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Contractor Final Report

TECHNICAL REPORT

BASIC RESEARCH IN AUTOMOBILE CRASHWORTHINESS-ANALYTICAL STUDIES

By: Richard P. Mayor, Calvin M. Theiss and Dieterich J. Schuring

CAL No. YB-2684-V-5

Prepared For:

Department of Transportation
Federal Highway Administration
National Highway Safety Bureau
Washington, D.C. 20591

INTERIM TECHNICAL REPORT
Contract No. FH-11-6918
November 1969

CORNELL AERONAUTICAL LABORATORY, INC.
OF CORNELL UNIVERSITY, BUFFALO, N Y 14221

ERRATA

BASIC RESEARCH IN AUTOMOBILE
CRASHWORTHINESS - ANALYTICAL STUDIES

CAL Report No. YB-2684-V-5

The following subroutine should be added to Appendix C: Program Lists beginning on page C-1.

```
CTVDYN


100 IF Z9<>0 THEN 150
110 PRINT "CTVDYN ";
120 PRINT "1700 11";
130 PRINT "  7 69"
140 GØ TØ 270
150 V5=T2/2
160 FØR K=1 TØ 3
170 FØR J=1 TØ 3
180 IF M(J,K)<=0 THEN 250
190 V6=F(J,K)/M(J,K)
200 B=(V6+A(J,K))*V5
210 S(J,K)=S(J,K)+(V(J,K)+0.5*B)*T2
220 V(J,K)=V(J,K)+B
230 F(J,K)=0
240 A(J,K)=V6
250 NEXT J
260 NEXT K
270 FØR J=1 TØ 2
280 C(1,J)=CØS(S(3,J))
290 C(2,J)=SIN(S(3,J))
300 NEXT J
310 RETURN
320 END
```

15 October 1970

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INTERIM TECHNICAL REPORT
BASIC RESEARCH IN AUTOMOBILE CRASHWORTHINESS-ANALYTICAL STUDIES

By

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AND
DIETERICH J SCHURING
VEHICLE RESEARCH DEPARTMENT

CAL REPORT NO. YB-2684-V-5
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DEPARTMENT OF TRANSPORTATION
FEDERAL HIGHWAY ADMINISTRATION
NATIONAL HIGHWAY SAFETY BUREAU
WASHINGTON D C 20591

FOREWORD

This report presents the results of three separate analytical investigations conducted within the Basic Research in Crashworthiness Program. The objective of the overall program has been to develop and test automobile structural configurations that will (1) reduce intrusions into the occupant compartment, (2) produce more nearly uniform crush characteristics, and (3) satisfy the strength and stiffness requirements of normal operating conditions. The results of the research will be used to explore the feasibility of an automobile crashworthiness standard to reduce penetration of the passenger compartment by outside objects and at the same time, to utilize more efficiently energy absorption principles.

The overall objectives were approached by making modifications on recent production automobiles. Four different structural concepts, related to front and/or side impacts, have been developed. The analytical investigations that are reported herein provide techniques that are useful in the development and evaluation of vehicle structural modifications.

The findings of the overall program are presented in the following series of reports:

> "Basic Research in Automobile Crashworthiness - Testing and Evaluation of Forward Structure Modification Concept", CAL Report No. YB-2684-V-1;

> "Basic Research in Automobile Crashworthiness - Testing and Evaluation of Engine Deflection Concept", CAL Report No. YB-2684-V-2;

> "Basic Research in Automobile Crashworthiness - Testing and Evaluation of Modifications for Side Impacts", CAL Report No. YB-2684-V-3;

"Basic Research in Automobile Crashworthiness - Testing and Evaluation of Rear Engine Concept", CAL Report No. YB-2684-V-4;

"Basic Research in Automobile Crashworthiness - Analytical Studies", CAL Report No. YB-2684-V-5;

"Basic Research in Automobile Crashworthiness - Summary Report", CAL Report No. YB-2684-V-6.

The reported research was performed under Contract No. FH-11-6918 with the National Highway Safety Bureau, Federal Highway Administration, U. S. Department of Transportation. The opinions, findings and conclusions expressed in this report are those of the authors and not necessarily those of the National Highway Safety Bureau.

