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COMPUTATION OF RIGID BODY ROTATION IN THREE-DIMENSIONAL SPACE FROM BODY-FIXED ACCELERATION MEASUREMENTS

By

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1. INTRODUCTION

Morris (1) has shown that, in principle, three-dimensional kinematics of a rigid body can be completely determined by making six acceleration field measurements on the body. In practice, the computed angular accelerations based on these measurements are subject to errors due to cross-axis sensitivity and misalignment of the linear accelerometers. Padgaonkar et al (2) showed that angular acceleration could be determined reliably with 9 linear acceleration measurements, using a special configuration of these accelerometers. The body-fixed angular velocity can be computed by direct integration of body fixed angular acceleration. The numerical problem of computation of rigid body rotation in the 3-dimensional space from a body-fixed angular velocity remains to be solved. It is conventional to express three-dimensional finite rotations in terms of Euler angles. However, integration of Euler angular rates involves singularities and computational difficulties. An alternate method, using the orientation vector as a measure of rigid body rotation is developed. If at any time t, the inertial reference frame is rotated about an axis parallel to Θ by an angle equal to the magnitude of Θ, it would be brought into coincidence with the body-fixed frame. Bortz (3) used a differential equation to describe the orientation vector. The method generates an orthogonal transformation matrix and does not require constant updating, resulting in a much smaller rate of growth of error in the computation of finite rotations. For these reasons, the method is found to be suitable for computing the magnitude of the rotation vector Θ, its unit base vector components Θ_i, Θ_j, Θ_k and the corresponding quaternions, from linear accelerometer data.

2. FINITE ROTATIONS FROM FILM DATA

A method has been developed to compute large three-dimensional angular rotation of the head from the high speed movie films taken during the experimental runs. A target in the form of a 7.6 cm cube was mounted on the head and was viewed by 3 movie cameras placed along the three orthogonal axes of the inertial frame. A body-fixed frame embedded in the rotating cube was followed by the cameras, providing a transformation matrix relating the body-fixed frame to the inertial reference frame. The transformation matrix was the normalized using an iterative scheme. Quaternions, rotation vector magnitude and unit rotation vector components were then computed from an almost orthogonal transformation matrix. However, it should be noted that the computation of these quantities for 25 time points of a experimental run required approximately 1000 vanguard film readings from the 3 views. The computation of rigid-body rotation from 3-dimensional film data is a laborious process.

3. COMPARISON OF COMPUTED FINITE ROTATION

The computed finite rotations based on accelerometer data can be compared with those based on film data. Table 1 lists the 4 cadaveric runs for which head motion in three-dimensional space is compared. The cadaver was either restrained by a three-point belt or by a set of clamps which held the upper and lower legs tightly to the impact seat. They were all subjected to -Gx acceleration levels of 5 to 7g. The various kinematics quantities are shown in Figures 1 through 31.

1. Morris, J.R.W., "Accelerometry - A Technique for the Measurement of Human Body Movement", J. of Biomechanics, 1973, Vol. 6, pp. 729-736.

2. Padgaonkar, A.J., Kriegor, K.W. and King, A.I "Measurement of Angular Acceleration of a Rigid Body Using Linear Accelerometers", Journal of Applied Mechanics, Sept. 1975, pp. 552-556.

3. Bortz, J.E., "A New Mathematical Formulation for Strapdown Inertial Navigation", IEEE Trans. on Aerospace and Electronic Systems, 1971, pp. 61-66.

TABLE 1

LIST OF IMPACT EXPERIMENTS

RUN NO.	MODE	CAD. NO.	PEAK ACCELERATION
308	3 pt. Belt Frontal	3788 M	5.5
309	Clamped Frontal	3788 M	5.0
329	3 pt. Belt Frontal	3846 F	7.0
332	Clamped Frontal	3797 F	6.0

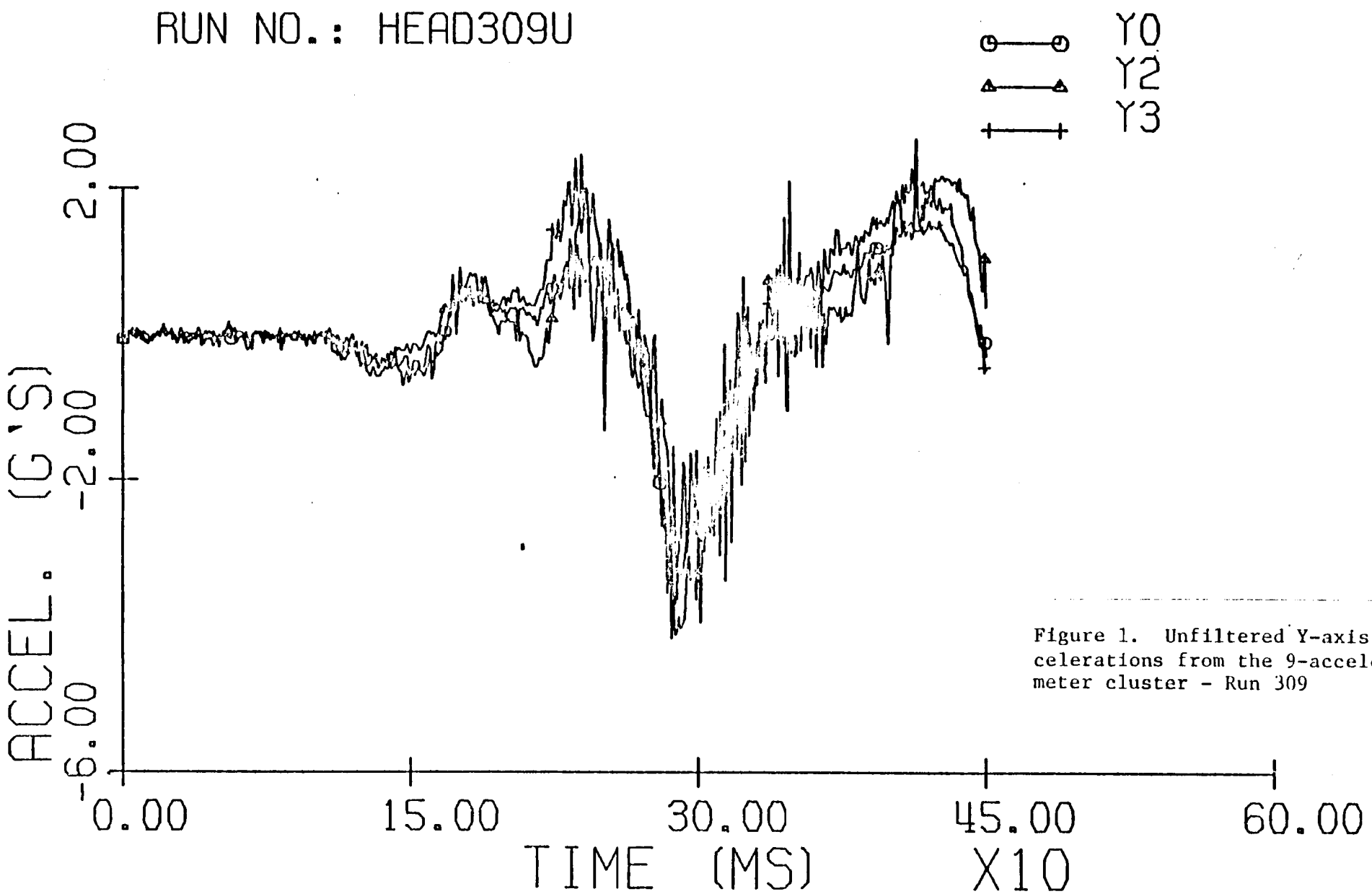

Figure 1. Unfiltered Y-axis accelerations from the 9-accelerometer cluster - Run 309

