness of SF1, given that SF2 is already present. As the group of neutral types of accidental situations we may take the whole group N_1 and not only the intersection of N_1 and N_2 because all vehicles of interest are equipped with SF2.

Table 4
Cross-tabular for calculating the odds-ratio

all vehicles, *equipped with SF2*, where SF1 equipment is known		type of accident	
		N_1	A
SF1	yes	18	176
	no	11	108
sum		29	284

So the effectiveness of SF1, given that SF2 is already on board of the vehicle is: (Equation 10)

$$eff(A) = 1 - \frac{176}{108} \Big/ \frac{18}{11} \approx 0.41\% \tag{10}$$

This result may very well be interpreted as given that the vehicle of interest is already equipped with SF2, there is more or less no additional gain of SF1.

Remember again, that we calculated the overall effectiveness for all the accidents within the data-base. So there may exist a small group of accidents, where the effectiveness of SF1 given SF2 is much larger. But then necessarily this group of accidents has to be comparatively small as the overall effectiveness is so close to zero. As well we want to point out, that the number of cases considerably decreased in comparison to Table (1) and (2). This is due to the fact we not only need the knowledge whether a vehicle is equipped with SF2 or not but rather we need cases where the vehicle indeed *is* equipped with SF2. This effect typically occurs when working with real world accident data.

The other way around we now want to calculate the effectiveness of SF2, given that SF1 is already existent. Accordingly to Table (4) as all vehicles of interest are equipped with SF1, N_2 may be considered to be a group of neutral accidental situations. Let us assume that we obtain from our data-base the following table.

Table 5
Cross-tabular for calculating the odds-ratio

all vehicles, *equipped with SF1*, where SF2 equipment is known		type of accident	
		N_2	A
SF2	Yes	18	176
	No	29	328
Sum		47	504

Again the same calculation leads to (Equation 11):

$$eff(A) = 1 - \frac{176}{328} \Big/ \frac{18}{29} \approx 13.55\% \qquad (11)$$

Now the situation is completely different. We obtain a rather high overall effectiveness of SF2, given that SF1 already is existent.

To sum it up: With an effectiveness of 5.9% and 8.6% we observe a moderate effectiveness of SF1 and SF2, considered as single safety functions. The combination of both safety functions shows with an effectiveness of 10.5% that this effectiveness is less than the sum of the single ones, but still larger than the effectiveness of only one of them. By evaluating the interactions we found out that: SF2 is even more effective if SF1 is already existent (13.6%), whereas if SF2 is already existent it does not change much if SF1 is existent as well (0.4%).

Again it is important to point out that for all calculations any external influences were ignored. We may not exclude the possibility that some of the computed results may be explained by some external variable(s). See subsection "logistic regression and […]" for more information on this.

EVALUATING INJURY MITIGATING AND INJURY AVOIDING EFFECTIVENESS

So far odds-ratios have only been used for evaluating the accident avoiding effectiveness, but as pointed out in the beginning of the paper the other types of effectiveness (e.g. injury avoiding and in jury mitigating) are of major interest as well. Typically these types of effectiveness can be quantified on the basis of in-depth accident studies and simulations based on accident-reconstructions. But as we intended to propose a general approach that as far as possible is independent on the type of safety function of interest, we will present such an approach in the following. This approach will only lead to lower and upper bounds of the wanted quantities.

As seen in the section above, odds-ratios are able to evaluate the accident avoiding effectiveness of some safety configurations within a certain type of accident situations called A. To evaluate the effectiveness of a safety function on different severity levels of injuries, A has to be split up in n different subgroups, enumerated according to an increasing severity of the accident. Thus $A1$ stands for all accidents within category A with material damage only, $A2$ may stand for all accidents within category A with slightly injured passengers only, up to An which stands for accidents of category A and with fatally injured passengers. As the described classification of the accidents is vehicle-related, only the occupants of the vehicle of interest are relevant for the classification Ax, $x=1,\ldots,n$, and not for example the most severely injured person involved in an accident.