This report has been reviewed and is approved by:

Edwin A. Kidd

Edwin A. Kidd, Head
Transportation Research Dept.

SUMMARY

Three separate analytical investigations that are useful in the development and evaluation of vehicle structural modifications were developed within this research program. Presented below is a brief summary of each analytical study.

Limit Analysis of Planar Frames

The feasibility of using a two-dimensional plastic structures program to analyze the types of structures developed and tested in the program is established. Specifically, an existing computer program is used to determine the collapse behavior of a bumper and support system that was developed and employed in the vehicle modification phase of the program. This computer program could be used to optimize the behavior of other structures that were employed in the various vehicle modifications.

Lumped Mass, Nonlinear Spring Program

A computer program capable of calculating time histories of up to 30 masses connected by nonlinear springs has been developed. Emphasis in developing the program has been on simplicity of operation and rapid turnaround time. Five sample problems are solved to illustrate the type of problems solved by the program, to detail preparation of input data, and to compare the program solutions with exact solutions. The program description includes a flow chart, a program listing, and an error analysis of the numerical integration scheme.

Analysis of Side Impact Dynamics

The program provides a two-dimensional simulation of a vehicle and its occupant being struck on the side by the front of another vehicle. The mathematical model was developed primarily to study the influences of vehicle collapse characteristics and occupant restraint systems on occupant safety. Numerous computer runs are provided for vehicles that have structures exhibiting assumed "production" and modified "prototype", force-deflection characteristics.

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LIMIT ANALYSIS OF PLANAR FRAMES

by

Richard P Mayor

November 1969

CAL REPORT YB-2684-V 5 PART I

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LIST OF FIGURES

1. INTRODUCTION

The initial configurations for the experimental vehicle modifications were selected and designed by approximate hand calculations. As experimental evidence concerning loads and behavior of the structures during impact accumulates, a corresponding upgrading of the analytical and design capabilities is indicated.

In lieu of developing a computer program from the beginning, it appeared that the most fruitful approach would be to adapt an existing program. A comprehensive survey of the available computer programs was undertaken. The primary objective of the survey was to find a program that was well-documented to minimize difficulty in adapting the program to the CAL System 360/65. Although other computer programs were found that can handle effects that are important in collision phenomena, the program selected was one originally developed by Wang (Reference 1) and subsequently modified and expanded by Harrison (Reference 2). This program essentially performs a static limit analysis of planar frames. No available program was found for three-dimensional structures.

2. METHOD OF SOLUTION

Only minor changes were required to make the program operational. Specifically, these were: (1) the use of double precision arithmetic, (2) more intermediate output for checking, and (3) a slight change in the criterion for detecting collapse of the structure. Otherwise, the description and user instructions are unchanged from their documentation in Reference 2. A review of the technique used in the program will be useful in relating results to the structures used in vehicle modifications.

Figure I-1(a) shows a beam, clamped at both ends, with a concentrated load. From the input data the stiffness matrix is assembled by the matrix-displacement method. By solving a set of simultaneous linear equations, the moment influence coefficients are determined. That is, the moment due to a unit load is calculated for each possible plastic hinge location. With these influence coefficients, a search is conducted to determine where the moment will first equal the moment capacity, or plastic moment. Once this location is determined, the original structure is modified to one that has an actual hinge at the required location and an externally applied load equal to the plastic moment as shown in Figure I-1(b). In operation, the program performs the necessary bookkeeping to modify the original stiffness matrix to one corresponding to the structure with an additional hinge. The solution then proceeds in a cyclical manner, finding additional hinges, until collapse occurs. Collapse is detected by the occurrence of very large deflections (see Figure I-1(c)).

Example: Mod. 2 Bumper

As an example of the information furnished by the use of the program, an idealization of the Mod. 2 bumper was analyzed, Reference 5. This bumper was tested in both a head-on and 15° oblique full scale tests. The structure and member sizes of the bumper are given in Figure I-2. For purposes of analysis the supports are assumed to be fixed; in actual construction the members are welded to a heavy plate which in turn is connected to the engine support and longitudinal members.

The load-deflection curves, plotted from the computer results, are shown in Figure I-3. The similarity between the curves for the disparate loading conditions can be observed. The following table quantitatively compares performance data for the two load conditions.