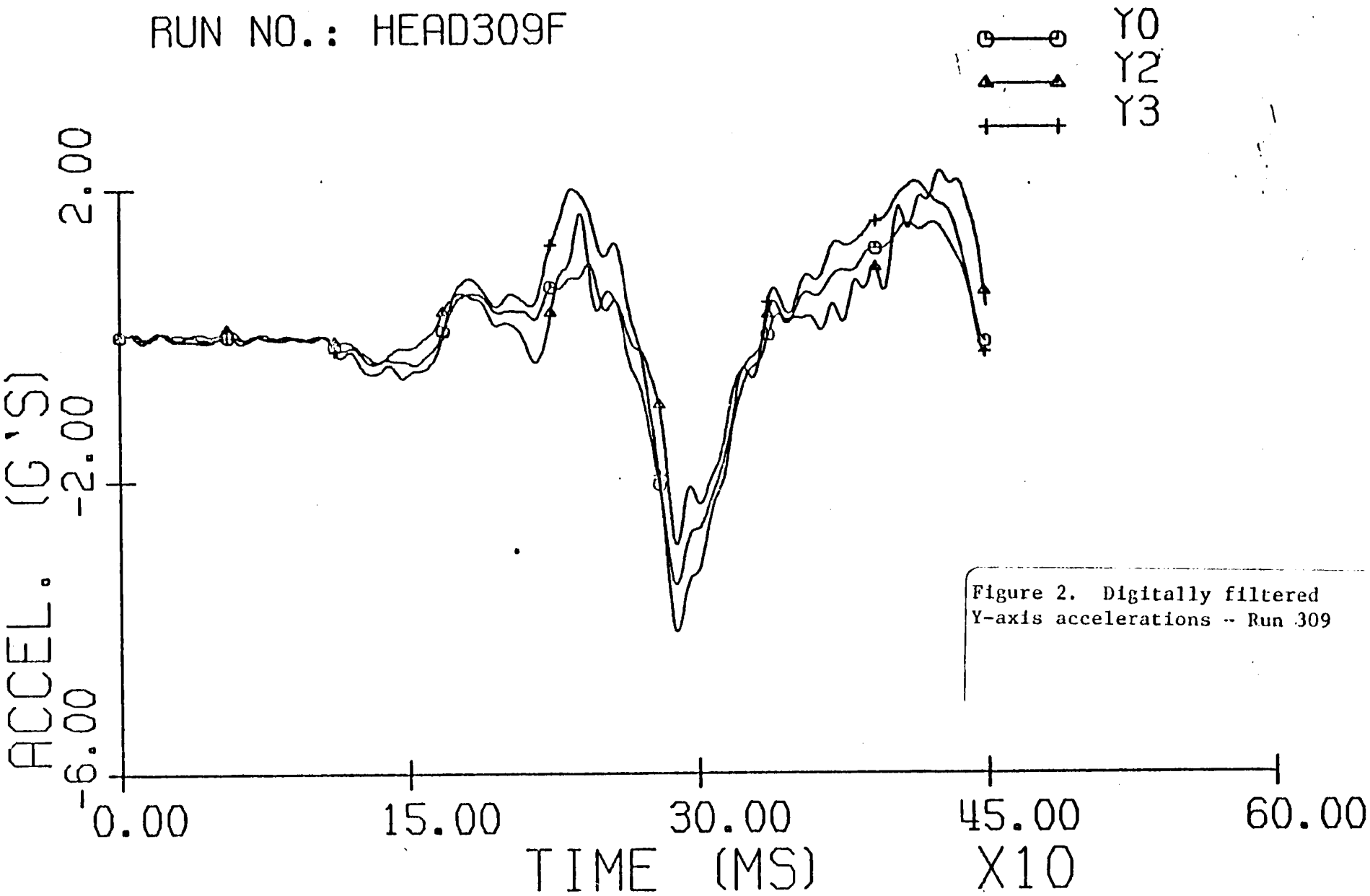

Figure 2. Digitally filtered Y-axis accelerations -- Run 309

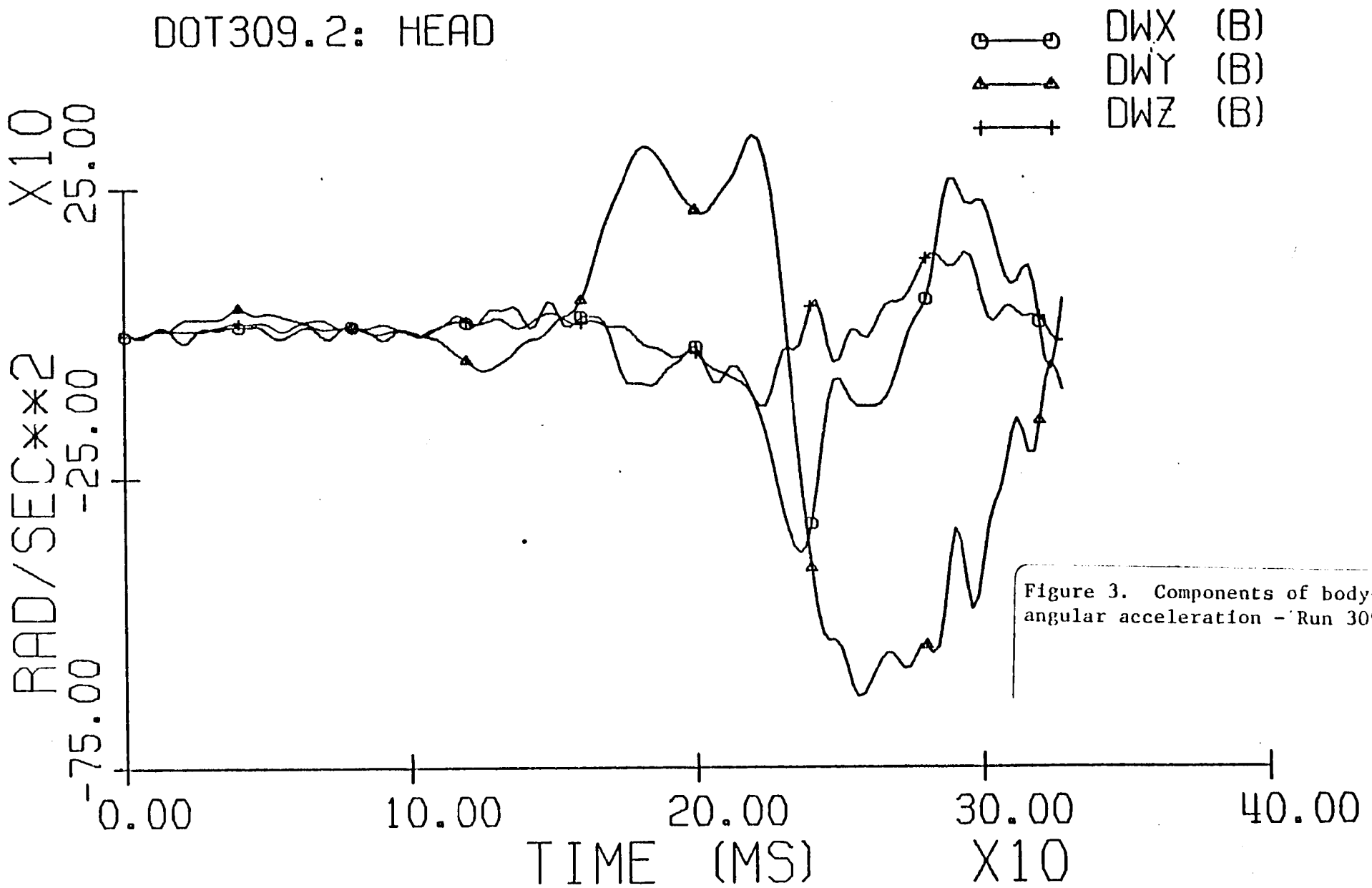

Figure 3. Components of body-fixed angular acceleration - Run 309

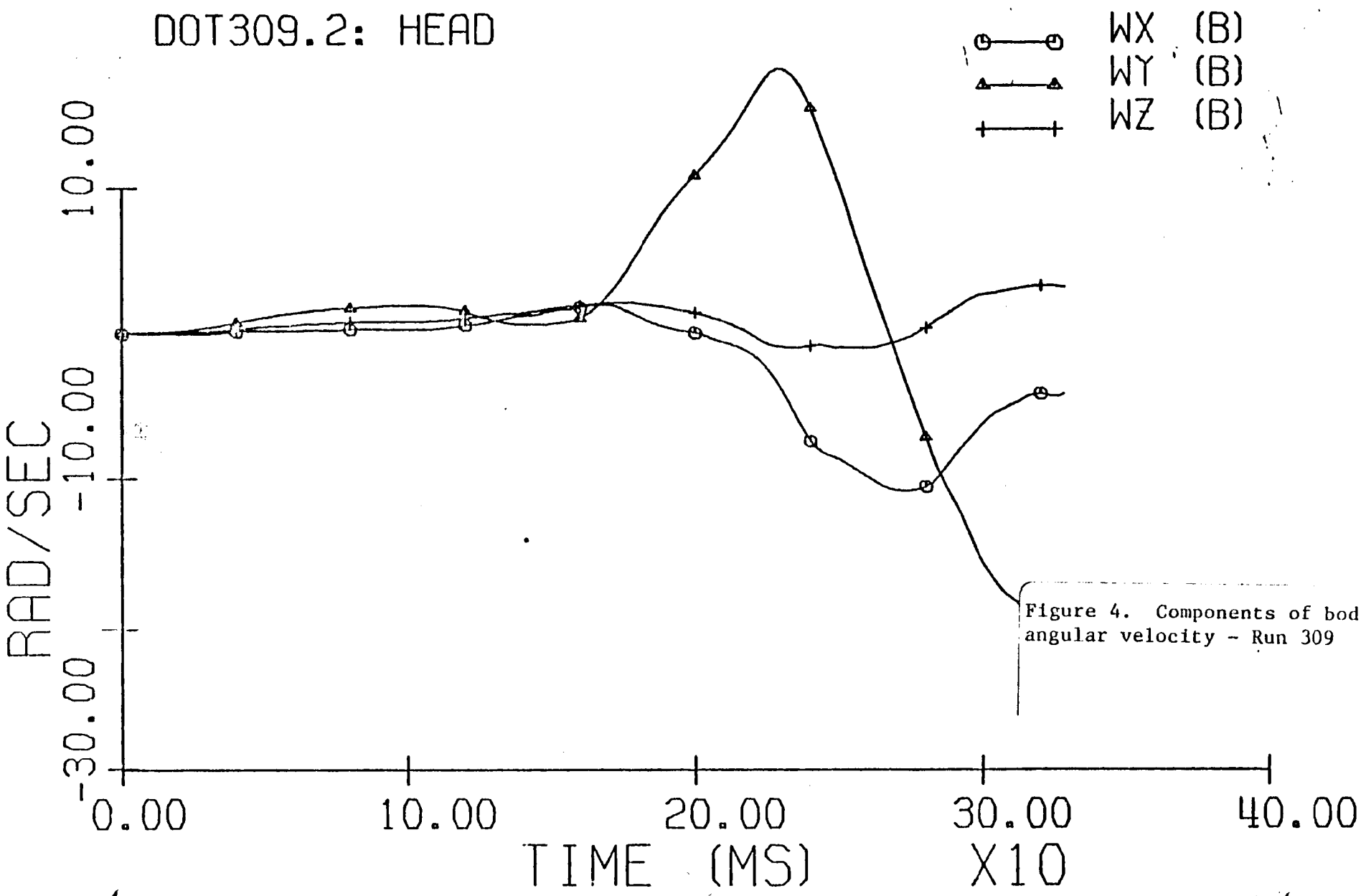

Figure 4. Components of body fixed angular velocity - Run 309

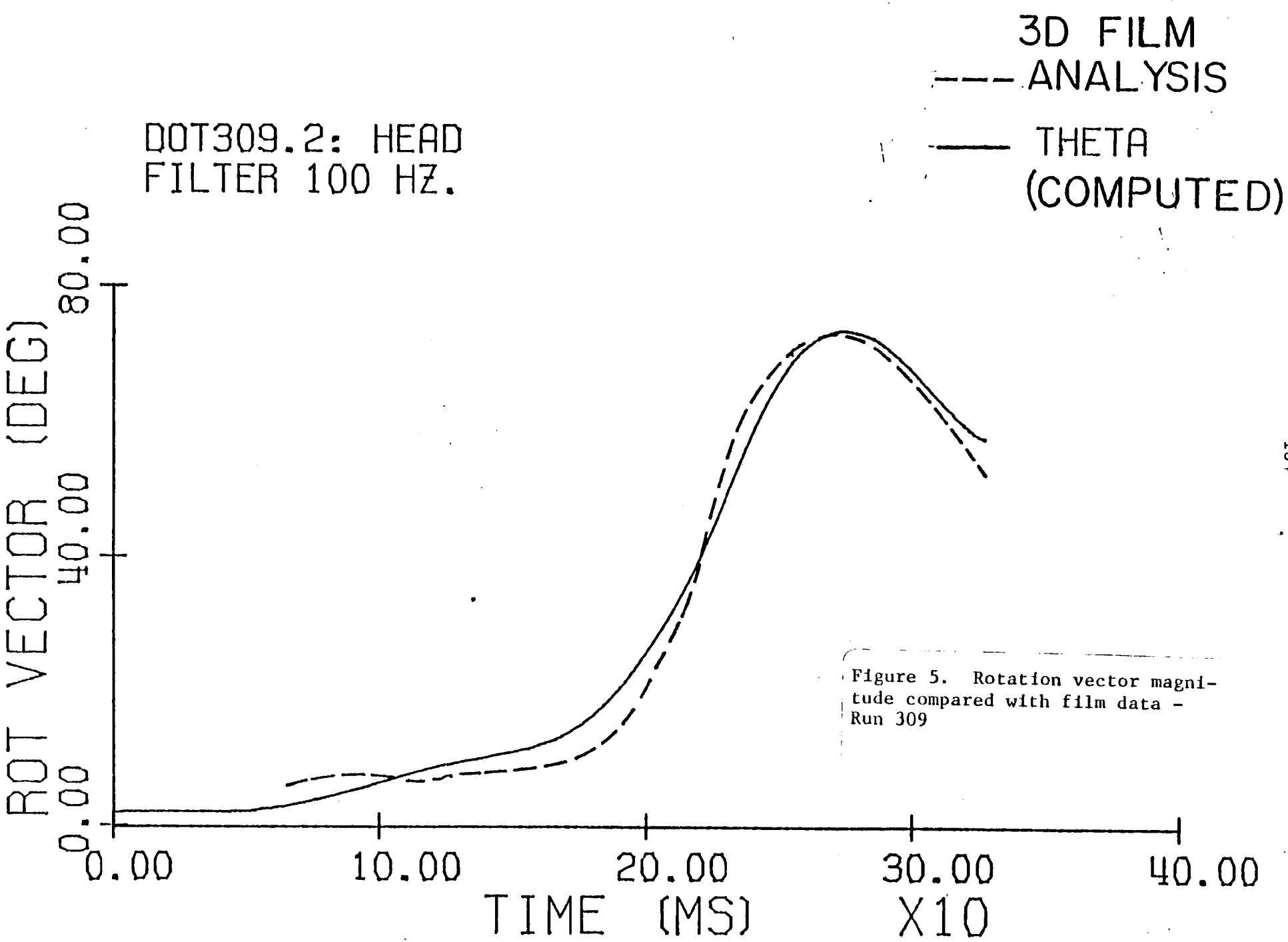

Figure 5. Rotation vector magnitude compared with film data - Run 309

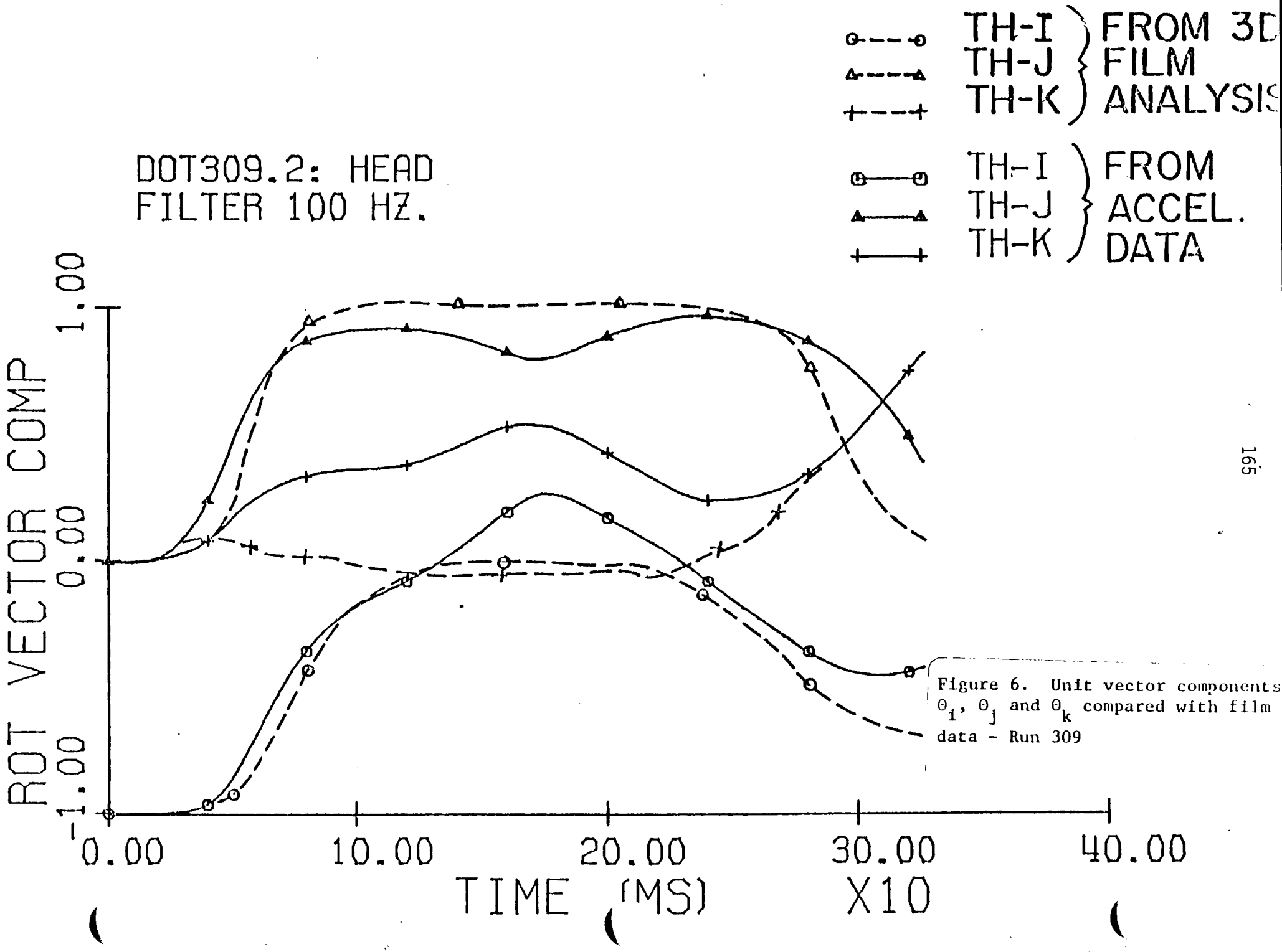

Figure 6. Unit vector components θ_i, θ_j and θ_k compared with film data - Run 309

