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# **The VeCor Program**

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16. Abstract This report describes the VeCor program for assessment and correction of nonoblique crash test data. VeCor was developed for the National Highway Traffic Safety Administration, Office of Crashworthiness Research.  VeCor uses kinematic and dynamic relationships and user-supplied target information to find the best combination of calibration, bias, and time shifts for each signal to correct the test data set. The solution is based on a weighted least squares optimization, where the user has control over the weighting of each target relationship.					
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# 1 Introduction

The **VeCor** program was developed to provide an automated tool for the assessment and correction of systematic errors in the NHTSA vehicle crash test data. **VeCor** is used to provide a better understanding of the effectiveness of crash test instrumentation and to allow the data to be used in sensitive modeling applications [4, 5].

**VeCor** considers simultaneously all of the impact direction signals in a test and finds the set of calibration, bias, and time shift errors which provide the best fit to physical and kinematic relationships and to user-supplied supplementary target data. The user can control the solution by the type of target data provided and by overriding the default target confidence ranges.

**VeCor** can be used with very little supplementary information or with a large amount. The more and more accurate information provided the better. Nominally, to uniquely determine the amount of each error in each signal requires three distinct relationships for each signal. Using default tail bias, end of test velocity scatter, and momentum relationships, this can be achieved by supplying relative crush measurements and a target velocity and displacement for each vehicle (from film analysis). When less than this much information is available, **VeCor** generates a reasonable solution by tempering the available information with confidence ranges for the calibration, bias, and time shift values themselves. **VeCor** provides a “report card” showing the best fit solution and will, on request, create signal data files corrected for these errors.

Currently, **VeCor** treats only the nonoblique Vehicle-to-Barrier (VTB) and Vehicle-to-Vehicle (VTV) test configurations. Only the impact axis accelerometers and the total barrier force are considered. These tests provide the most relationships for data assessment. Extensions of the **VeCor** methodology to other test configurations and possibly Biomechanics data are worth considering if the number of relationships is determined to be adequate for meaningful solutions.

**VeCor** is written in portable Fortran 77 and is available for a number of computing platforms. **VeCor** uses the standard NHTSA UDS data file format for signal input and output.

## 2 VeCor Users Guide

The inputs to VeCor are the data files for the raw signals being considered and the desired supplementary information and confidence ranges. The nature of the equations involved in the minimization requires a search for the best time shift solution, and the user can also control the region over which the search is performed and the number of starting points for the search. The solution report card can be directed to a specified file and the generation of corrected data files can be suppressed. VeCor is driven by a formatted input file, as described below.

The VeCor methodology assumes that serious non-systematic errors (signal clips, channel failure, etc.) have been detected and that such data has been excluded from the VeCor inputs.

VeCor provides the following default target relationships:

Relationship	Code	Description
Tail Bias	DS AC DE AC	Acceleration and force should be close to zero before time-zero and at the end of the test
Velocity Scatter	VS	Velocities within each vehicle should be nearly equal at the end of the test
Momentum	MO	Momentum change and impulse should be nearly balanced at separation
Initial Response		Initial response to the impact should occur after time-zero

Any of these except the last can be altered or suppressed by the user.

The user can also specify the following types of target relationships:



Relationship	Code	Description
Acceleration Constant	AC	Targets an acceleration at a specified time
Velocity Constant	VC	Targets a velocity at a specified time
Displacement Constant	DC	Targets a displacement at a specified time
Displacement Relative	DR	Targets a relative displacement between two signals at a specified time
Timing Relative	TR	Orders initial response time of specified signals

In addition there are default relationships for the signal calibration, bias, and time shift “error” values which express the expectation that they are probably within certain limits. These targets allow reasonable solutions to be computed even when the amount of information would be insufficient for a unique solution otherwise.

The user can control the solution by altering the default confidence ranges applied to each relationship (except the timing relationships which are implemented as constraints). These ranges determine the weight applied to the differences between the specified relationship values and the actual data. Time spans for these targets also allow the user to account for uncertainty in the exact time when a stated relationship should hold, and to compensate for signal noise and higher frequency content which can distort the solution. The target contribution is computed as an RMS value for the specified time span.