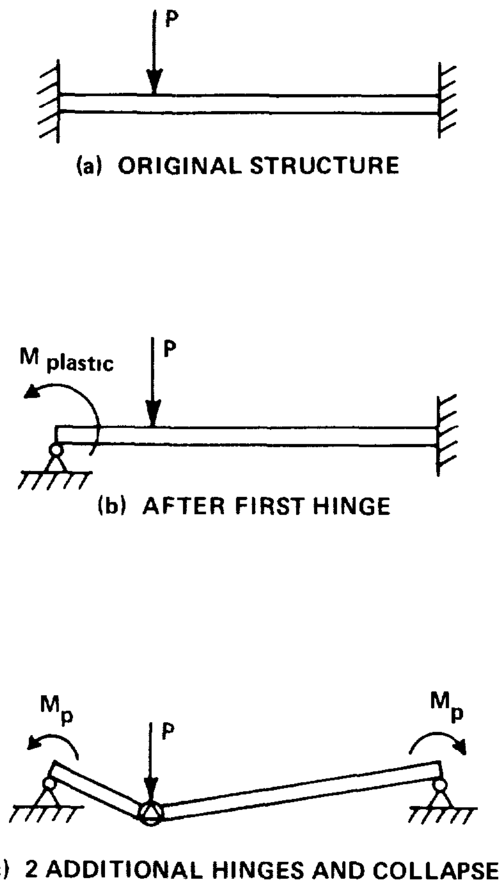
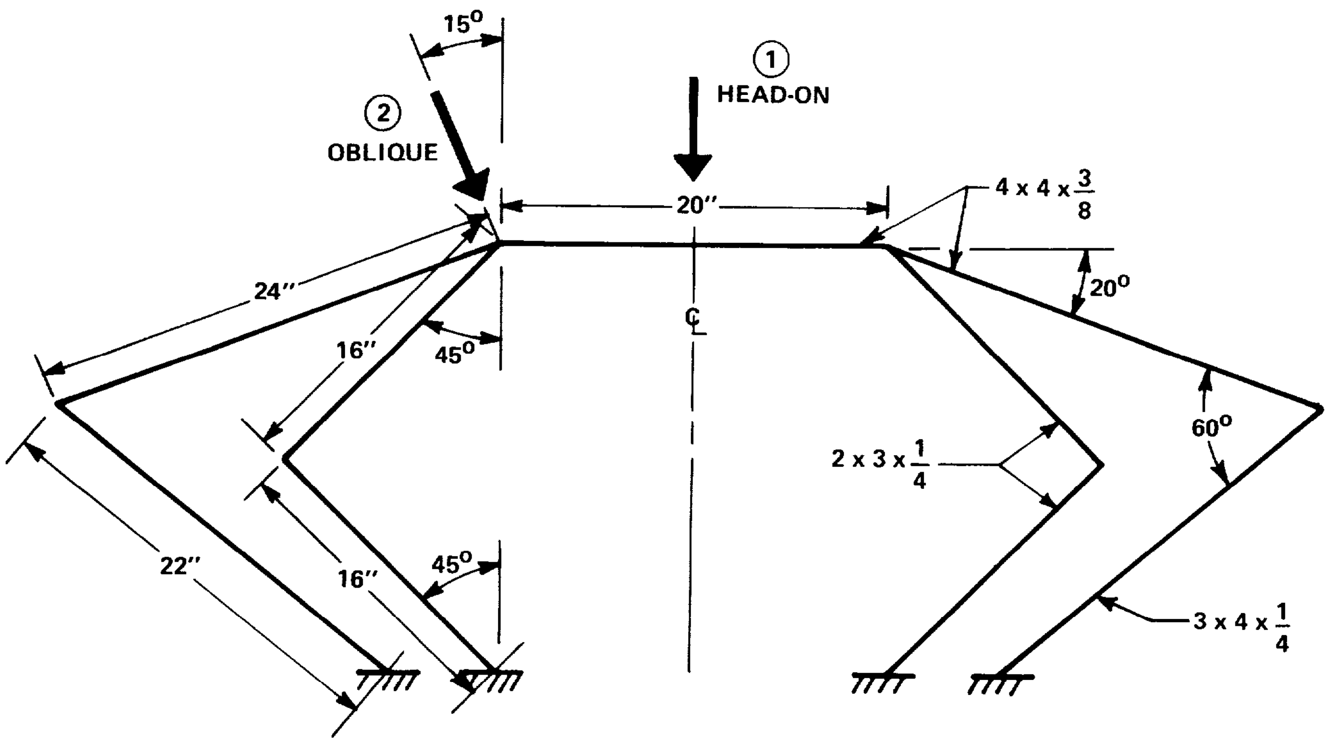