Assume a crash of type A would lead for a vehicle not equipped with a certain safety function to a classification of Ax. If the vehicle would have been equipped with this safety function the accident outcome would only be of type Ay with some $y < x$. This is a typical case of injury mitigation. Thus, every injury mitigation will appear somehow as accident avoidance within a certain subgroup. Therefore the so far developed methodology can be applied in principle.

If in the above situation $y = 1$ it is not only injury mitigation but injury avoidance, so injury avoidance may be handled similar to injury mitigation. As well it may be possible that an accident without injured persons involved is not reported in the data-base at hand anymore. In this case injury avoidance appears as accident avoidance. This means that if we are able to properly deal with accident avoiding and injury mitigating effectiveness, then we implicitly have covered injury avoiding effectiveness.

We would like to point out that injury mitigating in general does not change the total number of accidents within the data-base. It just leads to a different distribution over the considered categories of accident severity. Those injury mitigations that lead to fact that the accident is not reported to the data-base anymore are interpreted as accident avoidances for this paper.

It is rather clear that from real world accident data one typically cannot decide for a given accident of category A to what extend the severity would be reduced by the safety function of interest. This implies that an accident of category Ax may be mitigated with some probability to Ay for any $y < x$. It seems unrealistic to assume that we can obtain reliable information on these probabilities.

The described mitigating effectiveness of a safety function leads to the following situation. For all accidents with a certain severity that belong to group Ay we have on one hand a reduction of the number of cases because of injury mitigation (due to the safety function) to accident categories with lower severity. On the other hand we have, again due to the injury mitigating effectiveness of the safety function, that accidents from category Ax with $x > y$ are mitigated to category Ay. This leads to an increase of the number of accidents within category Ay. The reduction on one hand and the increase on the other hand are only observable as a superposition and this makes the quantification of the injury mitigating effectiveness of a safety function rather delicate. Just for the most severe accidents which belong to category An we observe a possible reduction of the number of cases, only. This is because we assume that the safety function of interest does never increase the severity of an accident. So when looking at the effectiveness within a certain group Ax one always has to keep this in mind. Ignoring these facts may lead to completely misleading results.

Let us discuss the following two ways to handle this dilemma of quantification of injury mitigating effectiveness.

One idea could be not to calculate the effectiveness within each group Ax but instead within the aggregated groups Ax^+ which are defined as $Ax^+ := Ax \cup \ldots \cup An$. I.e. Ax^+ contains all accidents of type A and with accident severity larger or equal to x. Then every injury mitigation from a group Ax to a group Ay will be observed within each effectiveness of the groups Ax^+ to $A(y+1)^+$. The idea would be to look at the distribution of the observed crude effectiveness over the different groups Ax^+ for $x=1,\ldots n$. The effectiveness within such an Ax^+ can be computed from equation (8) with A replaced by Ax^+. For every calculation of an effectiveness for a subgroup of A such as Ax or Ax^+ also the type of neutral accident has to be classified accordingly and is denoted by Nx or Nx^+.

Using this approach we are able to circumvent the dilemma of quantification of injury mitigating effectiveness by only considering the groups Ax^+ for some x. Recall that it then is not possible to have downshifts to this group from more severe accidents by injury mitigating. Even though following this proposal we are not able to calculate the effectiveness within a group Ax instead of Ax^+.

The other approach to be described quantifies lower and upper bounds for the effectiveness within the group of interest, taking into account possible shiftings from groups of more severe accidents to the accident group of interest. Let us assume that we are interested in comparing the effectiveness of two safety functions SC I and SC II on accidents of type A with a specific severity Ax.