The relationships should hold within a tolerance of  $\pm$  the confidence range value with 95% certainty. When significant vibrations are present, the vibration amplitude should be contained within the confidence range to avoid excessively weighting the error contribution from the vibration. Generally, this is only a consideration for acceleration relationships.

The time spans should be as long as the relationship is believed to hold to provide VeCor with the most information. While velocity or displacement values estimated from a film analysis are really instantaneous values, the rigid body assumption that is the basis of the velocity scatter relationship is valid after separation has occurred and transient vibrations have dissipated. Relative crush values should be valid over this same time span. Momentum should balance from the same starting time but not necessarily all the way to the end of the signals, since the contribution of unknown drag forces will affect the velocities at late times. For this reason, it is valuable to provide VeCor with a momentum interval based on knowledge of the time of separation.

Time spans for “instantaneous” relationships should also be large enough to account for uncertainty in the measurement time, especially when significant vibrations are present. This must compromise between wanting to average out at least one cycle of high frequency vibration (usually only significant for accelerations) and the rate of change of the quantity being specified.

The time spans should also avoid the extreme tails of the signals where time shifts may cause zero-filled measurements to distort the errors in the relationships. The default time shift limits are  $[-.01, .01]$ .

The equivalent of three distinct relationships per signal are needed, in theory, to uniquely specify the three errors. In practice, the quality of the information available and the presence of other than the three types of errors considered make it worthwhile to provide VeCor with as much information as possible (using appropriate confidence ranges). Nevertheless, data corrected with a minimum of supplementary measurements may be sufficiently close to the true behavior for a wide range of quantitative applications, and the general severity of the errors indicated by the VeCor solution will be indicative of the accuracy of the test as a whole.

The recommended minimum information for accurate VTB and VTV test assessment is the following:

- Pre- and post-test relative displacements (crush) between the accelerometers (including each accelerometer in at least one measurement).
- Target velocity and displacement for each vehicle at roughly 20 msec before the end of the signals (from film analysis).
- Time of separation between the vehicles or the vehicle and barrier.

Combining this information with the default tail bias, velocity scatter, momentum, and timing relationships should give a reliable solution for all errors, except, possibly, the barrier force errors since there is no equivalent of a target or relative displacement. When less than this information is used, the default scaling, bias, and time shift target relationships will tend to distribute the computed error solution evenly over the signals.

When the amount of information is insufficient to uniquely specify the VeCor solution, a zero time shift solution, obtained by limiting the time shift search domain to a value less than the data time step, may be of value. The zero time shift solution is also useful for assessing the sensitivity of the error measure to the time shifts.

Within the time shift domain VeCor performs a combination grid/gradient search. The number of grids per time shift axis can be set to control the computation time *vs.* solution accuracy trade-off. The number of actual search starting points will be on the order of the number of grids per axis raised to the number of signals power (minus the points excluded by the relative timing constraints). Even a few hundred points takes a couple of minutes of CPU time, so no more than two grids per axis are practical for interactive runs with five or more signals. On the other hand, the time shift “space” is reasonably smooth for typical tests and relationships, and exact solutions for simulated test data have been found from a single search point in all trials.

VeCor can be used with signals which lack a segment before time-zero but a degraded solution may result. Extremely noisy data may also degrade the VeCor solution, although longer time spans on the relationships should alleviate this somewhat.

VeCor accepts english or metric units data. The units type of the run is determined by the units type of the data files, which must be consistent. The metric units are:

MILLIMETERS, KILOMETERS/HOUR, G'S, KILOGRAMS, NEWTONS, SECONDS

and the english units are:

INCHES, MILES/HOUR, G'S, POUNDS (mass and force), SECONDS.

## 2.1 VeCor Input File

VeCor is driven by a formatted input file, organized in five and ten column fields. The input file contains the signal file names, specifications for the signal scaling, bias, and time shift errors, specification of the relationship errors to be applied, and information controlling the time shift search grid and the program outputs. Extensive checking is performed on the inputs to avoid undetected errors.

