PERCEPTUAL RISK ESTIMATE (PRE): AN INDEX OF THE LONGITUDINAL RISK ESTIMATE

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Paper Number 11-0121

ABSTRACT

Background: In order to help reduce rear-end collisions, a forward vehicle collision warning system has been developed and deployed. The effectiveness of the system largely depends on how early the warning can be given. However, we also need to consider that too early warning may cause a nuisance because the driver may not feel any avoidance maneuver is necessary at the timing. If the system can alert the driver by detecting the absence of braking at the normal timing based on his/her longitudinal risk estimate, the warning can be acceptable without nuisance. In order to achieve the goal, we aimed to develop an index of the driver's perceptual estimate of longitudinal risk. Method: First, we hypothesized that a driver judges when to brake based on two kinds of perception: kinematic perception to approach a lead vehicle and dynamic perception when the lead vehicle decelerates. Each perception was derived from previous studies of human perception. Then, an index of the longitudinal risk estimate reflecting these kinds of perception was proposed. The index is formulated as "perceptual relative velocity" divided by "perceptual distance." Both elements are corrected from their physical value so as to reflect their perceptual magnitude. The perceptual distance is the exponent of the distance between the subject vehicle and the lead vehicle. The perceptual relative velocity is the velocity difference of these two vehicles that is corrected by the subject vehicle's velocity and their relative acceleration. The hypothesis was tested on test track using two actual vehicles and found that PRE also matches to steering avoidance timing, and as we expected, the parameters were different from the ones for brakes. Finally, PRE was compared with other indices (TTC, THW, Risk Perception (RP), and KdB_c) and it was shown that the PRE is a comprehensive and enhanced model of those indices.

INTRODUCTION

Rear-end collisions are sometimes caused by human error such as distraction and inattention. In order to reduce those accidents, a forward vehicle collision warning system has been developed and deployed. The effectiveness of the system largely depends on how early the warning can be given; however, we also need to consider that a too early warning may cause a nuisance because the driver may not feel any avoidance maneuver is necessary at the timing. If the system can alert the driver by detecting the absence of braking at the normal timing based on his/her longitudinal risk feeling, the warning can be acceptable without nuisance. In order to achieve the goal, we aimed to develop an index of the driver's perceptual estimate of longitudinal risk.

First, we tested two major indices if they match to drivers' brake timing: 1. TTC (Time To Collision, i.e., relative distance divided by relative velocity) is the time to collision if the subject vehicle and the lead vehicle keep constant velocity. 2. KdB_c is an index for the approach and the proximity of the lead vehicle based on a driver's visual input on the retina [1].

As a result, a constant TTC did not match drivers' brake timing at low relative velocity; drivers' brake timing showed larger TTC. Also both the constant TTC and KdB_c estimated the risk lower than drivers' perception when the lead vehicle decelerated, thus they alert later than drivers' brake timing.

In this research, we hypothesized that a driver judges when to brake based on two kinds of perception: kinematic
perception to approach a lead vehicle and dynamic perception when the lead vehicle decelerates.

**KINEMATIC PERCEPTION**

For the kinematic perception, it was hypothesized that a driver’s brake timing was judged by Weber’s Law. It states that the change in a stimulus that will be just noticeable is a constant ratio of the original stimulus. Applying the law to the situation of approaching a lead vehicle, we hypothesized the following relations:

(i) When the original relative distance is small, the subject vehicle’s driver notices approaching a lead vehicle and brakes even if the relative distance becomes a little bit shorter.

(ii) On the other hand, when the original relative distance is large, the driver notices approaching the lead vehicle and brakes if the relative distance changes larger.

In these cases, the ratio of the changed distance to the original distance will be the same. This relationship can be formulated as follows:

\[
\frac{\Delta D}{D} = \frac{V_r}{D} = \text{const.} \quad (\text{Eq.1})
\]

Where,

- \(D\): Original relative distance [m]
- \(\Delta D\): Distance changed in \(\Delta t\) [m]
- \(\Delta t\): Duration of time [s]
- \(V_r\): Relative velocity [m/s]

However, Eq. 1 is known as inverse TTC, and does not match the brake timing as mentioned earlier. It was hypothesized the reason of the discrepancy comes from perceptual errors in distance and relative velocity.

In the distance perception, the relation between actual distance \((D_{real})\) and perceived distance \((D_{percep})\) can be shown as Eq. 2 [2-3]. When the exponent \(n\) is not equal to 1, distance error becomes larger along with the actual distance.

\[
D_{percep} = (D_{real})^n \quad (\text{Eq.2})
\]

In the relative velocity perception, the lead vehicle’s optic flow will be influenced by the subject vehicle velocity. It was hypothesized based on a previous study by Gray et al. [4] that the relative velocity is perceived higher when the subject vehicle velocity becomes higher.

\[
V_{r,percep} = V_{r,real} + aV_{s,real} \quad (\text{Eq.3})
\]

Where,

- \(V_{r,percep}\): Perceived relative velocity [m/s]
- \(V_{r,real}\): Actual relative velocity [m/s]
- \(V_{s,real}\): Actual subject vehicle velocity [m/s]

Based on Eqs. 1-3, we hypothesized brake timing of kinematic perception as follows:

\[
\frac{V_r + aV_s}{D^n} = \text{const.} \quad (\text{Eq.4})
\]

A driver is assumed to brake when the situation satisfies Eq. 4. In order to validate the hypothesis, we measured the brake timing of a driver in various combinations of subject vehicle velocity and relative velocity. An expert test driver was used for the experiment. He was asked to brake when he did not want to approach the lead vehicle anymore.