In doing so at first completely ignore possible injury mitigation from more severe accident categories and compute the effectiveness within the group Ax just using the observed number of cases. We have argued above that one has to subtract from the observed number of accidents within category Ax the number of mitigated accidents from more severe accidents to that category in order to obtain the pure effectiveness of the safety configuration on accidents of severity Ax. Ignoring this subtraction would lead to an underestimation of the effectiveness of the safety configuration on Ax and therefore yields a lower bound $eff_{min}(Ax)$ of the effectiveness on Ax. In a second step we will obtain an upper bound $eff_{max}(Ax)$ for this effectiveness. The exact procedure will be described below. Having done this we end up with an interval $[eff_{min}(Ax) , eff_{max}(Ax)]$ which contains the true but unknown effectiveness of the safety configuration on Ax.

The main idea is to estimate how many accidents at most may have been downshifted from more severe accidents to the category Ax, then to subtract this number from the observed number of cases within category Ax and finally calculate on this basis the upper bound $eff_{max}(Ax)$. For the estimation of the number of accidents which at most may have been downshifted, we first need the following abbreviations (Equations 12):

$$z := x+1$$
$$e_z := eff(Az^+)$$
$$eqr_{Az+} := \frac{\text{No. of vehicles with SC I in } Az^+}{\text{No. of vehicles with SC II in } Az^+} \quad (12)$$
$$|Az^+| := \text{No. of vehicles within } Az^+$$
$$|Az_{\mathrm{I}}^+| := \text{No. of vehicles with SC I in } Az^+$$
$$|Ax_{\mathrm{I}}| := \text{No. of vehicles with SC I in } Ax$$

Then the corrected amount of vehicles equipped with SC I in *Ax* which is needed to calculate the upper bound of the effectiveness reads as follows. (Equation 13)

$$|Ax_{\mathrm{I,corr}}| := |Ax_{\mathrm{I}}| - |Az^+| \cdot \frac{eqr_{Az+}}{1+eqr_{Az+}} \cdot \frac{e_z}{1-e_z} \quad (13)$$

which is equivalent to

$$|Ax_{\mathrm{I,corr}}| := |Ax_{\mathrm{I}}| - |Az_{\mathrm{I}}^+| \cdot \frac{e_z}{1-e_z}$$

If now calculating $eff_{max}(Ax)$ using the value $|Ax_{\mathrm{I,corr}}|$ instead of $|Ax_{\mathrm{I}}|$ then one assumes that all accidents that have been avoided due to SC I out of $A(x+1)^+$ have been downshifted to group *Ax*.

It is worth mentioning that the interval $[eff_{min}(Ax), eff_{max}(Ax)]$, which contains the wanted effectiveness of the safety function is by no means a statistical confidence interval which contains the wanted value only up to some probability.

DATA EXAMPLE (3)

To explain the described procedure let us assume we want to evaluate a secondary safety function SF3 within our fictive data-base. We suppose SF3 to be only relevant to frontal impacts. There have been 4940 vehicles identified, where the equipment with SF3 is known. For an evaluation of the effectiveness of this safety function, a class of neutral accidental situation is needed. We chose those vehicles that were hit from behind since our safety function of interest addresses frontal impacts only.

We again want to evaluate the overall effectiveness, so the accident type of interest *A* is interpreted as any accident within the database.

As the safety function of interest is a secondary one, we need to evaluate the injury mitigation effectiveness and the injury avoiding effectiveness. According to the section above a more detailed classification of *A* has to be created.

The existing data (that is accident type *A*) is divided in only two subgroups. *A2* contains every vehicle, where at least one occupant is fatally or severely injured, *A1* contains the remaining accident-involved vehicles.

Table 6.
Cross-tabulars for calculating odds-ratios

all vehicles within data-base		type of accident	
		N	*A*
SF3	no	178	1384
	yes	449	3383
sum		627	4767

vehicles without any severely injured occupant		type of accident	
		N1	*A1*
SF3	no	162	1097
	yes	408	2796
sum		570	3893

vehicles with at least one severely injured occupant		type of accident	
		N2	*A2*
SF3	no	16	287
	yes	41	587
sum		57	874

With this data the effectiveness of each group may be calculated similar to equation (5) as it is shown in (Equation 14):

$$eff(Ax) = 1 - \frac{\dfrac{\text{No. of cars with SF1 within } Ax}{\text{No. of cars without SF1 within } Ax}}{\dfrac{\text{No. of cars with SF1 within } Nx}{\text{No. of cars without SF1 within } Nx}}, \quad (14)$$

for x equal to 1 or 2.