Up to one force signal and 20 acceleration signals can be input to VeCor. For VTB runs the force signal, if any, must be the first signal, specified as signal 0. For VTV runs the Vehicle 1 signals must precede the Vehicle 2 signals. Signals must be numbered sequentially. VeCor checks that all signals are from the same test, the same axis, and have the same time step size.

The user can specify the coordinate system for each signal. The Vehicle 1 coordinate system is considered the '+' system by default, and it is assumed that the barrier force or Vehicle 2 systems are oppositely directed '-' systems (a '+' orientation must be specified for principally positive barrier forces).

The target relationships are specified in successive records by their type code (*e.g.*, AC for Acceleration Constant), the signal or vehicle number, the time span, the expected value, and the confidence range. These specification records can also be used to alter (or suppress) the default tail bias, velocity scatter, and momentum errors. The relative timing constraints are specified by type TR and are a sequence of (up to 21) signals that should experience the initial response in that order. Up to 100 of each target relationship type, including default targets, can be specified (each adjacent pair of signals in a relative timing error counts as one target).

A template and description of the VeCor input file follows.

```

VeCor Input File
ID Code___ Run Title-----
Pulse Information
   J +/-      TSL      RS      RB      RT      S      B      T
   j +/-      tsl      rs      rb      rt      s      b      t
Pulse j file name
. .
. .
. .
Error Information
Type | Spec's | | | |
AC   j    t1   tu   a   ra
DS AC  j    t1   tu   a   ra
DE AC  j    t1   tu   a   ra
VC   j    t1   tu   v   rv
DC   j    t1   tu   d   rd
DR j1j2  t1   tu   d   rd
VS  v#    t1   tu           rv
MO           t1   tu           rv
TR  j1  j2  j3  j4  j5  j6  j7  j8  j9  j10 j11 j12 j13 j14 j15
Solution Information
NGA
Output Information
NO Output file name

```

Columns:

1	1	2	3	4	5	6	7	8							
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 2.1: VeCor input file template.

## VeCor Input File Description

- The first line, `VeCor Input File`, is ignored by the program.
- `ID Code` is a ten character identifying code for the run that appears in all outputs.
- `Run Title` is a seventy character title for the run.
- The `Pulse Information` line and the column header line that follows it are ignored by the program.
- Each signal has a two line entry. The first line should have the information:
  - `j` Signal number (signals must be in order; the force signal must be signal 0; Vehicle 1 signals must precede Vehicle 2).
  - `+/-` Identifies the coordinate system of the signal (Vehicle 1 is + and Vehicle 2 or the force is - by default - specify + for positive barrier forces) (col 10).
  - `ts1` Time shift limit for the signal error solution (sec) (.01 default provides search on [-.01, .01]).
  - `rs` Scaling error confidence range (default=0.5) (scaling is the inverse of calibration).
  - `rb` Bias error confidence range (default=5 g's for an acceleration signal or  $5 \times$  the vehicle weight in force units for a force signal).
  - `rt` Time shift confidence range (sec) (default=0.02).
  - `s` Scaling factor expected value (default=1.0).
  - `b` Bias expected value (default=0.0).
  - `t` Time shift expected value (sec) (default=0.0).

The second line for each signal is the signal's data file name. `VeCor` translates data file name paths to the local platform format (relative paths work best) to allow the same `VeCor` input file to be used on different platforms.

- The `Error Information` line and the following header line are ignored by the program.
- The `Error Information` section contains the information supplied by the user about the data, including the override of any default target relationship specifications. The code for the entry class appears in the first five columns. The two character codes must appear in columns 4 and 5, as shown, and the `DS` and `DE` flags must appear in columns 1 and 2. The other values must also appear within their column ranges (they are shown right-justified in their ranges). The codes categories and any default values are presented below. The actual specification lines can appear in any order in the `VeCor` input file. The default target relationships, `DS AC`, `DE AC`, `VS`, and `MO` need not appear except to alter their specifications. The start and end time of the signals are indicated by  $t_s$  and  $t_e$ .