Figure 1 shows that the TTC and the relative distance when the test driver braked. Smaller distances and TTCs were found in the area of lower subject vehicle velocity (closer to the origin) and larger distances and TTCs in the area of higher velocity (away from the origin). From the figure, it was found that the driver did not brake at a constant TTC but the brake timing differs by the subject vehicle’s velocity and the relative velocity.

**Figure 1. TTC - Distance graph of the brake timing**

Figure 2 shows the brake timing in distance against the relative velocity. The brake timing dots were well matched to the line of Eq. 4 (bold dashed line for high velocity and bold dash-dotted line for low velocity). Here, the parameters of Eq. 4 \((a, n,\) and the constant value of the right hand) were optimized to minimize the modeling error. The optimized parameter values were used for the following analyses.
subject vehicle’s velocity and the relative velocity are the same.

Moreover, if there is a preceding car in front of the lead vehicle (two-car ahead of the subject vehicle) and the preceding car decelerates, the driver of the subject vehicle may press the brake earlier by foreseeing the lead vehicle’s deceleration. We hypothesized from a previous study by Sasaki et al. [5] that such “foreseen” deceleration would affect the brake timing in addition to the lead vehicle’s deceleration.

\[ V_{r_{Bon}} = V_{r_{percep}} + RT(A_{p_{real}} + Af) \quad Eq.5 \]

Where,

- \( V_{r_{Bon}} \): Foreseen relative velocity when the driver pressed brake [m/s]
- \( V_{r_{percep}} \): Perceived relative velocity [m/s]
- \( RT \): Reaction time from releasing gas pedal to braking [s]
- \( A_{p_{real}} \): Deceleration of the lead vehicle [m/s²]
- \( Af \): Foreseen deceleration of the lead vehicle [m/s²]

We confirmed Eq. 5 by measuring the brake timing on a test track where drivers followed the lead vehicle that randomly and frequently decelerated and accelerated between 30 to 100 km/h.

Figure 4 shows the result. The horizontal axis shows the left-hand value of Eq. 5 and the vertical axis shows the foreseen relative velocity when the driver braked. Here, \( V_{r_{Bon}} \) was the relative velocity when the drivers braked, \( V_{r_{percep}} \) was the relative velocity when they released the gas pedal, and \( A_{p_{real}} \) was the deceleration of the lead vehicle at the timing. The outlined diamond plots show the brake timing if \( Af \)s are equal to zero, and the filled diamond plots show the brake timing where \( Af \)s were estimated to satisfy Eq. 5, thus the plots lie on the diagonal line by definition.

The average of the estimated \( Af \)s was rather small: 0.13 m/s² (SD = 0.22 m/s²). This small value may be due to the experiment situation where there was no other car than the subject vehicle and the lead vehicle, and drivers knew the experiment setting (the lead vehicle frequently accelerated and decelerated). The effect of \( Af \) should be examined in the real traffic environment.
PERCEPTUAL RISK ESTIMATE

Based on Eqs. 1 to 5 in the previous sections, the following Eq. 6 can be derived to model the brake timing with both kinematic and dynamic perception:

\[ \frac{V_r + \alpha V_s + RT(A_p + A_f)}{D^*} = \text{PRE} \quad \text{Eq.6} \]

We confirmed Eq. 6 on a test track with three drivers. It was found that the drivers' brake timings were well matched to the hypothesis; they braked when the ratio of the numerator and the denominator of Eq. 6 was at a certain value for various combinations of the subject vehicle’s velocity, the relative velocity, and the lead vehicle deceleration (Figure 5). Thus, Eq. 6 was confirmed to be able to show drivers’ brake timing.

We examine the meaning of Eq. 6. The numerator can be interpreted as “perceptual relative velocity” that includes foreseen lead vehicle movement. The denominator is interpreted as “perceptual distance,” which is corrected from their physical value so as to reflect their perceptual magnitude. It can be thought that a driver brakes when the ratio of the perceptual relative velocity to the perceptual distance becomes a certain threshold. Therefore, the value of Eq. 6 can be thought of as the driver’s longitudinal risk estimate. We call the value of Eq. 6 Perceptual Risk Estimate (PRE). It can also be thought that the inverse of Eq. 6 is perceptual TTC, where a driver brakes when the perceptual TTC reaches a certain value. The parameter values of Eq. 6 can be calculated by optimization of measured brake timings.