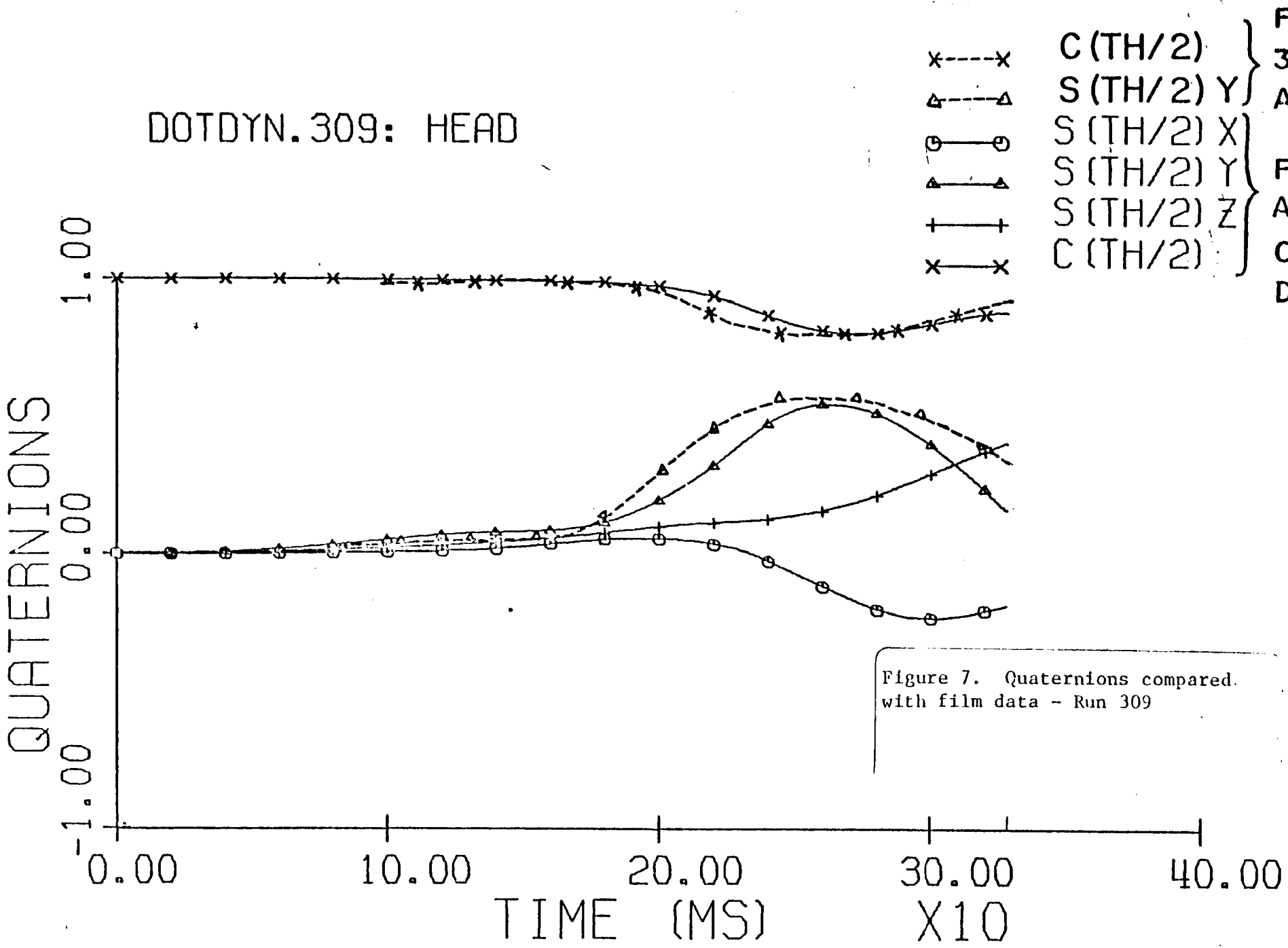

Figure 7. Quaternions compared with film data - Run 309

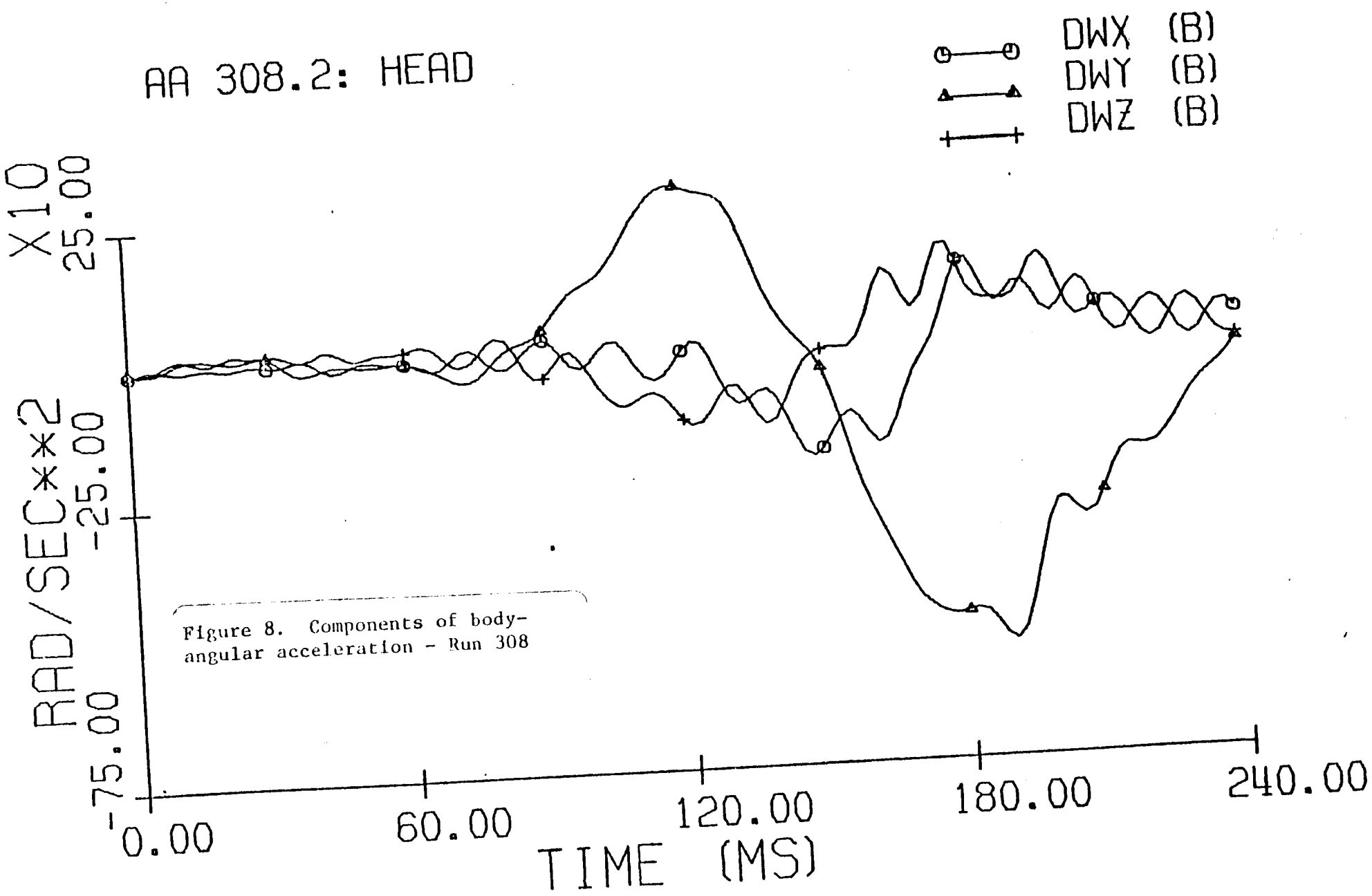

Figure 8. Components of body-angular acceleration - Run 308

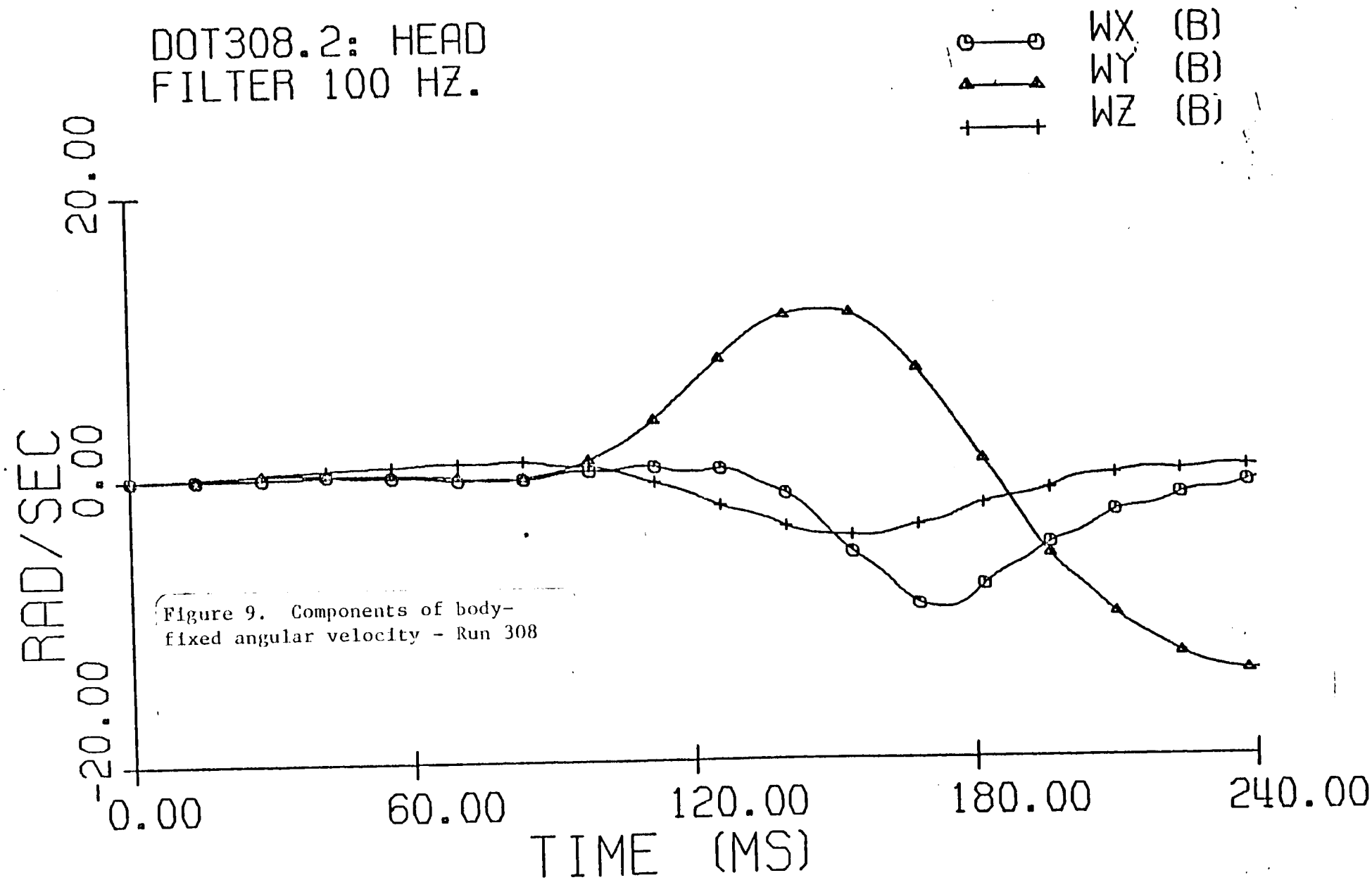

Figure 9. Components of body-fixed angular velocity - Run 308

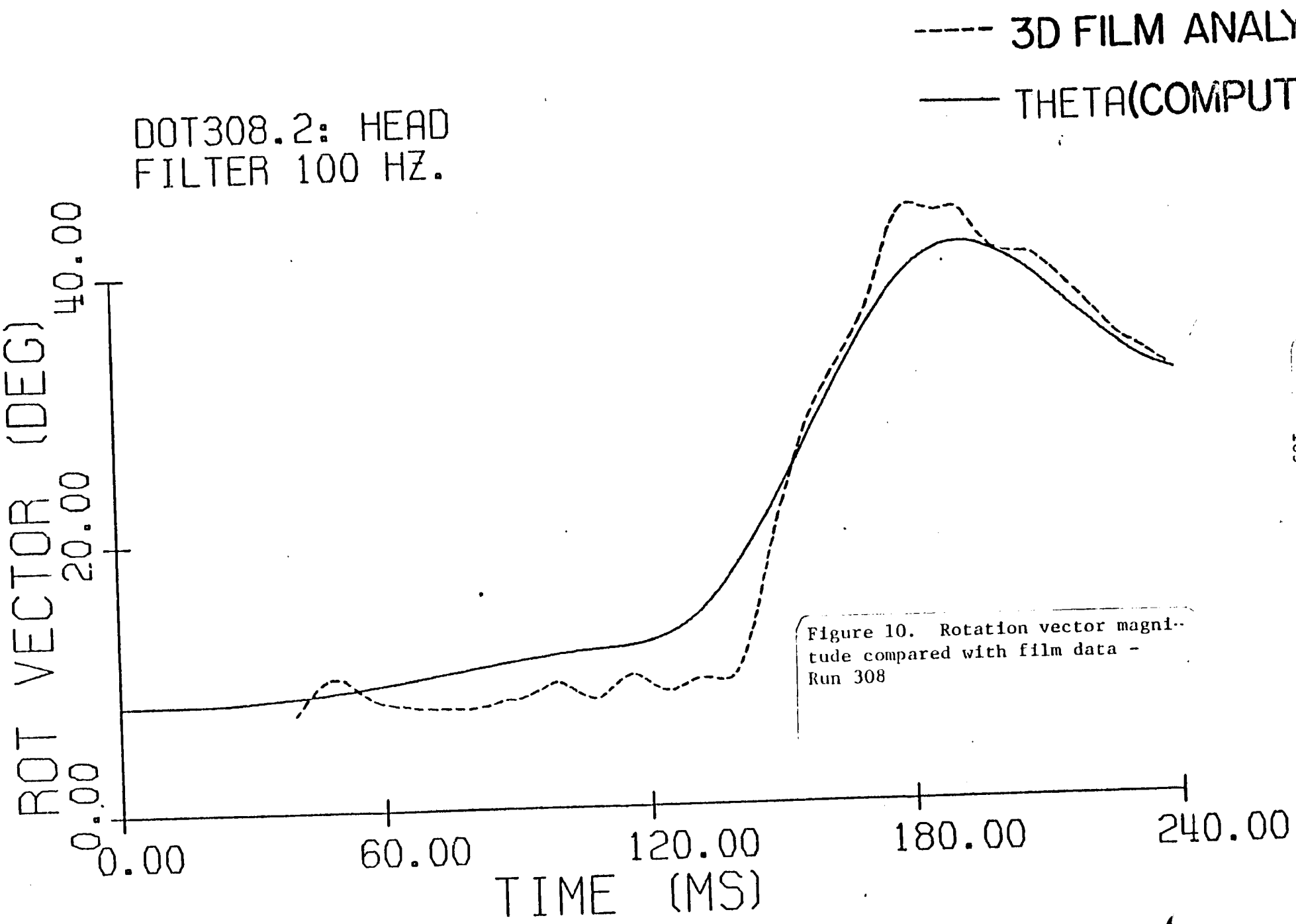

Figure 10. Rotation vector magnitude compared with film data - Run 308

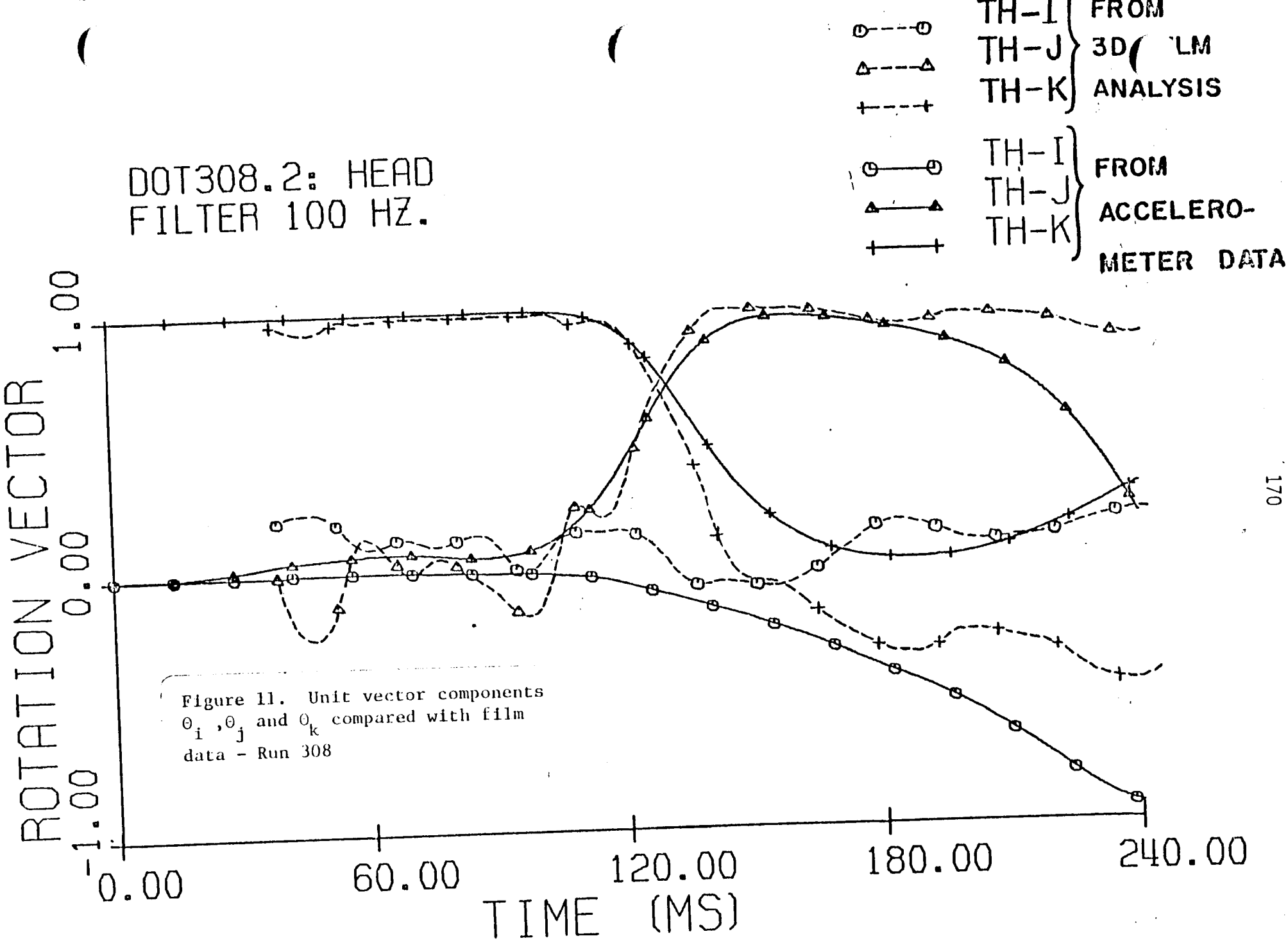

Figure 11. Unit vector components Θ_i, Θ_j and Θ_k compared with film data - Run 308