- AC Acceleration Constant target relationship.
- j Signal number.
  - t1 Time span lower value (sec).
  - tu Time span upper value (sec).
  - a Target acceleration value (g's) (default=0.0).
  - ra Confidence range (g's) (default=2.0+ RMS noise).
- DS AC Default Start Acceleration Constant target relationship.
- j Signal number.
  - t1 Time span lower value (sec) (default= $t_s + \text{ts1}$ ).
  - tu Time span upper value (sec) (default=0.0).
  - a Target acceleration value (g's) (default=0.0).
  - ra Confidence range (g's) (default=2.0+ RMS noise).
- DE AC Default End Acceleration Constant target relationship.
- j Signal number.
  - t1 Time span lower value (sec) (default= $t_e - \text{ts1} - .03$ ).
  - tu Time span upper value (sec) (default= $t_e - \text{ts1}$ ).
  - a Target acceleration value (g's) (default=0.0).
  - ra Confidence range (g's) (default=3.0+ RMS noise).
- VC Velocity Constant target relationship.
- j Signal number.
  - t1 Time span lower value (sec).
  - tu Time span upper value (sec).
  - v Target velocity value (default=0.0).
  - rv Confidence range (default=1.0 mph).
- DC Displacement Constant target relationship.
- j Signal number.
  - t1 Time span lower value (sec).
  - tu Time span upper value (sec).
  - d Target displacement value (default=0.0).
  - rd Confidence range (default=1.0 inch).
- DR Displacement Relative target relationship.
- j1 First signal number (col 7–8).
  - j2 Second signal number (col 9–10).
  - t1 Time span lower value (sec) (default= $t_e - \text{ts1}$ ).
  - tu Time span upper value (sec) (default= $t_e - \text{ts1}$ ).
  - d Target displacement value (in coordinate system of signal j1) (default=0.0).
  - rd Confidence range (default=1.0 inch).

VS Velocity Scatter default target relationship.

- v# Vehicle number (default=1 if only one vehicle).
- t1 Time span lower value (sec) (default= $t_e - \text{ts1} - .02$ ).
- tu Time span upper value (sec) (default= $t_e - \text{ts1}$ ).
- rv Confidence range (default=1.0 mph).

MO Momentum default target relationship.

- t1 Time span lower value (sec) (default= $t_e - \text{ts1} - .02$ ).
- tu Time span upper value (sec) (default= $t_e - \text{ts1}$ ).
- rv Confidence range (default=1.0 mph for the effective system mass).

TR Timing Relative Constraint relationship.

- j1–j21 Signal numbers in order of initial response (15 shown, up to 21 allowed).

- The **Solution Information** line is ignored by the program. The solution information consists of:

NGA Number of Grid points per time shift Axis for the time shift search procedure to start from (default selected for roughly 100 grid points total).

- The **Output Information** line is ignored by the program. The output information consists of:

NO Indicates No corrected data file output requested (default=YES).

Output file name File for VeCor report output (defaults to the screen).