So we obtain the effectiveness within each group as follows (Equation 15).

$$\begin{aligned} eff(A) &= 1 - \frac{3383}{1384} \Big/ \frac{449}{178} \approx 3.10\% \\ eff_{\min}(A1) &= 1 - \frac{2796}{1097} \Big/ \frac{408}{162} \approx -1.20\% \\ eff(A2) &= 1 - \frac{587}{287} \Big/ \frac{41}{16} \approx 20.18\% \end{aligned} \quad (15)$$

As it is explained in the previous section we so far only obtain a lower bound of the effectiveness within group *A1*.

For calculating the upper bound of the effectiveness within group *A1* we need to apply formula (14) and we get (Equation 16)

$$\begin{aligned} |A1_{1,new}| &= |A1_1| - |A2_{\mathrm{I}}^+| \cdot \frac{eff(A2^+)}{1-eff(A2^+)} \\ &= 2796 - 587 \cdot \frac{20.18\%}{1-20.18\%} \\ &\approx 2647.6\,, \end{aligned} \quad (16)$$

which leads to the upper bound of the effectiveness within group *A1* of (Equation 17)

$$eff_{\max}(A1) = 1 - \frac{2647.6}{1097} \Big/ \frac{408}{162} \approx 4.17\% \ . \quad (17)$$

We end up with (Equations 18)

$$\begin{aligned} &eff(A) \approx 3.10\% \\ &eff(A1) \in [-1.20\% \ ; \ 4.17\%] \\ &eff(A2) \approx 20.18\% \end{aligned} \quad (18)$$

How can these results be interpreted?

At the first glance a non-zero overall effectiveness seems strange as we are investigating a secondary safety functions which is supposed not to have an accident avoiding effectiveness. On the other hand it is likely that some cases where injuries were avoided entirely due to the safety function have not been reported in the data base. This is equivalent to an accident avoiding effectiveness.

The effectiveness within the group of vehicles with slightly or not injured occupants (*A1*) lies within the interval [-1.20% ; 4.17%] (cf. equation (19)). We therefore can not exclude a zero-effectiveness within this group. This would mean a similar behaviour of SC I compared to SC II on vehicles involved in accidents with slightly or not injured occupants.

The effectiveness of SC I compared to SC II within the group of accident-involved vehicles with at least severely injured occupants is rather high. Just recall that the calculated effectiveness is the effectiveness within the entire group *A2* and not only within a subpopulation of "sensitive" cases.

To put it all into a nutshell we can state that the main difference of these two safety equipments seems to be an injury mitigating effectiveness that is by far most effective for avoiding severe accidents. So obviously odds-ratios may be used to estimate the injury mitigating effectiveness of a safety equipment.

A last word of caution: It is advisable to compare the different equipment-rates within the various types of neutral accidental situations. It may occur that the calculated effectiveness within some groups of accidental situations results from an increased equipment-rate within the neutral subgroup of accidents instead of a reduction of the equipment-rate within the entire group only. In this case the selection of neutral accidental situations needs to be crosschecked or at least explained.

In our example the equipment-rates within the group of neutral accidental situations differ only slightly (Equation 19)

$$\begin{aligned} eqr_N &= 449/178 \approx 2.522 \\ eqr_{N1} &= 408/162 \approx 2.519 \\ eqr_{N2} &= 41/16 \approx 2.563 \ , \end{aligned} \quad (19)$$

which seems to be quite reasonable.

If we just do a small modification of Table 6 in order to obtain the following Table 7,

Table 7.
Cross-tabulars for calculating odds-ratios

vehicles with at least one severely injured occupant		Type of accident	
		N2	*A2*
SF3	no	**20**	287
	yes	41	587
sum		57	874

we end up with a more or less vanishing effectiveness of $eff(A2) \approx 0.23\%$. But this is only due to the change of the equipment-rate within the group *N2*, which is $eqr_{N2} \approx 2.050$ and significantly less than the rates eqr_N and eqr_{N1}.