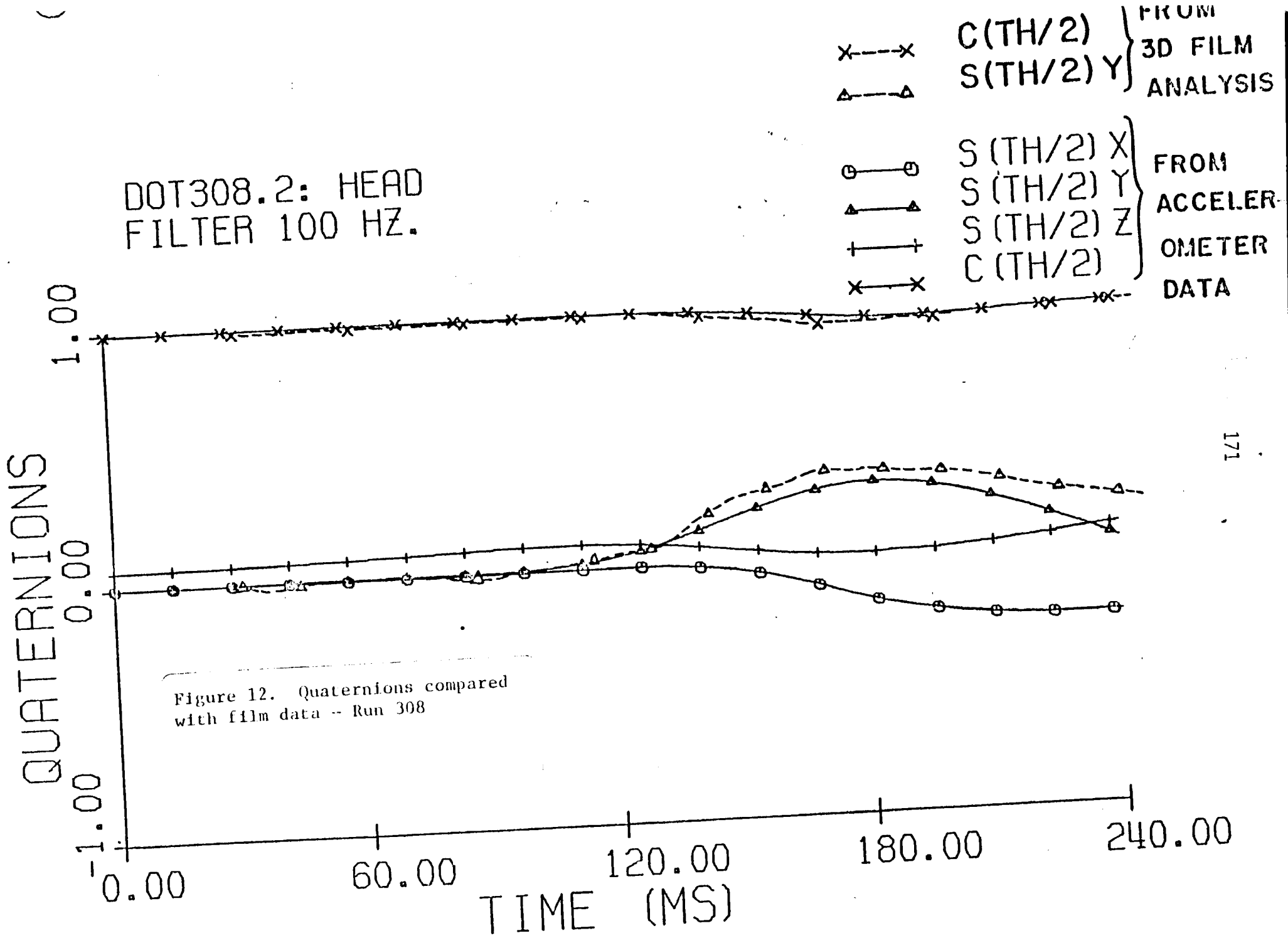

Figure 12. Quaternions compared with film data -- Run 308

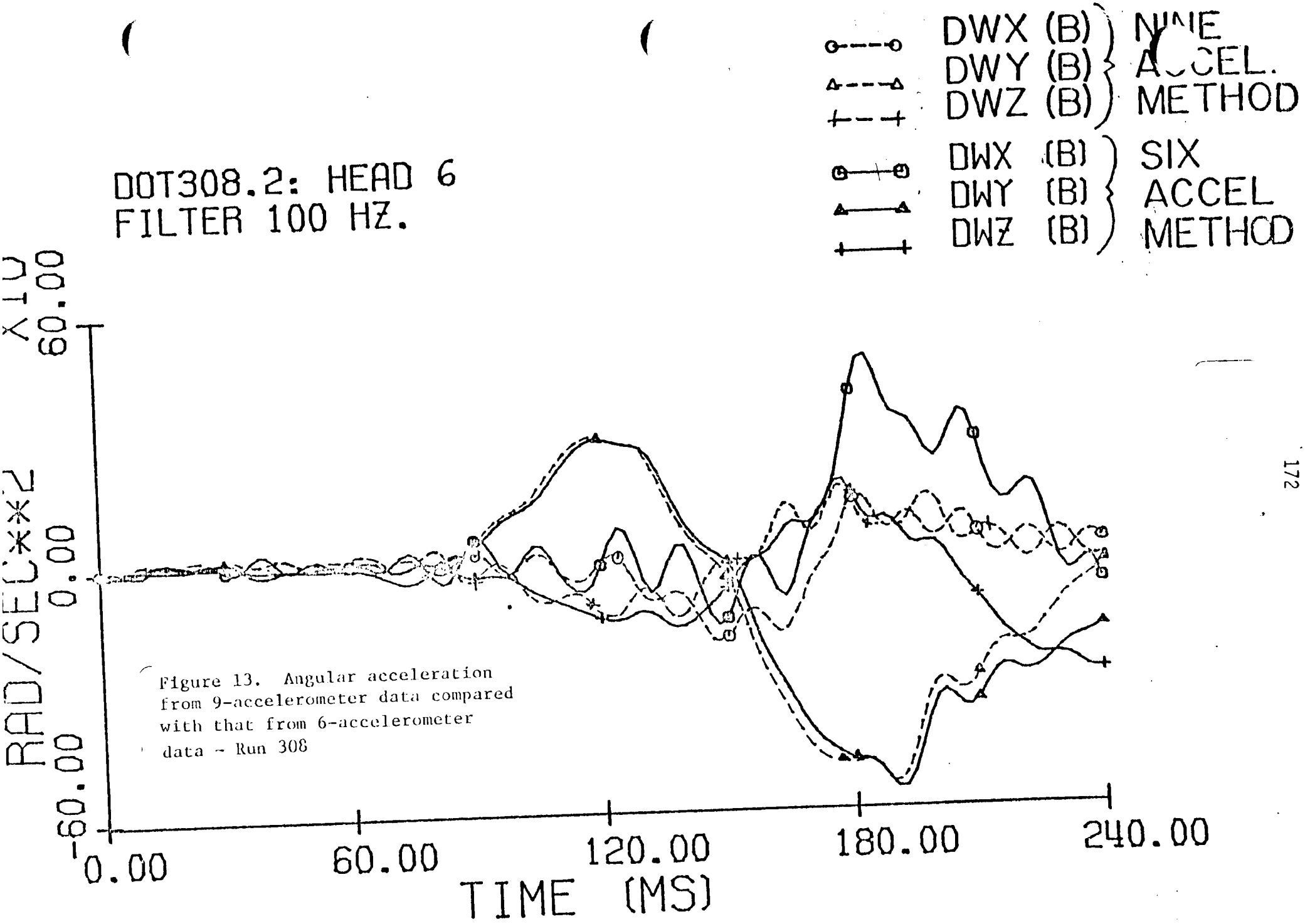

Figure 13. Angular acceleration from 9-accelerometer data compared with that from 6-accelerometer data - Run 308

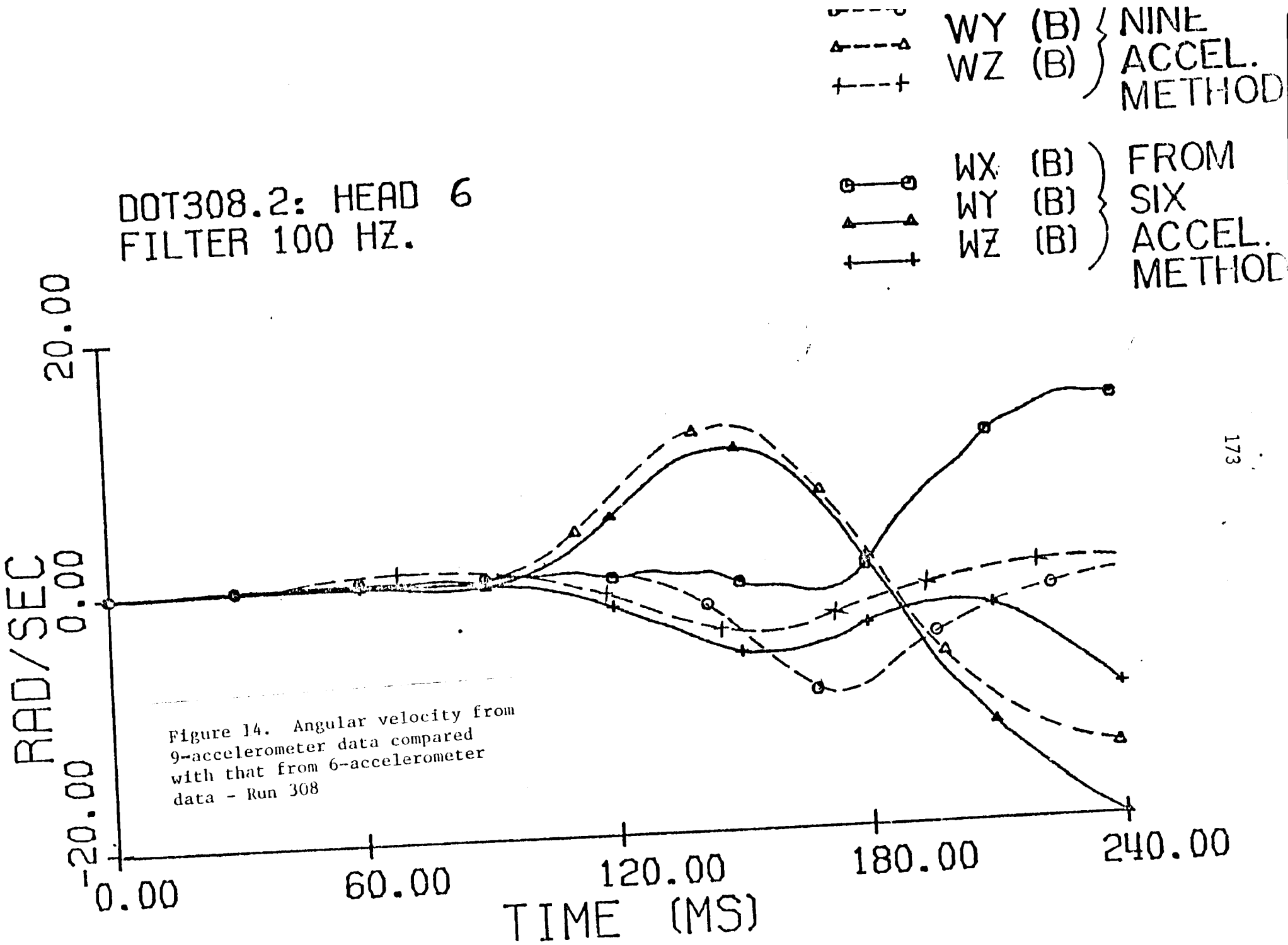

Figure 14. Angular velocity from 9-accelerometer data compared with that from 6-accelerometer data - Run 308

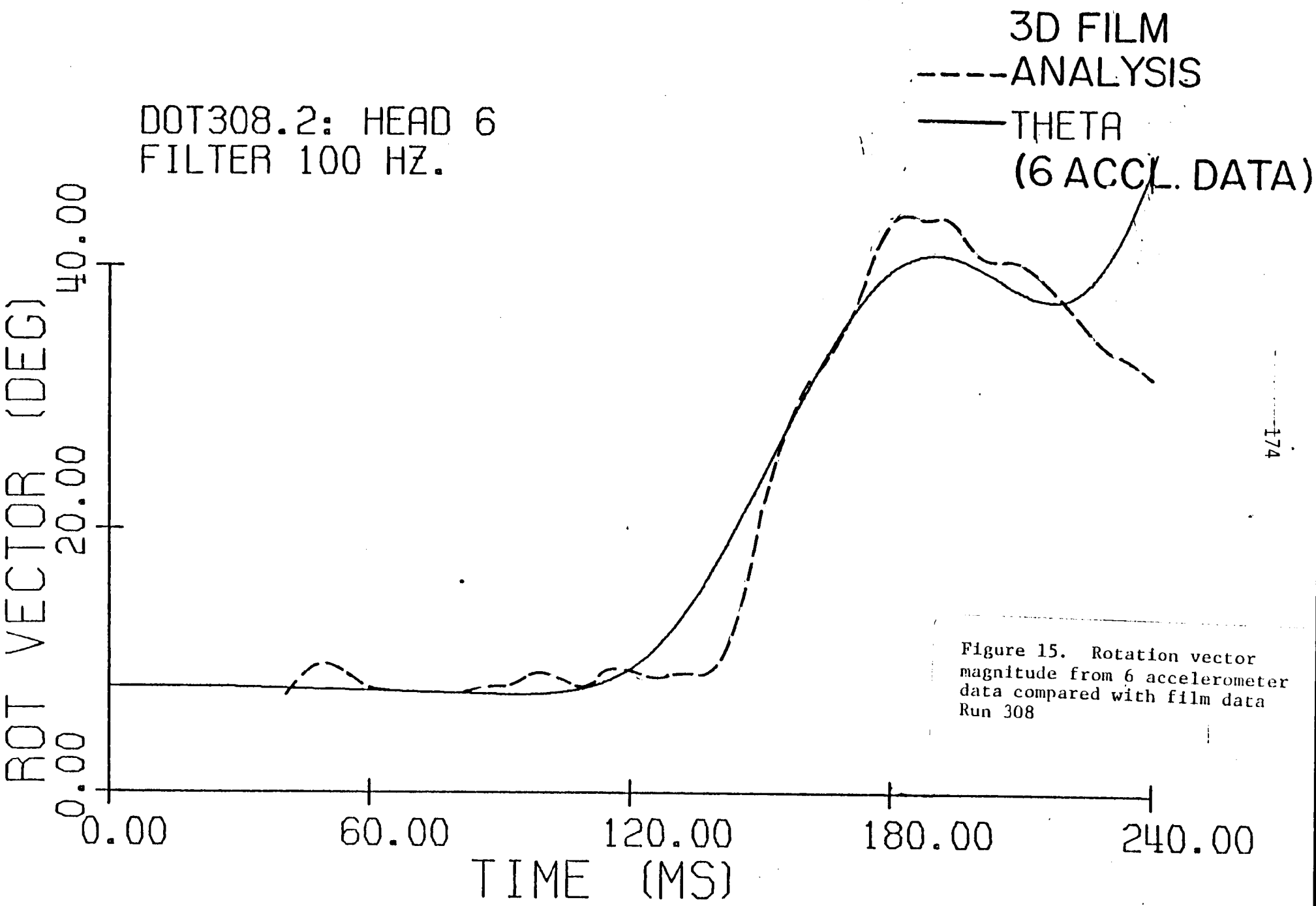

Figure 15. Rotation vector magnitude from 6 accelerometer data compared with film data Run 308