## VeCor Input File Notes

- Changes to `t1` and `tu` for default errors (`DS AC`, `DE AC`, `VS`, or `MO`) are cumulative, and occur if either `t1` or `tu` are nonzero.
- Default errors can be effectively suppressed by specifying a `t1` value of 1.0 (or any positive value) and leaving `tu` blank (a negative time span forces the associated weight to zero).
- `Error Information` entry lines can be “commented out” by placing the letter `S` in column 1.
- Expected values and the `rb` value should be in the coordinate system and units of the corresponding data.
- There is a tendency to use small confidence ranges that can lead to distortion of the solution (in some cases the default ranges are probably too small). It is safer to start with larger ranges and only reduce them as long as the solutions remain well behaved.
- For each acceleration target relationship, an RMS noise estimate is computed for the time span and  $\frac{3}{2}$  times that value is added to the (default or specified) acceleration confidence range to compensate for the effects of the noise.
- The `AC` error can be used for the external force (signal 0); the expected value and range should be given in force units.
- The default `MO` error is suppressed automatically if only one vehicle and no external force are present.
- The vehicle weights used in the momentum target are taken from the corresponding field in the data file.
- Judicious use of the `TR` constraints can greatly reduce the solution time because the time shift space is truncated by the constraints. It is safest to only include `TR` constraints that hold with a very high certainty.
- Only the first (`j1`) signal specified in a `TR` constraint can be the force signal (signal 0); other 0 or blank entries cause any remaining signal numbers to be ignored.
- The timing sequences specified in `TR` errors should be checked for consistency; impossible sequences or sequences which can't be satisfied within the `ts1` time shift limits will cause an “insufficient time shift limits” error to be printed.
- Time shift limits which are insufficient to allow the initial response time to occur after time-zero will cause an “insufficient time shift limits” error to be printed.
- To limit VeCor to a single search originating at the zero time shift point, set the `NGA` input value to 1.
- When the `NGA` value is insufficient for any of the grid points to fall within the `TR` timing constraints, it is possible to generate solutions with large error measures which violate some of the constraints.



### 3 VeCor Formulation

VeCor solves a quadratic minimization problem [1, 2, 3] for the best calibration, bias, and time shift corrections in each signal of a nonoblique crash test. The error minimized is the sum of weighted errors for default and user-specified target relationships.

The relevant quantities for each signal are:

Quantity	Symbol
Measured acceleration	$\tilde{a}(t)$
Measured velocity	$\tilde{v}(t)$
Measured displacement	$\tilde{d}(t)$
Corrected acceleration	$a(t)$
Corrected velocity	$v(t)$
Corrected displacement	$d(t)$
Calibration error	$C$
Scaling error	$S (= 1/c)$
Bias error	$B$
Time shift error	$T$

where the measured velocity and displacement are computed by integrating the measured acceleration. Note that the error values are the errors in the measured data, not the values required to correct the data.

The basic error relationship for an acceleration signal is

$$\tilde{a}(t) = C(a(t - T) + B)$$

which is rearranged and integrated to give

$$\begin{aligned} a(t) &= (\tilde{a}(t + T))S - B \\ v(t) &= (\tilde{v}(t + T) - \tilde{v}(T))S - tB + \tilde{v}(0) \\ d(t) &= (\tilde{d}(t + T) - \tilde{d}(T) - \tilde{v}(T)t)S - \frac{1}{2}t^2B + t\tilde{v}(0) + \tilde{d}(0) \end{aligned}$$

where the initial velocity and displacement values are assumed correct. Conversion factors between the units are ignored here. The relations for force and impulse parallel those for acceleration and velocity and are not presented here.

### 3.1 Target Relationships

The VeCor solution for the signal errors is based on a (typically overdetermined) set of target equations. These target equations are derived from various relationships believed to hold, within some confidence range, for the signals. The confidence ranges determine the relative weighting used for the various target relationships in computing the optimal solution. The solution method is a least squares minimization [1, 2, 3] so a squared error measure and its partial derivatives with respect to the scaling and bias errors are needed (the time shift errors are not amenable to direct quadratic minimization techniques so their gradient terms are not used, as discussed in the next section).

The various target relationships and their errors and partial derivatives are presented below. The signal target relationships are shown in the form of a target at a single instant,  $t$ ; in practice they are treated as sums of errors over small time spans (with the error contribution normalized for the length of the time span to give an RMS contribution), which helps compensate for signal noise. The weighting value,  $\omega$ , is computed as  $1/\delta^2$  for a confidence interval of  $\delta$ . A subscript of  $j$  indicates a value corresponding to signal  $j$ .