The other way around is possible as well: A not reliable classification of the group of neutral accidents may also lead to an observed effectiveness even though only the equipment-rates within the group of neutral accidents differ (so-called pseudo effectiveness). As an example, consider Table 8 which again is obtained by applying only slight modifications compared to Table 6.

Table 8.
Cross-tabulars for calculating odds-ratios

all vehicles within data-base		type of accident	
		N	*A*
SF3	no	178	1384
	yes	**459**	**3527**
sum		637	4911

vehicles without any severely injured occupant		type of accident	
		N1	*A1*
SF3	no	162	1097
	yes	408	2796
sum		570	3893

vehicles with at least one severely injured occupant		type of accident	
		N2	*A2*
SF3	no	16	287
	yes	**51**	**731**
sum		67	1018

In this case we end up with the effectiveness within each group as it is shown in (Equation 20)

$$\begin{aligned} &eff(A) \approx 1.17\% \\ &eff(A1) \in [-1.20\% \ ; \ 5.45\%] \\ &eff(A2) \approx 20.09\% \end{aligned} \tag{20}$$

So the results seem to be comparable to the results obtained from the original example shown in Table (6). But a closer look on the equipment-rates within the group of neutral accidental situations shows clearly a problem concerning the classification of this group (Equation 21):

$$\begin{aligned} &eqr_N = 459/178 \approx 2.579 \\ &eqr_{N1} = 408/162 \approx 2.519 \\ &eqr_{N2} = 51/16 \approx 3.188 \end{aligned} \tag{21}$$

So the observed effectiveness may be due to the high equipment-rate within the group of neutral accidents. When looking at the equipment-rates within the groups *A*, *A1* and *A2* we get (Equation 22)

$$\begin{aligned} &eqr_A = 3527/1384 \approx 2.548 \\ &eqr_{A1} = 2796/1097 \approx 2.549 \\ &eqr_{A2} = 731/287 \approx 2.547 \end{aligned} \tag{22}$$

Summarizing, it seems to be clear that the observed effectiveness of 20% within the group *A2* is due to the high equipment-rate within the group *N2* and nothing else. As a result we may state that if and only if this high equipment-rate is reliable the observed effectiveness is reliable as well.

LOGISTIC REGRESSION AND THE QUANTIFICATION OF THE INFLUENCE OF ADDITIONAL EXTERNAL FACTORS

In drawing conclusions from a statistical analysis one always has to be careful. A causal relationship between two variables always leads to some kind of statistical dependence between these two quantities. The opposite assertion, namely that an existing statistical dependence between two quantities leads to a causal relationship between the corresponding variables, not necessarily is true. The easiest example one may think of is as follows. Assume that one variable Z has a causal relationship to the variables X and Y which are of interest to the investigator. If one consider or observe the variables X and Y only, then there typically will show up some kind of dependence between them. But the true story is that both variables depend on the third one Z. In the context of this paper this could mean that if the driver populations of vehicles equipped and not-equipped with a specific safety equipment are completely different or even disjoint then the observed effectiveness of this safety equipment completely may be due to the difference in the driver population. One easily can think of other examples which in some and even in relevant cases may lead to a significant misinterpretation of the results. In pure statistical theory one therefore usually assumes that the test conditions of the two experiments are completely equal except for the variable of interest. In our context this means that we e.g. assume that all loss of control accidents are almost similar except the equipment with an electronic stability program. Having such an ideal situation at hand, all observed differences in accident outcome between equipped and non-equipped vehicles is due to the electronic stability program for sure. But the above mentioned theoretical assumption is far from being realistic when investigating real world accident data. In reality the equipment of vehicles not only differs up to a single safety function and the driver population rarely is the same for different vehicles. Therefore methodology is needed to deal with this situation.