Since PRE can describe perceptual longitudinal risk, it is natural to consider that the index also can express the timing when a driver starts steering avoidance maneuver, although the timing, i.e., the parameters of PRE, may be different from those of braking. We again tested on the test track and found out that PRE can be used for predicting steering avoidance timing, and as we expected, the parameters were found different from the ones for brakes. The relationships among the parameters in Eq. 6 between braking and steering avoidance are as follows:

\[ \alpha_{\text{brk}} > \alpha_{\text{steer}} \]
\[ RT_{\text{brk}} > RT_{\text{steer}} \]

Figure 6 shows an example of braking and steering avoidance timings in different subject vehicle’s velocities.

![Figure 6. Relation between braking and steering avoidance timings of a driver at different subject vehicle’s velocities.](image)

![Figure 4. Test result to confirm Eq. 5](image)

![Fig. 5 Test result of three drivers’ brake timing to confirm Eq. 6](image)

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DISCUSSION

There are some other indices of collision risk and brake timing proposed so far. We examined such indices as TTC, TTC with relative acceleration, THW, Risk Perception (RP) [6], and KdB_c, and clarified the relation of those indices towards PRE.

1) TTC: The inverse TTC is formulated as \( V_r / D \), which is the simplest form of PRE (\( \alpha = 0 \), \( RT = 0 \), \( n = 1 \)). However, as mentioned in the introduction, the brake timing does not match a constant TTC in low relative velocity.

\[
TTC = \frac{D}{V_r} = \text{const.} \rightarrow \frac{1}{TTC} = \frac{V_r}{D} = \text{const.} \quad \text{Eq.7}
\]

2) TTC with relative acceleration: Considering relative acceleration, TTC can be shown as the upper formula of Eq. 8. The formula can be solved as the lower of Eq. 8, which can be interpreted as TTC adjusted by the relative acceleration. This is a form of PRE (\( \alpha = 0 \), \( RT = \kappa \), \( Ap + Af = Ar \), \( n = 1 \)).

\[
TTC = \frac{-V_r + \sqrt{V_r^2 + 2 \times D \times Ar}}{Ar} = \text{const.}
\]

\[
\rightarrow \frac{V_r + \kappa Ar}{D} = \text{const.} \quad \text{Eq.8}
\]

3) THW (Time Headway): The inverse THW is formulated as \( V_s / D \), which is a form of PRE (\( \alpha = 1 \), \( n = 1 \), \( RT = 0 \), no \( V_r \)). Because THW is constant if the subject vehicle’s velocity is the same, THW does not match the brake timing in various relative velocities.

\[
THW = \frac{D}{V_s} = \text{const.} \rightarrow \frac{1}{THW} = \frac{V_s}{D} = \text{const.} \quad \text{Eq.9}
\]

4) RP (Risk Perception): Kondo et al. proposed Risk Perception that is the linear combination of TTC and THW. This index can be solved as a form of PRE (\( \alpha = \lambda \), \( n = 1 \), \( RT = 0 \)), i.e., TTC adjusted by the subject vehicle’s velocity.

\[
RF = \frac{a}{THW} + \frac{b}{TTC} = \text{const.} \rightarrow \frac{V_r + \lambda V_s}{D} = \text{const.} \quad \text{Eq.10}
\]

5) KdB_c: KdB_c is shown as the right side of the first formula of Eq. 11. It seems complicated, however, it can be solved as a form of PRE (\( \alpha = \mu \), \( RT = 0 \)), which is interpreted as TTC adjusted by the subject vehicle velocity and powered distance.

\[
KdB_c(d) = 10 \times \log \left( \frac{4 \times 10^7 \times V_r - d V_p}{D} \right) \text{sgn}(V_r-dV_p)
\]

\[
= -22.66 \times \log_{10} D + 74.71
\]

\[
\rightarrow \frac{V_r + \mu V_s}{D} = \text{const.} \quad \text{Eq.11}
\]

From Eqs. 7 to 11, the exponent \( n \) and the constant values are different. The most important point is not the specific values of the parameter in each index but the common feature of the indices where longitudinal risk estimate can basically be expressed as the ratio of the relative velocity to the relative distance, adjusted by the subject vehicle’s velocity, deceleration, and distance perception. Thus, we do not focus on showing concrete values of the parameters and the figure axes in this study.

Figure 7 shows the derived relation among the indices. As mentioned above, we found that the PRE is a comprehensive and enhanced model of those indices.

CONCLUSION

In this research we developed an index of perceptual risk estimate (PRE) that estimated the brake timing; the drivers braked when the PRE reached a certain threshold. PRE also can predict the timing when a driver starts a steering avoidance maneuver, although the timing, i.e., the parameters of PRE, was different from those of braking. Finally, PRE was compared with other indices (TTC, THW, Risk Perception (RP)),
and KdB_c) and it was shown that the PRE is a comprehensive and enhanced model of those indices.

REFERENCE