#### Acceleration Constant (AC)

This targets a signal acceleration,  $a_j(t)$ , to a specified constant value,  $a^*$ . Default acceleration start and end tail targets (DS AC and DE AC) are generated in this form with  $a^* = 0$  to include the expectation that the acceleration should be near zero before the impact and at the end of the event.

$$\begin{aligned} e_{AC} &= \omega [a_j(t) - a^*]^2 \\ &= \omega [\tilde{a}_j(t + T_j)S_j - B_j - a^*]^2 \end{aligned}$$

$$\begin{aligned} \frac{\partial e_{AC}}{\partial S_j} &= [2\omega (\tilde{a}_j(t + T_j))^2] S_j + [-2\omega \tilde{a}_j(t + T_j)] B_j + [-2\omega \tilde{a}_j(t + T_j)a^*] \\ \frac{\partial e_{AC}}{\partial B_j} &= [-2\omega \tilde{a}_j(t + T_j)] S_j + [2\omega] B_j + [2\omega a^*] \end{aligned}$$

### Velocity Constant (VC)

This targets a signal velocity,  $v_j(t)$ , to a specified constant value,  $v^*$ .

$$\begin{aligned} e_{\text{VC}} &= \omega [v_j(t) - v^*]^2 \\ &= \omega [(\tilde{v}_j(t + T_j) - \tilde{v}_j(T_j)) S_j - tB_j + \tilde{v}_j(0) - v^*]^2 \\ &\equiv \omega [V_S S_j - tB_j + V_C]^2 \end{aligned}$$

$$\begin{aligned} \frac{\partial e_{\text{VC}}}{\partial S_j} &= [2\omega V_S^2] S_j + [-2\omega t V_S] B_j + [2\omega V_S V_C] \\ \frac{\partial e_{\text{VC}}}{\partial B_j} &= [-2\omega t V_S] S_j + [2\omega t^2] B_j + [-2\omega t V_C] \end{aligned}$$

### Displacement Constant (DC)

This targets a signal displacement,  $d_j(t)$ , to a specified constant value,  $d^*$ .

$$\begin{aligned} e_{\text{DC}} &= \omega [d_j(t) - d^*]^2 \\ &= \omega \left[ \left( \tilde{d}_j(t + T_j) - \tilde{d}_j(T_j) - t\tilde{v}_j(T_j) \right) S_j - \frac{1}{2} t^2 B_j + \tilde{d}_j(0) + t\tilde{v}_j(0) - d^* \right]^2 \\ &\equiv \omega [D_S S_j - \frac{1}{2} t^2 B_j + D_C]^2 \end{aligned}$$

$$\begin{aligned} \frac{\partial e_{\text{DC}}}{\partial S_j} &= [2\omega D_S^2] S_j + [-\omega t^2 D_S] B_j + [2\omega D_S D_C] \\ \frac{\partial e_{\text{DC}}}{\partial B_j} &= [-\omega t^2 D_S] S_j + [\frac{1}{2} \omega t^4] B_j + [-\omega t^2 D_C] \end{aligned}$$

### Displacement Relative (DR)

This targets the relative displacement of two signals,  $d_{j_1}(t) - d_{j_2}(t)$ , to a specified constant value,  $d^*$ .

$$\begin{aligned} e_{\text{DR}} &= \omega [d_{j_1}(t) - d_{j_2}(t) - d^*]^2 \\ &= \omega \left[ \left( \tilde{d}_{j_1}(t + T_{j_1}) - \tilde{d}_{j_1}(T_{j_1}) - t\tilde{v}_{j_1}(T_{j_1}) \right) S_{j_1} - \frac{1}{2} t^2 B_{j_1} + \tilde{d}_{j_1}(0) + t\tilde{v}_{j_1}(0) - \right. \\ &\quad \left. \left( \tilde{d}_{j_2}(t + T_{j_2}) - \tilde{d}_{j_2}(T_{j_2}) - t\tilde{v}_{j_2}(T_{j_2}) \right) S_{j_2} + \frac{1}{2} t^2 B_{j_2} - \tilde{d}_{j_2}(0) - t\tilde{v}_{j_2}(0) - d^* \right]^2 \\ &\equiv \omega [D_{S_1} S_{j_1} - \frac{1}{2} t^2 B_{j_1} - D_{S_2} S_{j_2} + \frac{1}{2} t^2 B_{j_2} + D_C]^2 \end{aligned}$$