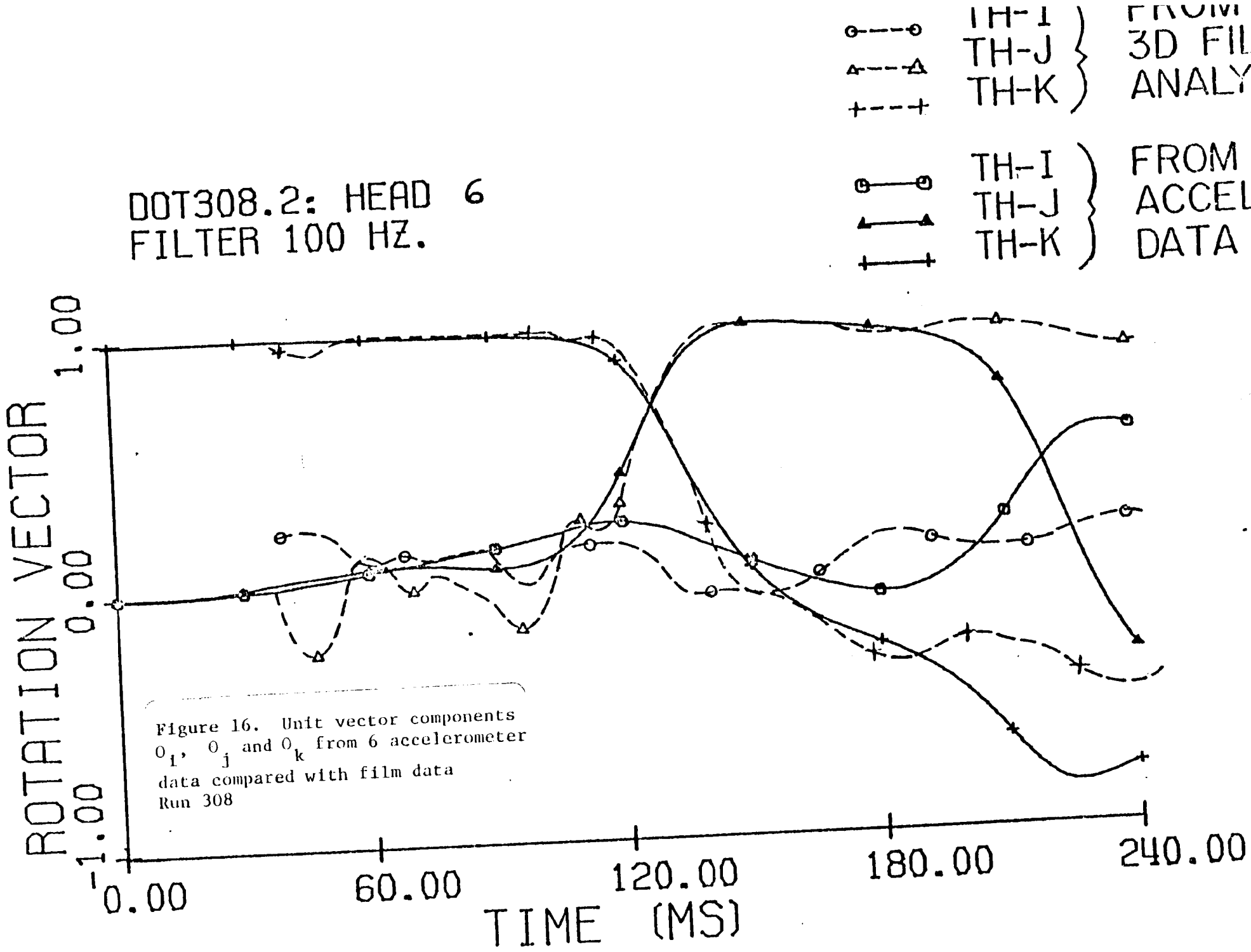

Figure 16. Unit vector components O_i, O_j and O_k from 6 accelerometer data compared with film data Run 308