Figure I-1 HINGE FORMATION LEADING TO COLLAPSE



E = 30 x 10⁶ PSI
YIELD = 65,000 PSI

Figure I-2 MOD. 2 BUMPER & LOADING CONDITIONS

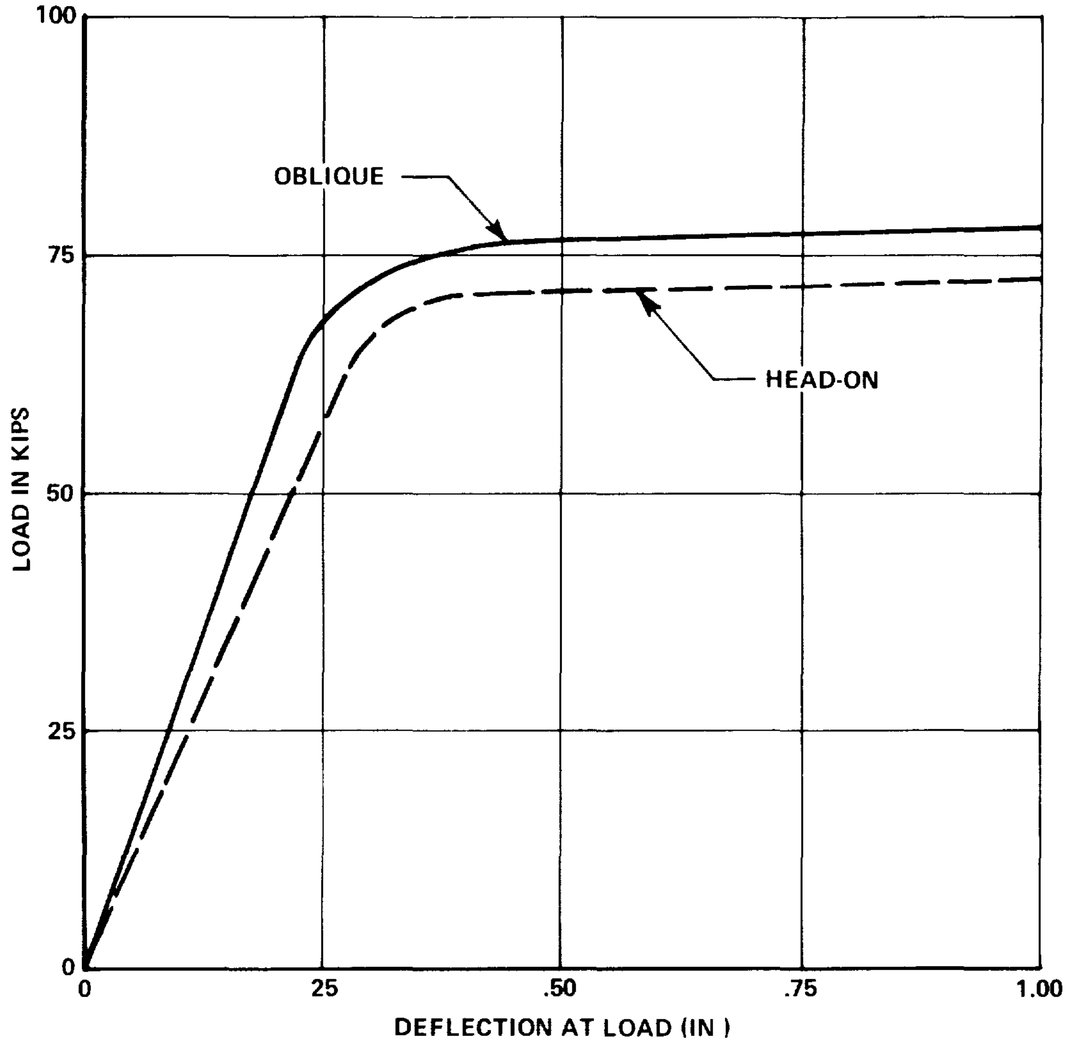


Figure I-3 THEORETICAL LOAD-DEFLECTION CURVES

Table I-1 Comparison of Two Load Conditions

	Head-On	Oblique
Stiffness (K/ft)	2730	3440
First hinge (K)	60.1	60.1
Collapse (K)	72.2	77.5
Rise time* (secs.)	.010	.009

* Estimated using a vehicle weight of 3600 lbs.