$$\begin{aligned}
\frac{\partial e_{\text{DR}}}{\partial S_{j_1}} &= [2\omega D_{S_1}^2] S_{j_1} + [-\omega t^2 D_{S_1}] B_{j_1} + [-2\omega D_{S_1} D_{S_2}] S_{j_2} + [\omega t^2 D_{S_1}] B_{j_2} + [2\omega D_{S_1} D_C] \\
\frac{\partial e_{\text{DR}}}{\partial B_{j_1}} &= [-\omega t^2 D_{S_1}] S_{j_1} + [1/2 \omega t^4] B_{j_1} + [\omega t^2 D_{S_2}] S_{j_2} + [-1/2 \omega t^4] B_{j_2} + [-\omega t^2 D_C] \\
\frac{\partial e_{\text{DR}}}{\partial S_{j_2}} &= [-2\omega D_{S_1} D_{S_2}] S_{j_1} + [\omega t^2 D_{S_2}] B_{j_1} + [2\omega D_{S_2}^2] S_{j_2} + [-\omega t^2 D_{S_2}] B_{j_2} + [-2\omega D_{S_2} D_C] \\
\frac{\partial e_{\text{DR}}}{\partial B_{j_2}} &= [\omega t^2 D_{S_1}] S_{j_1} + [-1/2 \omega t^4] B_{j_1} + [-\omega t^2 D_{S_2}] S_{j_2} + [1/2 \omega t^4] B_{j_2} + [\omega t^2 D_C]
\end{aligned}$$

### Velocity Scatter (VS)

This targets all signal velocities in a vehicle,  $v_j(t)$ ,  $j \in J$ , to their average velocity,  $\bar{v}$ . The default velocity scatter target is applied over a short time span near the end of the test data.

$$\begin{aligned}
e_{\text{VS}} &= \omega \sum_{j \in J} \left[ v_j(t) - \frac{1}{|J|} \sum_{k \in J} v_k(t) \right]^2 \\
&= \omega \sum_{j \in J} \left[ (\tilde{v}_j(t + T_j) - \tilde{v}_j(T_j)) S_j - t B_j + \tilde{v}_j(0) - \right. \\
&\quad \left. \frac{1}{|J|} \sum_{k \in J} [(\tilde{v}_k(t + T_k) - \tilde{v}_k(T_k)) S_k - t B_k + \tilde{v}_k(0)] \right]^2 \\
&\equiv \omega \sum_{j \in J} \left[ V_{S_j} S_j - t B_j - \frac{1}{|J|} \sum_{k \in J} [V_{S_k} S_k - t B_k] + V_C \right]^2
\end{aligned}$$

$$\begin{aligned}
\frac{\partial e_{\text{VS}}}{\partial S_j} &= [2\omega V_{S_j}^2] S_j + [-2\omega t V_{S_j}] B_j + \sum_{k \in J} \left[ \left[ -\frac{2}{|J|} \omega V_{S_k} V_{S_j} \right] S_k + \left[ \frac{2}{|J|} \omega t V_{S_j} \right] B_k \right] + \\
&\quad [2\omega V_{S_j} V_C] \\
\frac{\partial e_{\text{VS}}}{\partial B_j} &= [-2\omega t V_{S_j}] S_j + [2\omega t^2] B_j + \sum_{k \in J} \left[ \left[ \frac{2}{|J|} \omega t V_{S_k} \right] S_k + \left[ -\frac{2}{|J|} \omega t^2 \right] B_k \right] + [-2\omega t V_C]
\end{aligned}$$

where these partial derivatives are taken for all  $j \in J$ .