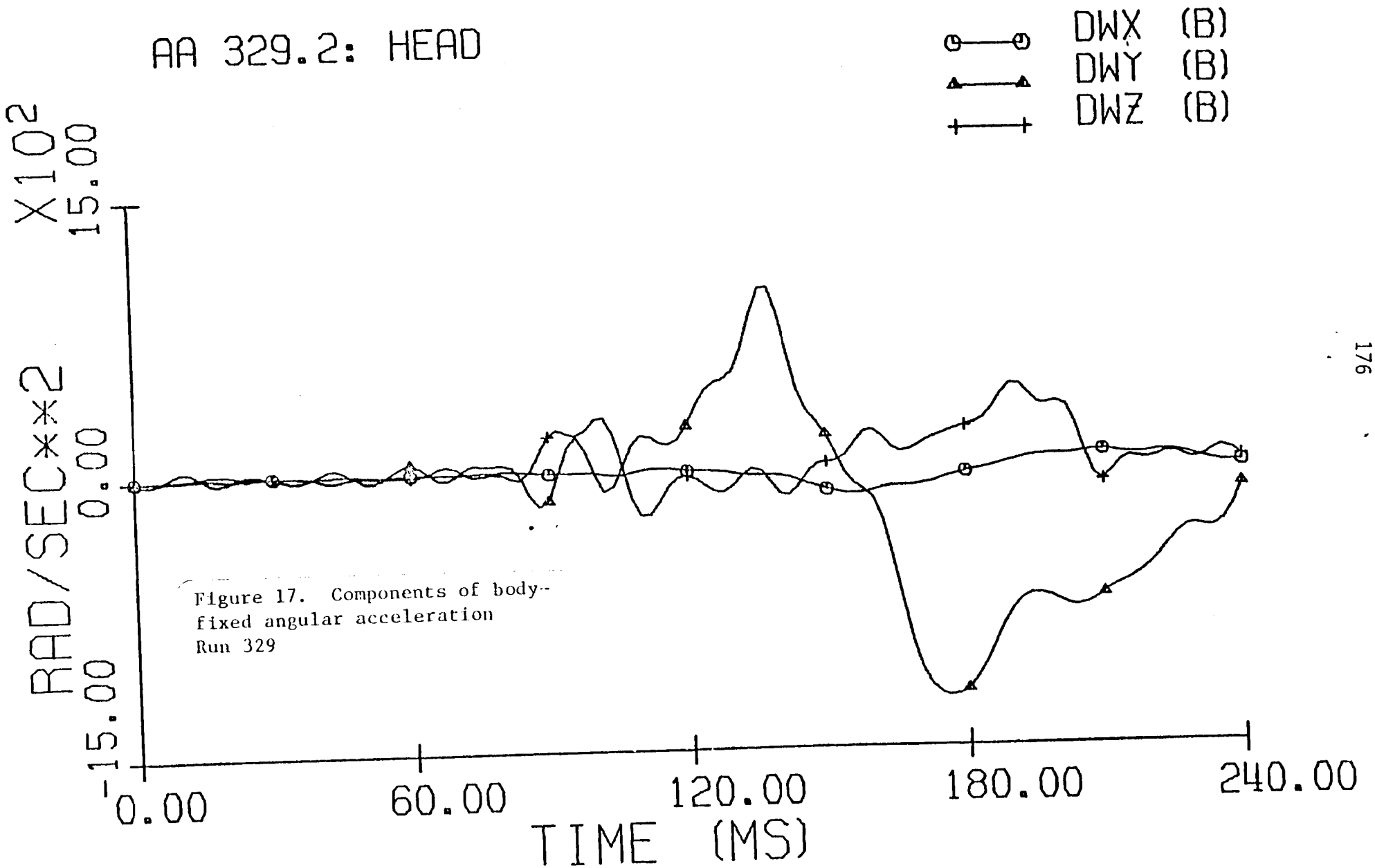

Figure 17. Components of body-fixed angular acceleration
Run 329

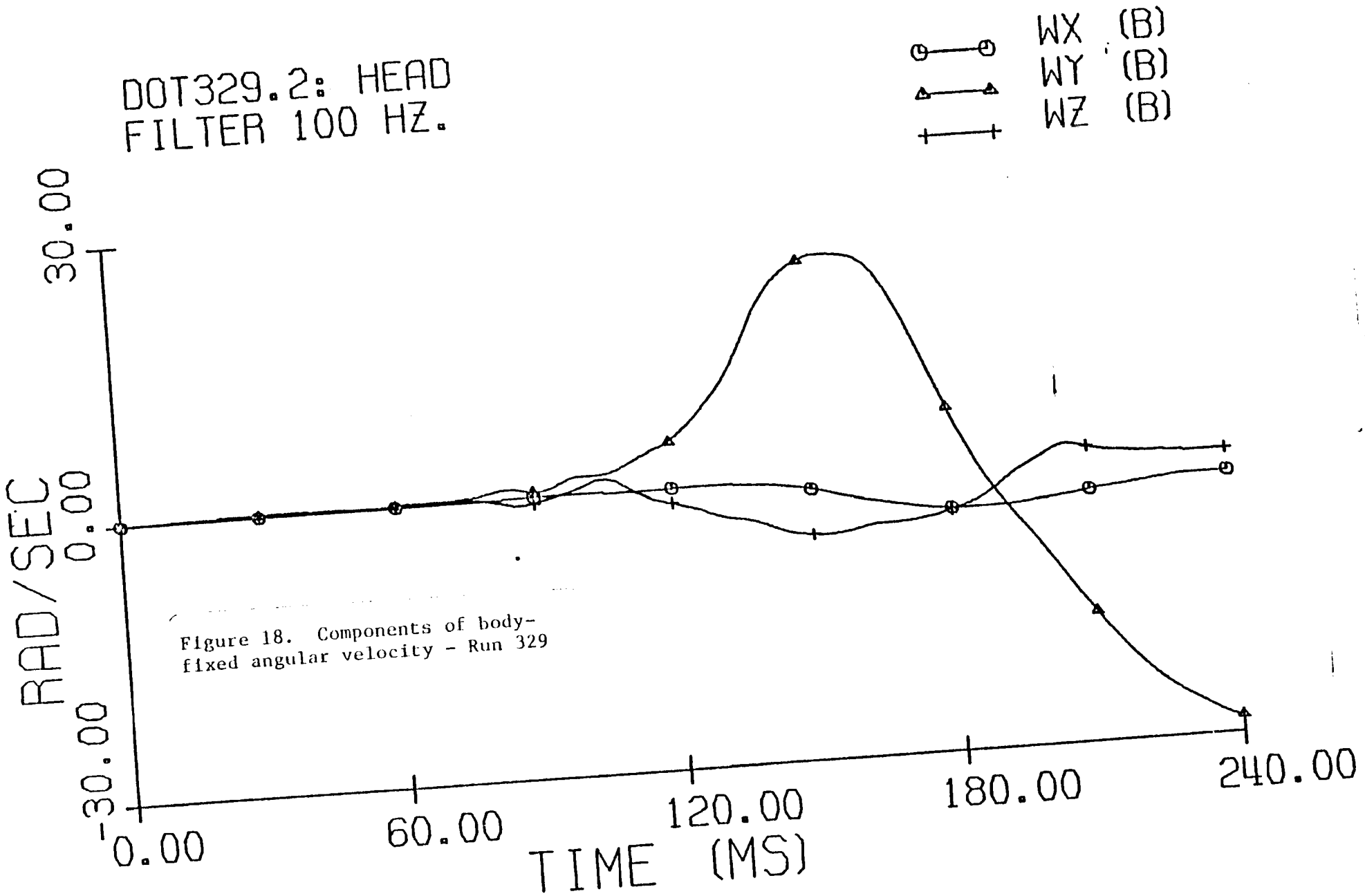

Figure 18. Components of body-fixed angular velocity - Run 329

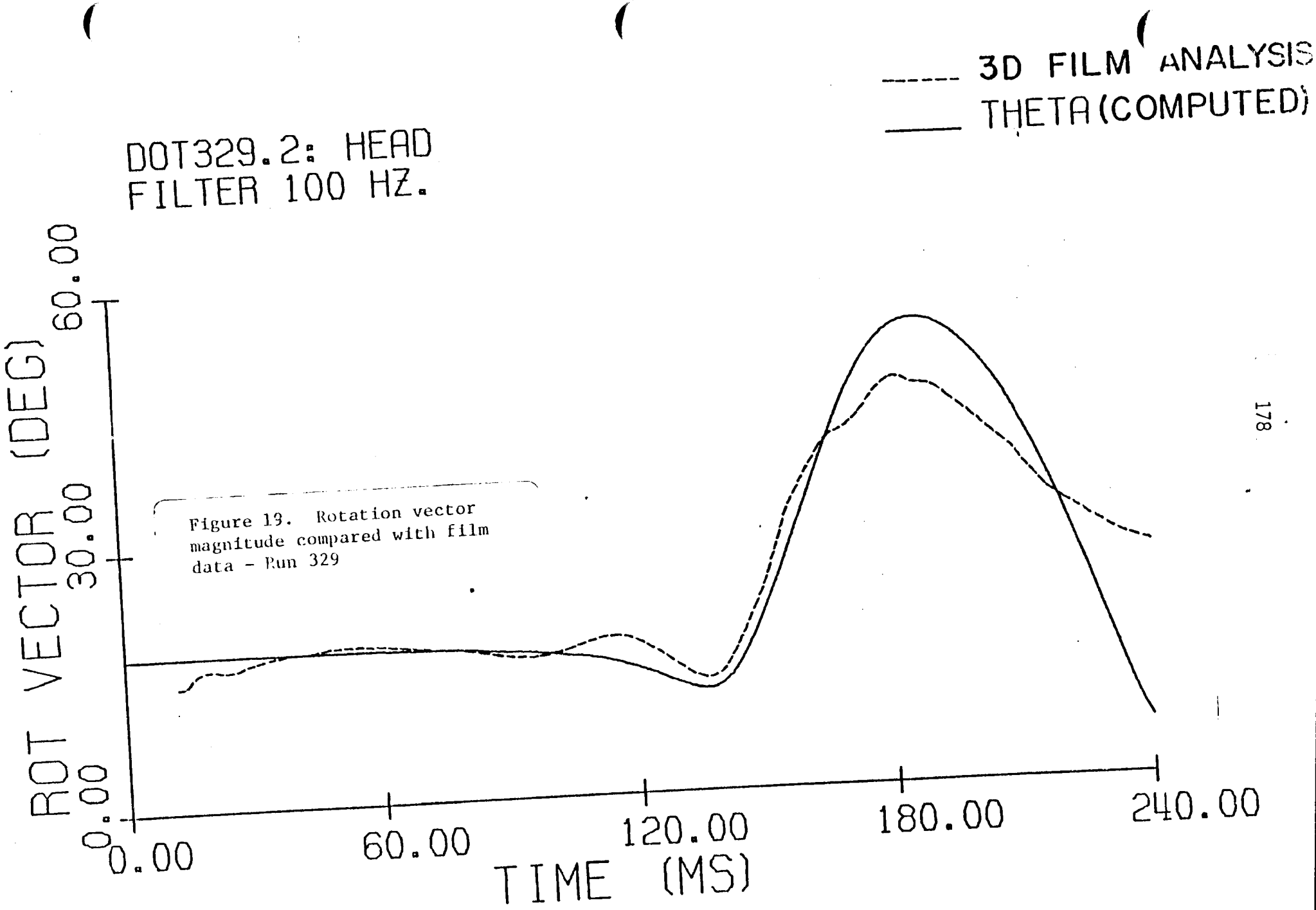

Figure 19. Rotation vector magnitude compared with film data - Run 329

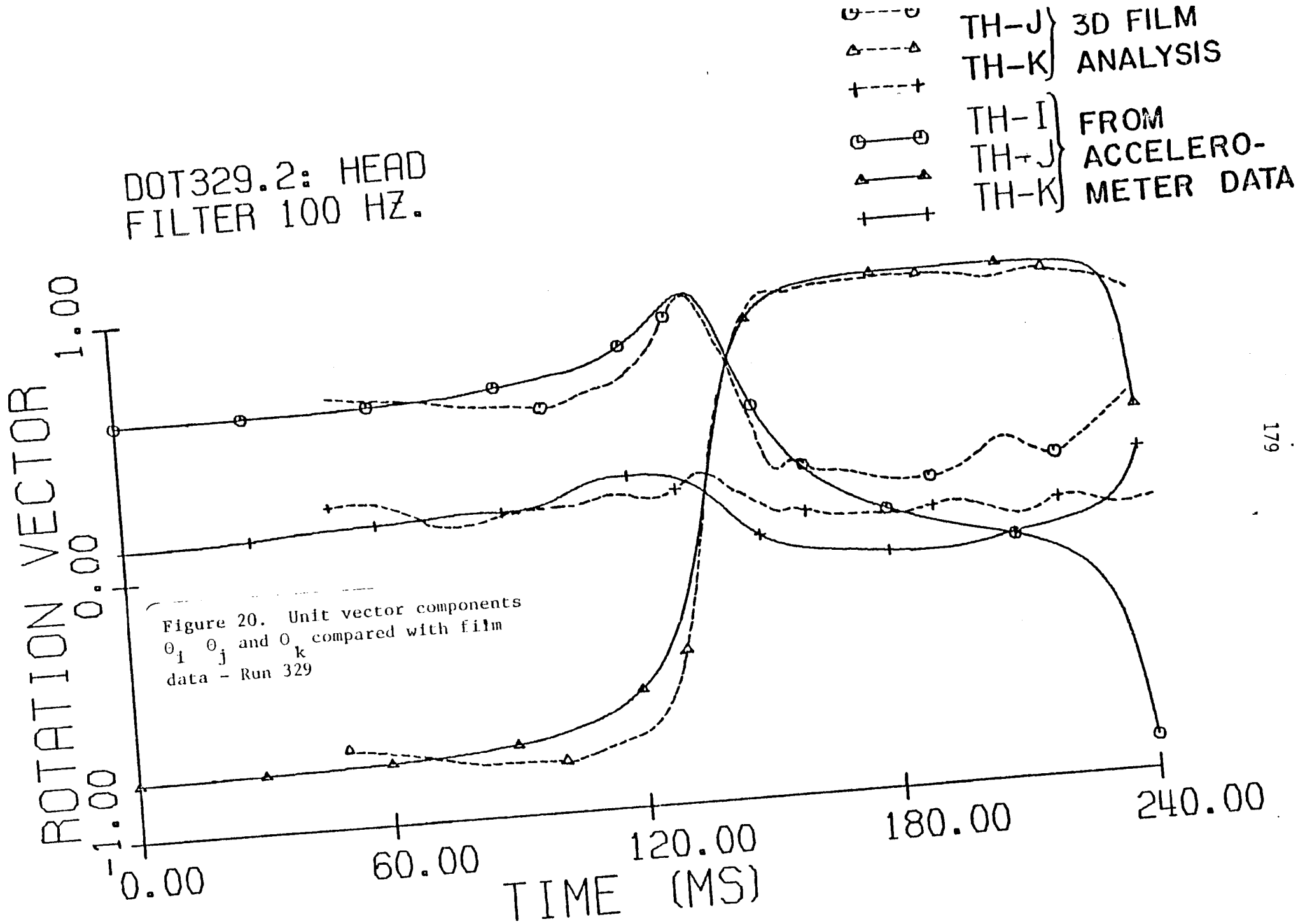

Figure 20. Unit vector components θ_i θ_j and θ_k compared with film data - Run 329