One simple idea is to create different categories of accidents in which all relevant external variables like driver's age and gender, size of the vehicle, weather conditions at the accident spot, accidental situation etc are as similar as possible. Within every group of such categorized accidents one may compute an odds-ratio as described above for example. The variation of the odds-ratio over the different categories easily may be interpreted as a quantification of the influence of the accident characteristics within a single category. This approach perfectly works if one has sufficient accident data at hand and not too many external variables in mind. If only one of these two hypotheses is not true one ends up with very few cases in each category which leads to non reliable statistical quantities within each category. Even if we only have five external variables in mind for which each of them may take five different values we at least need hundred thousand and more accidents in order to obtain reliable and interpretable results. Thus, even for a rather low number of external variables we are confronted with the so-called curse of dimensionality.

Another possibility in order to quantify the influence of external variables to the accident outcome is given by the statistical concept of logistic regression. A detailed explanation of the concept of logistic regression models may be found in any textbook of categorical data (cf. for example Agresti (1996)). A condensed explanation of the logistic regression approach in the context of accident research can be found for example in Kreiss et al. (2006). Before we start explaining a brief word of caution is in order. Logistic regression is not able to circumvent the above mentioned curse of dimensionality. The truth is that logistic regression is a statistical tool which is able to deal with a moderate and sometimes even high number of external variables by the price of assuming that the influence of the external variables is to some extend easily structured. From a principle point of view logistic regression assumes that the influence of the external variables to a slightly transformed output quantity is just as simple as a linear influence.

Let us describe the essentials of logistic modelling and assume that we have external variables $x_1, x_2, ..., x_d$ which could take values 0 or 1, in case of gender as an example, or could take numbers (like the age of the driver of the vehicle) and so on. One or more of the variables denotes the coding whether a specific safety function in the vehicle is on or off. Then logistic modelling for the probability $P(A \mid x_1, x_2, ..., x_d)$ of having an accident of type *A* given that the external variables take the specific values $x_1, x_2, ..., x_d$ reads as follows (Equation 23)

$$P(A|x_1,...,x_d) = \frac{\exp(\beta_0 + \beta_1 x_1 + ... + \beta_d x_d)}{1 + \exp(\beta_0 + \beta_1 x_1 + ... + \beta_d x_d)}. \quad (23)$$

For the so-called odds this means (Equation 24)

$$\begin{aligned} \text{logit}\, P(A|x_1,...,x_d) &= \ln \frac{P(A|x_1,...,x_d)}{1 - P(A|x_1,...,x_d)} \\ &= \beta_0 + \beta_1 x_1 + ... + \beta_d x_d, \end{aligned} \quad (24)$$

which just indicates the above mentioned linearity assumption of logistic modelling. Routine statistical theory immediately leads us to estimates of the parameters $\beta_1, \beta_2, ..., \beta_d$. The value β_k or equivalently $\exp(\beta_k)$, which is nothing else but an odds-ratio, represents the influence of the external variable number *k* having all other variables $x_i, i \neq k$, under control, i.e. having them similar for all accidents. It should be stressed again that a specific linear model for the influence of the external variables on the odds is assumed. In various situations this in fact may occur as a strong restriction. For example the logistic approach is not able to describe the behaviour of an external variable for which we have for low and high values a strong influence to the accident outcome and only a moderate influence for moderately large values of the external variable. One external variable for which this in fact is true is belt usage and height of the driver. The safety belt is designed for medium sized people and we indeed observe that rather small and rather tall drivers are less well protected by the safety belt than medium sized drivers.