### Momentum (MO)

This targets the net momentum change during the crash to the external impulse, if available. The default momentum target end time is a short time span near the end of the test data, but the time should be set to the time of separation, if known, for greater accuracy. Since

the velocities within a vehicle may not be quite the same at the end time, **VeCor** assigns each signal a weight  $w_j$  equal to the total vehicle weight divided by the number of signals for the vehicle. When an external force is present in a VTB test, **VeCor** scales the force by the vehicle weight to get an equivalent acceleration. In a VTV test the two vehicle momentum changes should cancel each other. Actual and equivalent motions are taken in the global coordinate system. Let  $J$  be the set of indices for the signals.

$$\begin{aligned}
 e_{\text{MO}} &= \omega \left[ \sum_{j \in J} w_j (v_j(t) - v_j(0)) \right]^2 \\
 &= \omega \left[ \sum_{j \in J} w_j ((\tilde{v}_j(t + T_j) - \tilde{v}_j(T_j)) S_j - t B_j) \right]^2 \\
 &\equiv \omega \left[ \sum_{j \in J} w_j (V_{S_j} S_j - t B_j) \right]^2
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial e_{\text{MO}}}{\partial S_j} &= \sum_{k \in J} [[2\omega w_k w_j V_{S_k} V_{S_j}] S_k + [-2\omega w_k w_j t V_{S_j}] B_k] \\
 \frac{\partial e_{\text{MO}}}{\partial B_j} &= \sum_{k \in J} [[-2\omega w_k w_j t V_{S_k}] S_k + [2\omega w_k w_j t^2] B_k]
 \end{aligned}$$

where these partial derivatives are taken for all  $j \in J$ .

## Scaling

This targets a signal scaling value,  $S_j$ , to an expected value,  $s^*$  (1 by default).

$$e_S = \omega [S_j - s^*]$$

$$\frac{\partial e_S}{\partial S_j} = [2\omega] S_j + [-2\omega s^*]$$

## Bias

This targets a signal bias value,  $B_j$ , to an expected value,  $b^*$  (0 by default).

$$e_B = \omega [B_j - b^*]$$

$$\frac{\partial e_B}{\partial B_j} = [2\omega] B_j + [-2\omega b^*]$$

## Time Shift

This targets a signal time shift value,  $T_j$ , to an expected value,  $t^*$  (0 by default).

$$e_T = \omega [T_j - t^*]$$

## 3.2 Solution Method

The VeCor solution is obtained by minimizing the sum of all the default and user-specified error contributions. Because the time shift errors do not appear linearly in the signal-based targets, the solution for the time shifts can not be obtained through direct quadratic minimization algorithms. Instead, VeCor performs a gradient search from a user-specified number of points in the time shift domain, where for each time shift point considered the optimal scaling and bias values are computed by quadratic minimization, solving the linear equations (using Gaussian elimination with row scaling and pivoting) for the zero of the gradient of the total error with respect to the scaling and bias values.

The gradient of the total error with respect to the time shifts is estimated by a single step finite difference approximation (only discrete time shifts in integral multiples of the signal time step are considered). A large penalty is added for violation of the relative timing constraints to keep the search from leaving the feasible region. The search has been successful from a single, central starting point in all trials with simulation data containing known errors.

If more than a “deterministic” amount of information is supplied (generally three relationships per signal specify the three errors), the solution cannot generally satisfy all of the specified relationships exactly. The scaling, bias, and time shift targets serve both to reduce solution sensitivity to inaccurate inputs and to allow solutions when only an indeterminate amount of information is available. In all cases the solution can be significantly affected by the weights (derived from confidence ranges) applied to each error.

# References

- [1] Lawson, C.L. and Hanson, R.J. (1974). *Solving Least Squares Problems*. Prentice–Hall, Inc., Englewood Cliffs, N.J.
- [2] Luenberger, D.G. (1973). *Introduction to Linear and Nonlinear Programming*. Addison–Wesley, Reading, Mass.
- [3] Shapiro, J.F. (1979). *Mathematical Programming: Structures and Algorithms*. Wiley, New York.
- [4] Mentzer, S.G. *et al* (1992). *The SISAME Methodology for Extraction of Optimal Structural Crash Models*. SAE Report No. 920358.
- [5] Mentzer, S.G. (1995). *The SISAME Program: Structural Crash Model Extraction and Simulation*. U.S. Department of Transportation Draft Report.