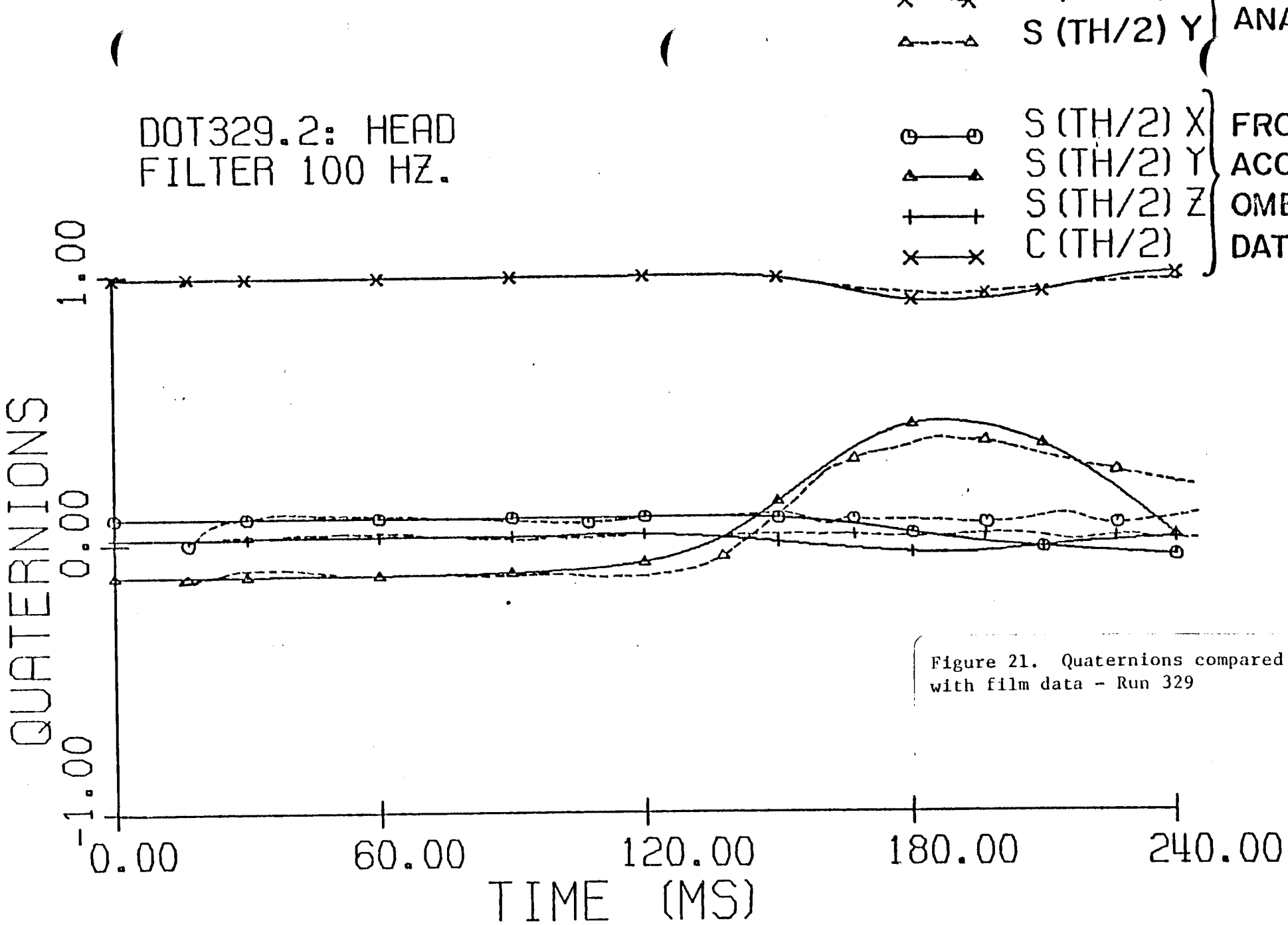

Figure 21. Quaternions compared with film data - Run 329

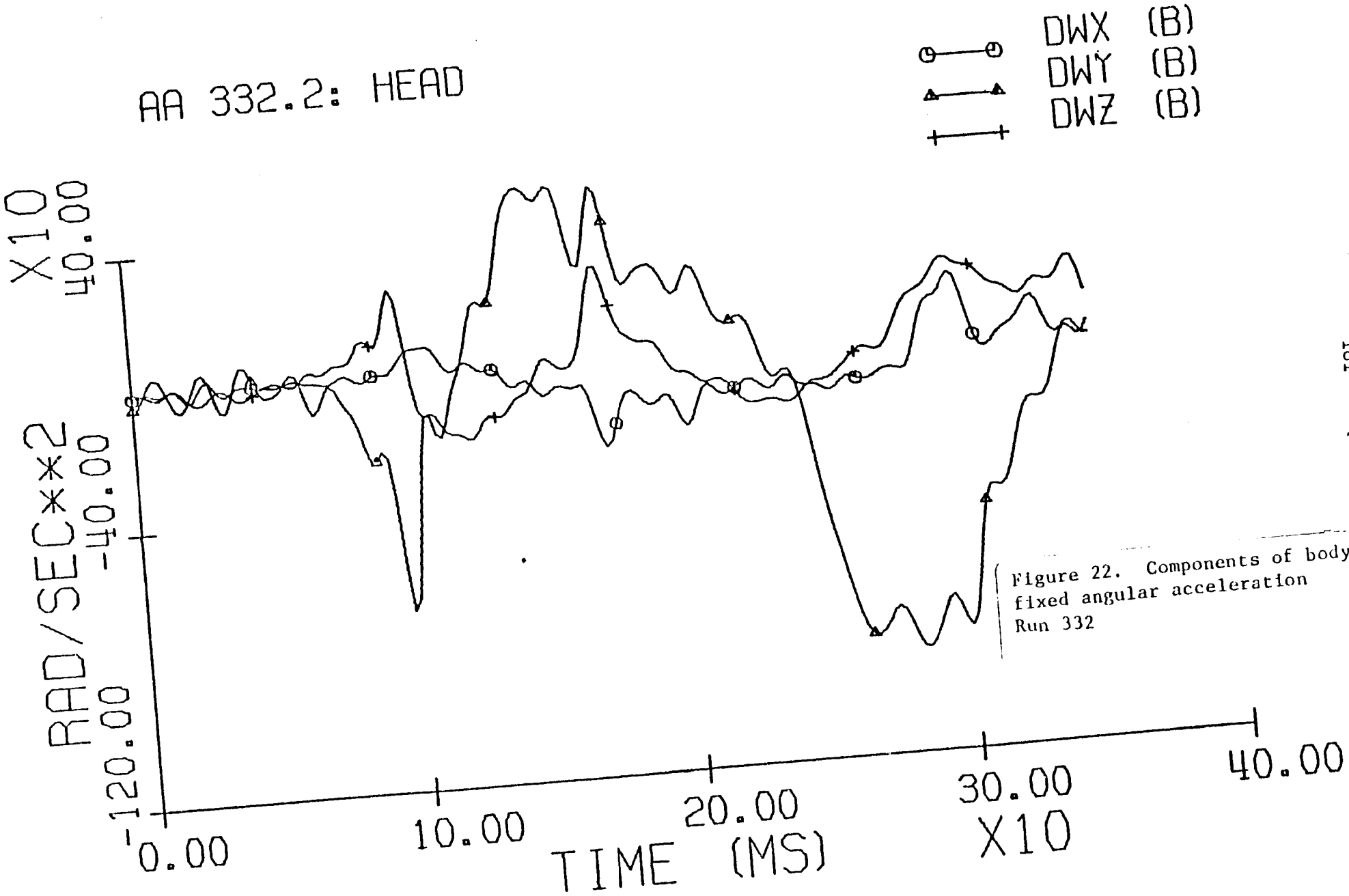

Figure 22. Components of body-fixed angular acceleration Run 332

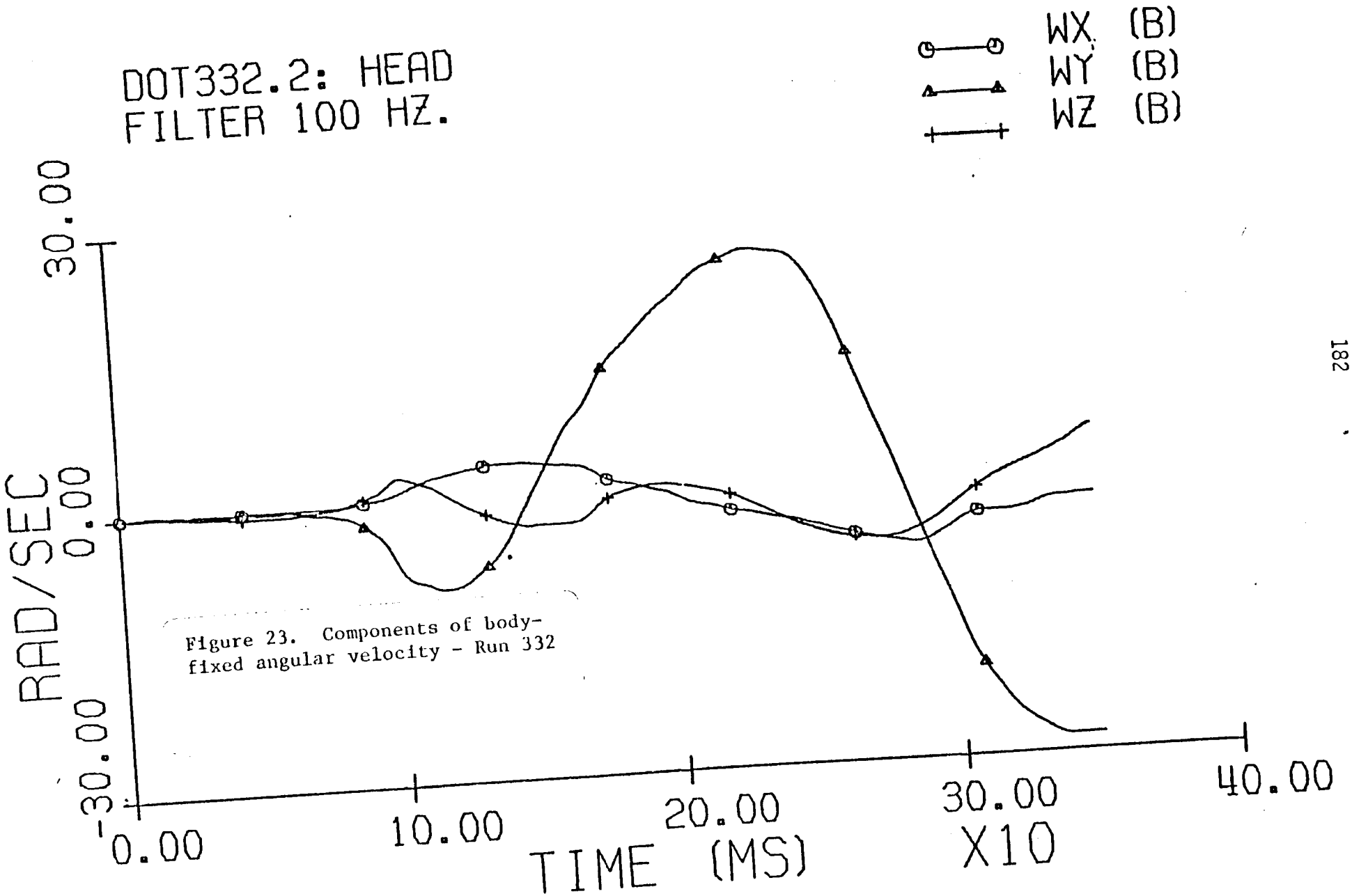

Figure 23. Components of body-fixed angular velocity - Run 332

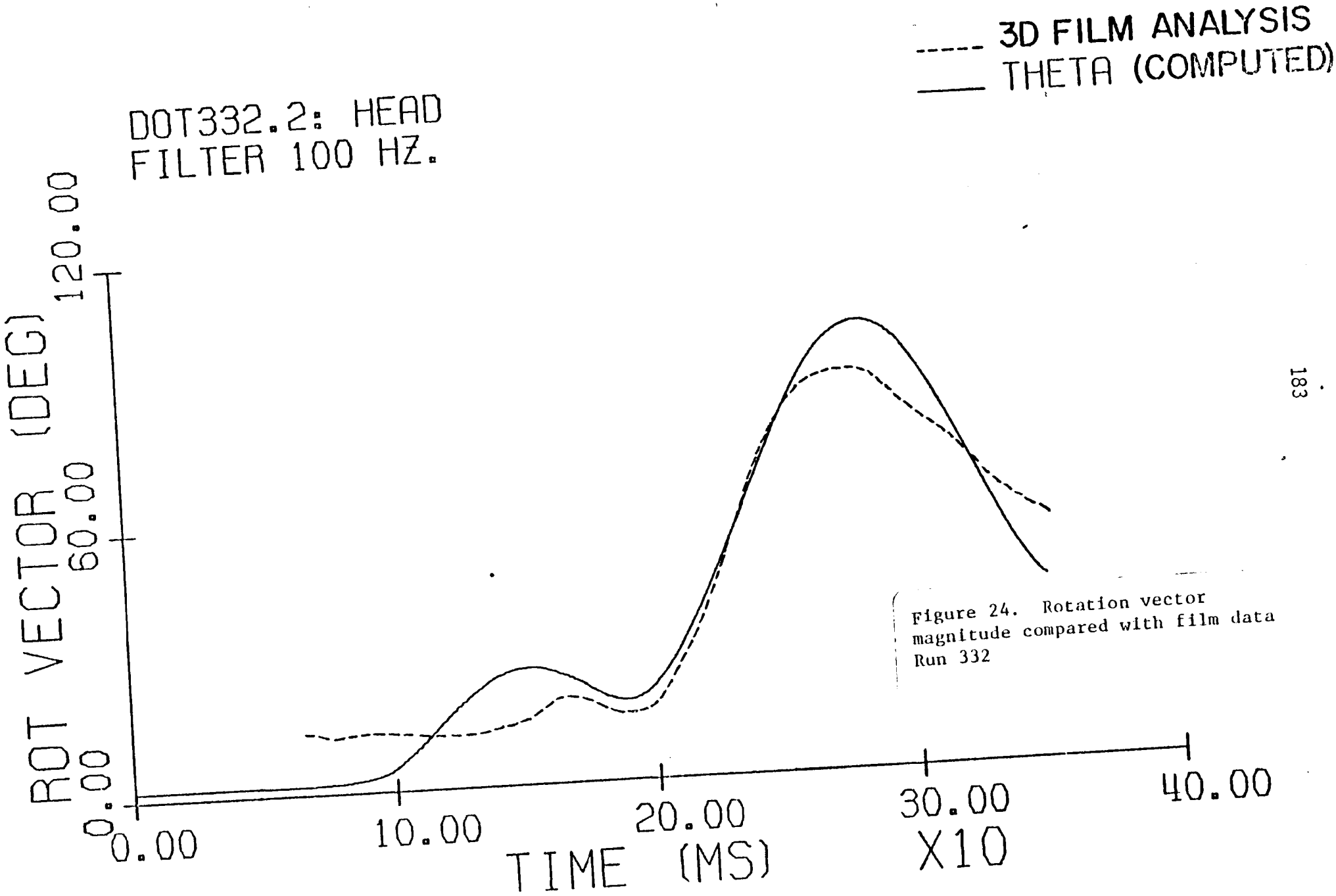

Figure 24. Rotation vector magnitude compared with film data Run 332

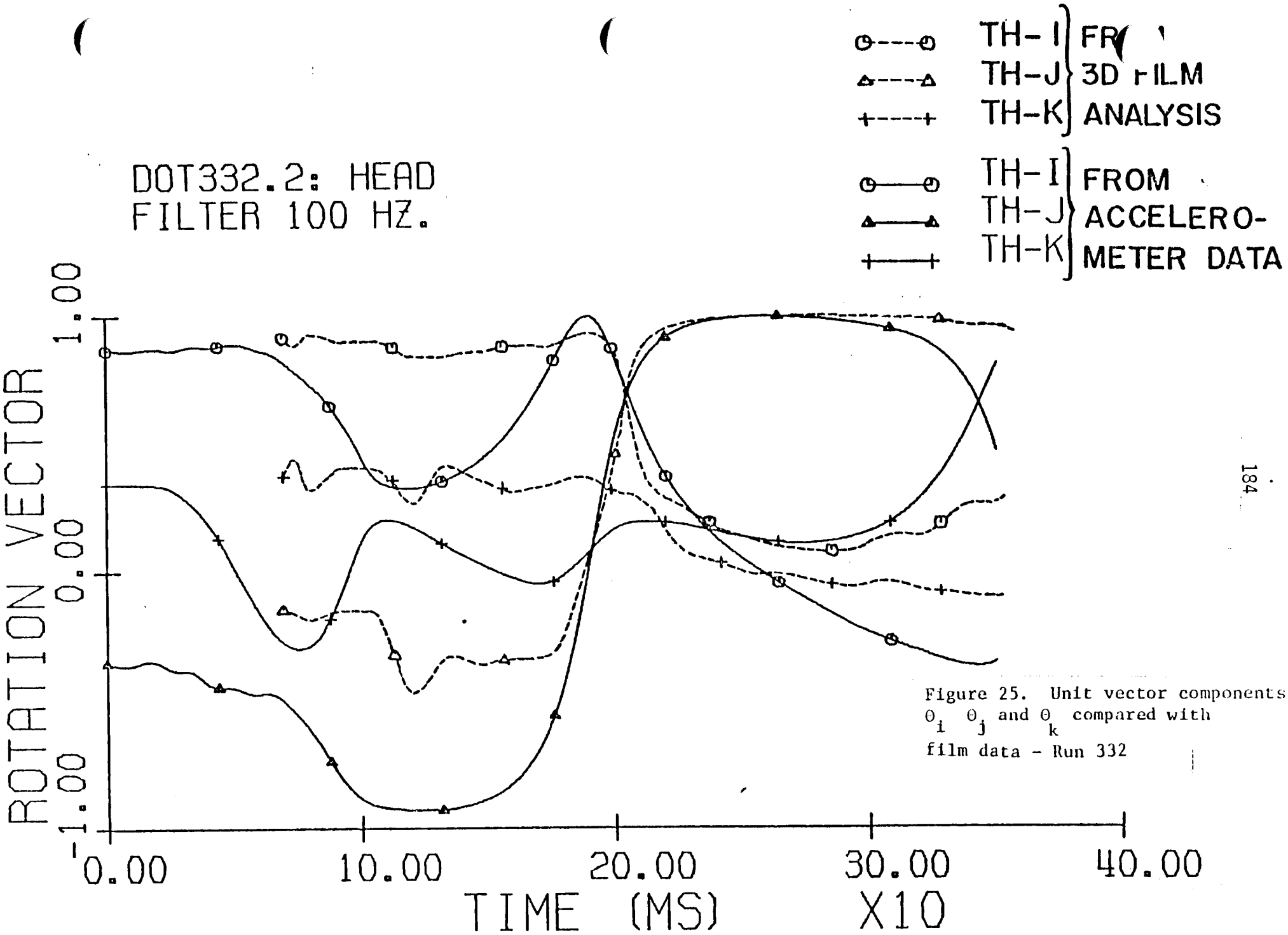

Figure 25. Unit vector components θ_i θ_j and θ_k compared with film data - Run 332

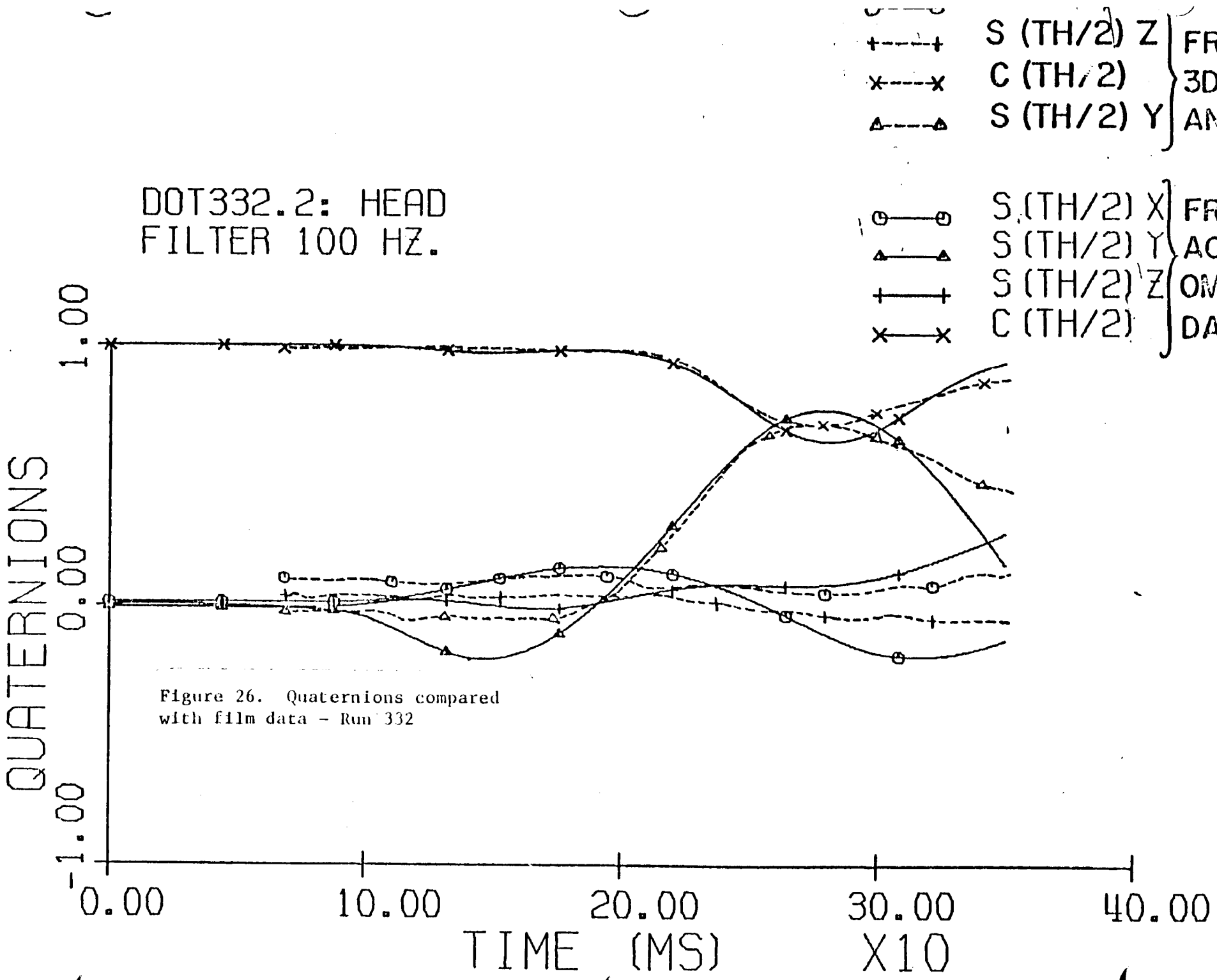

Figure 26. Quaternions compared with film data - Run 332

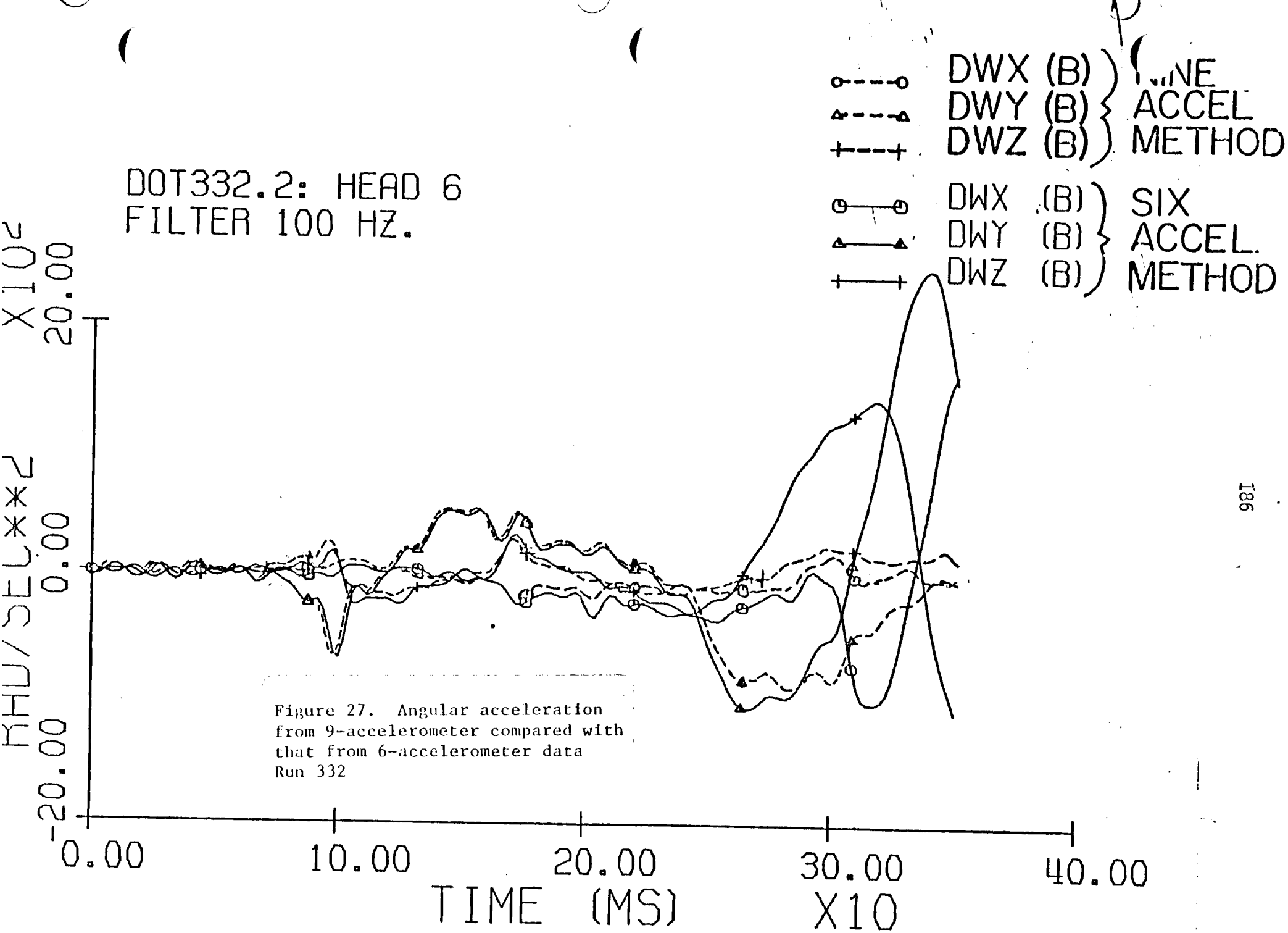

Figure 27. Angular acceleration from 9-accelerometer compared with that from 6-accelerometer data Run 332