Moreover it should be mentioned again that we really need a class of accidents neutral to the specific external variable we have in mind in order to be able to compute the above mentioned odds-ratio, namely the quantity $\exp(\beta_k)$, since we only observe specific realizations of the external variables only given that an accident what type ever has happened. In case that *A* stands for an arbitrary accident then the mentioned odds-ratio just quantifies the influence of the specific external variable to the overall accident outcome (overall influence of the specific variable). In case that *A* stands for a specific type of accidents (e.g. loss of control accidents or rear-end accidents) then the above described odds-ratio measures the influence of the specific external variable to accidents of the prescribed type only. Of course such an accident type specific influence can be extrapolated to an overall influence just by renormalizing, i.e. multiplying, the accident specific influence coefficient, i.e. the odds-ratio, by the percentage of accidents of type *A*. To be specific:

If the overall effectiveness of a safety equipment SF1 compared to safety equipment SF2 is wanted, specify a neutral type of accident *N* first. Define *A* as “not *N*” and then apply a logistic regression to gain the effectiveness of the safety function within *A*. It has to be kept in mind that cases that do not have either SF1 or SF2 equipment are not to be involved in the calculation for the logistic regression. The overall effectiveness then is (Equation 25)

$$OR = OR(A) \cdot \frac{\text{No. of cases within } A}{\text{No. of cases within } A \text{ or } N} \quad (25)$$

If instead of the odds-ratio one minus the odds-ratio (this is called the effectiveness of the specific external variable) is considered all stated arguments remain valid.

CONCLUSIONS

This paper shows, that the well researched and statistically sound method of odds-ratios is not only able to evaluate the accident avoiding effectiveness of a single safety function but as well may be used to evaluate the interactions of multiple safety functions as well. It even enables us to evaluate the injury avoiding and injury mitigating effectiveness with some limitations.

The crucial point within this kind of evaluation is the classification of a group of neutral types of accidental situations. The whole algorithm stands and falls with the reliability of this group!

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REFERENCES

Agresti, A. (1996). An Introduction to Categorical Data Analysis. Wiley Series in probability and Statistics (New York).

Evans, L. (1998). Antilock brake systems and risk of different types of crashes in traffic. ESV-paper No. 98-S2-O-12, 16th ESV Conference, Windsor (Canada).

Hannawald, L., Kauer, F. (2004). ACEA Equal effectiveness study.

Hautzinger, H. (2003). Measuring the Effect of Primary Safety Devices on Accident Involvement Risk of Passenger Cars – Some Methodological Considerations. SARAC-paper.

Kreiss, J.-P., Schüler, L., Langwieder, K. (2005).
The effectiveness of primary safety features in passenger cars in Germany. ESV-paper No. 05-0145. 19th-ESV-Conference, Washington D.C. (USA).

Kreiss, J.-P., Schüler, L., Zangmeister, T. (2006). Statistical Evaluation of the Effectiveness of Safety Functions in Vehicles based on Real-World Accidents (State-of-the-Art). TRACE-paper.

Kullgren, A., Lie, A., Tingvall, C., (1994). The Effectiveness of ABS in Real Life Accidents. ESV-paper No. 94-S9-O-11. 15th-ESV-Conference, Munich (Germany).

Martin, J.-L., Derrien, Y., Laumon, B. (2003). Estimating Relative Driver Fatality and Injury Risk According to Some Characteristics of Cars and Drivers Using Matched-Pair Multivariate Analysis.
ESV-paper No. 364. 18th-ESV-Conference, Nagoya (Japan).

Otto, S. (2004). Quantifizierung des Einflusses aktiver Sicherheitssysteme auf die Unfallwahrscheinlichkeit und Identifikation von sicherheitsrelevanten Attributen basierend auf Realunfalldaten.
Diplomarbeit, Universität Dortmund.

Page, Y., Cuny, S. (2004). Is ESP effective on French Roads?
1st-International ESAR, Hanover (Germany).

Page, Y., Cuny, S., Foret-Bruno J.-Y. (2005). Are expected and observed effectiveness of emergency brake assist in preventing road injury accidents consistent?
ESV-paper No. 05-0268. 19th-ESV-Conference, Washington D.C. (USA).

Tingvall C., Krafft M., Kullgren A., Lie A. (2003). The Effectiveness of ESP (Electronic Stability Programme) in Reducing Real Life Accidents. ESV-paper 261. 18th-ESV-Conference, Nagoya (Japan).