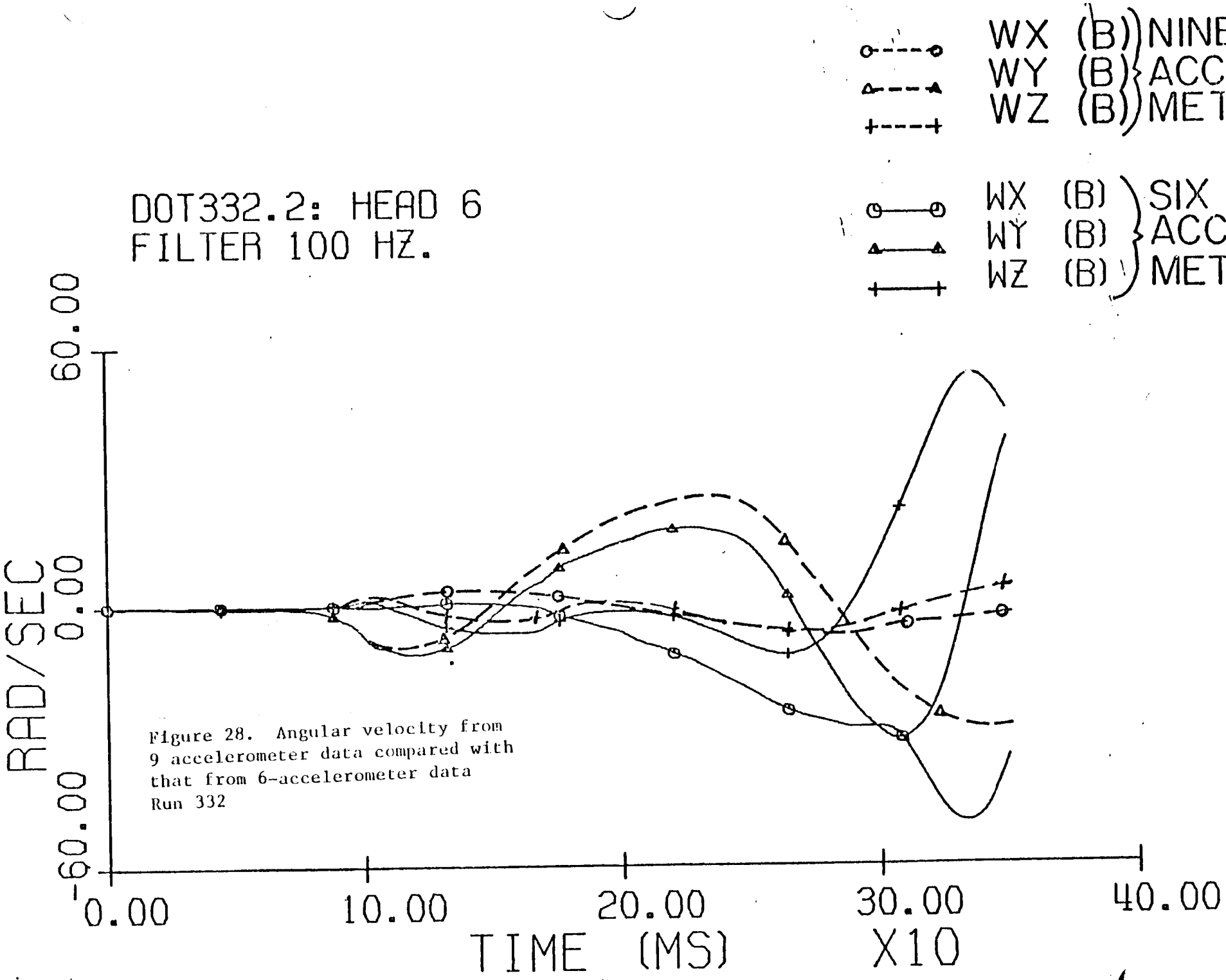

Figure 28. Angular velocity from 9 accelerometer data compared with that from 6-accelerometer data Run 332

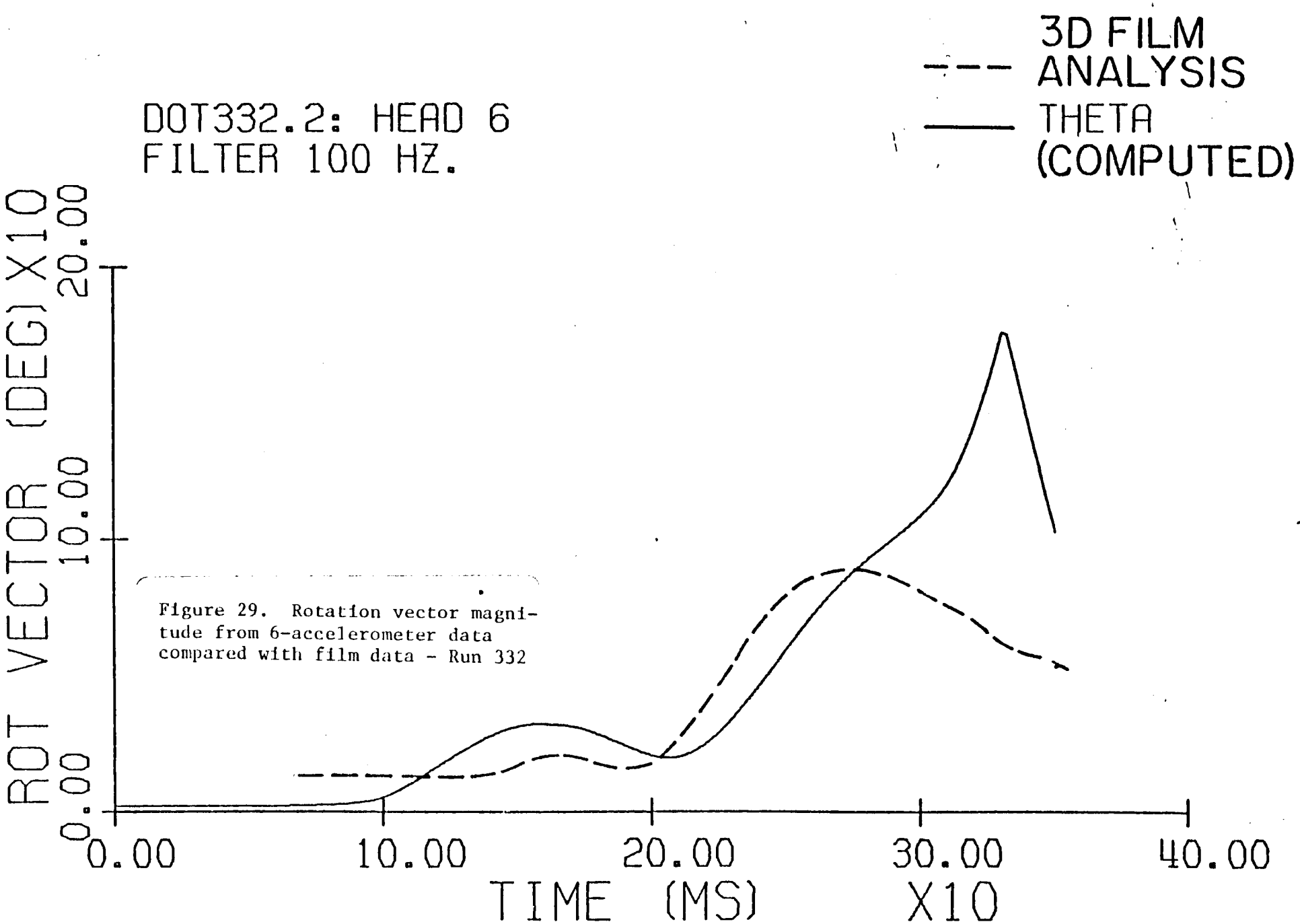

Figure 29. Rotation vector magnitude from 6-accelerometer data compared with film data - Run 332

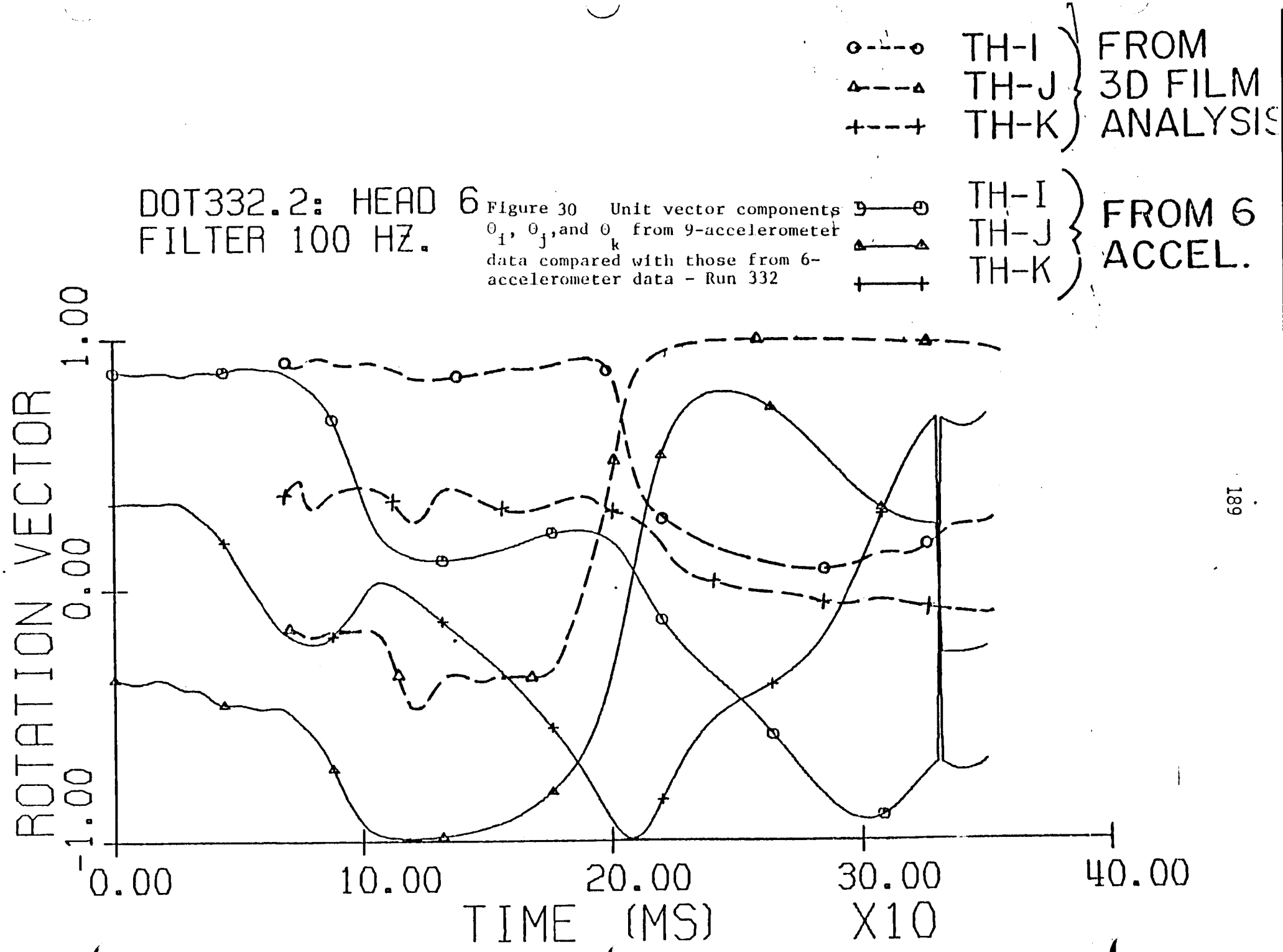

Figure 30 Unit vector components θ_i, θ_j, and θ_k from 9-accelerometer data compared with those from 6-accelerometer data - Run 332

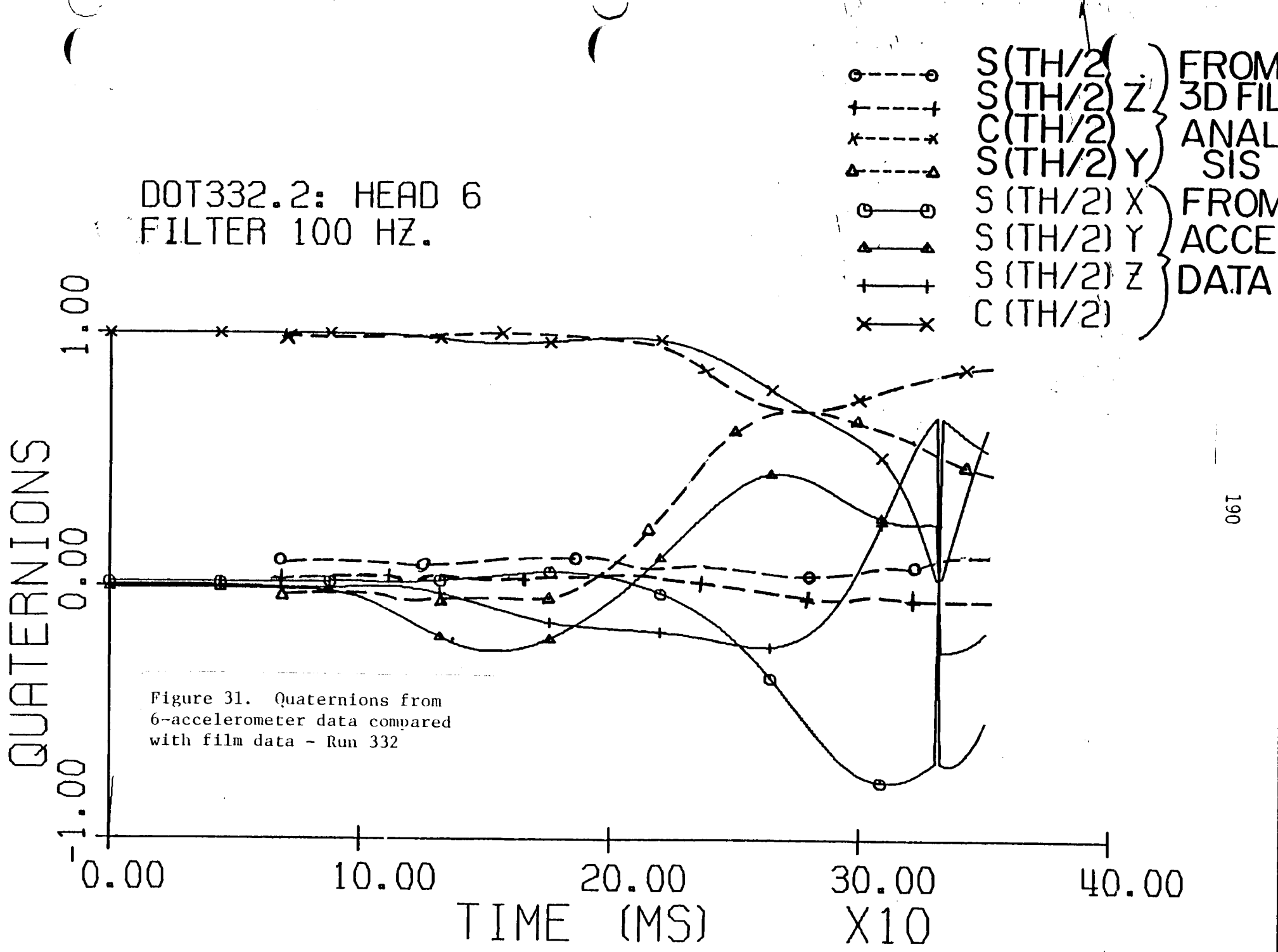

Figure 31. Quaternions from 6-accelerometer data compared with film data - Run 332