Once the data for a specific structure have been prepared, it is a simple matter to produce the above results for additional loading conditions (i.e., additional directions of impact). In contrast to other techniques of determining collapse loads, the method of this program is to "track" the structure up to collapse. This permits construction of the complete force-deflection curve and estimates of the initial stiffness of the vehicle. The stiffness is necessary not only for realistic estimates of rise time, but in determining relative severity in a two-vehicle collision (Reference 3).

Figures I-4 and I-5 show the collapse modes for the two loading conditions. The circled numbers at the joints of the figures refer to the sequence of hinge formation. A remarkable result of the analysis of the head-on collision, Figure I-4, is that hinge formation initiates at the joint furtherest away from the impact point and progresses forward. The rise time of the load and the small relative differences in loads between hinge formation establish that all hinges for both loading conditions form in less than two millisecs. Thus, for practical purposes hinge formation is instantaneous and the actual sequence could easily escape detection on high-speed film data. Figure I-6 presents photographs taken from the film coverage of the head-on collision, which should be compared with the predicted mode of Figure I-4. Comparable photographs are not available

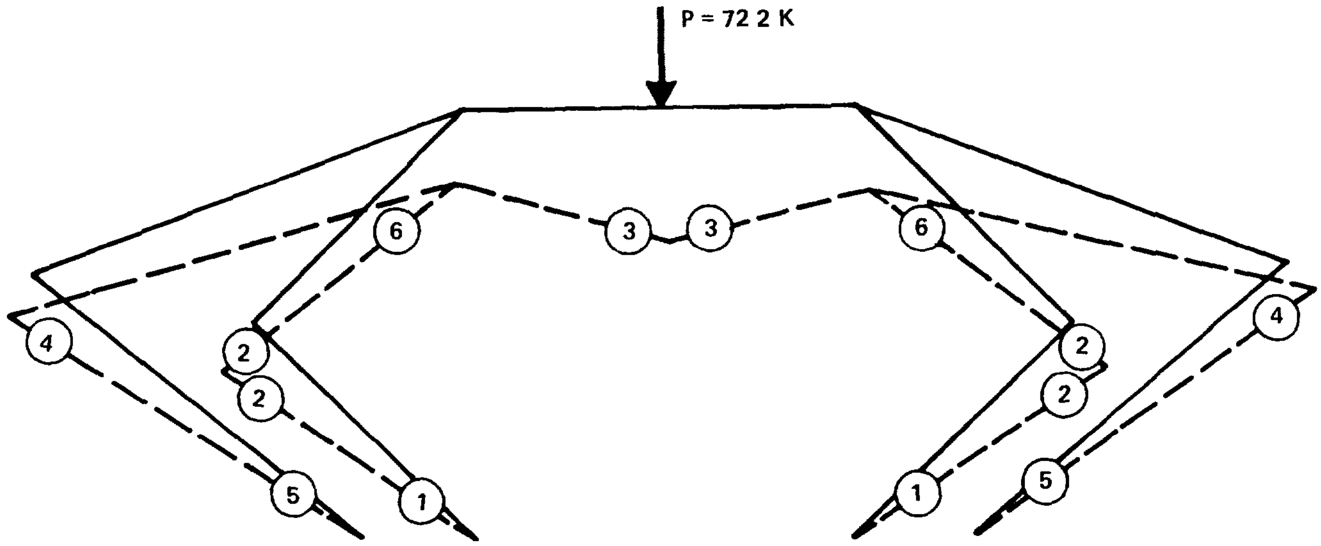


Figure I-4 HEAD-ON COLLISION COLLAPSE MECHANISM AND SEQUENCE OF HINGE FORMATION

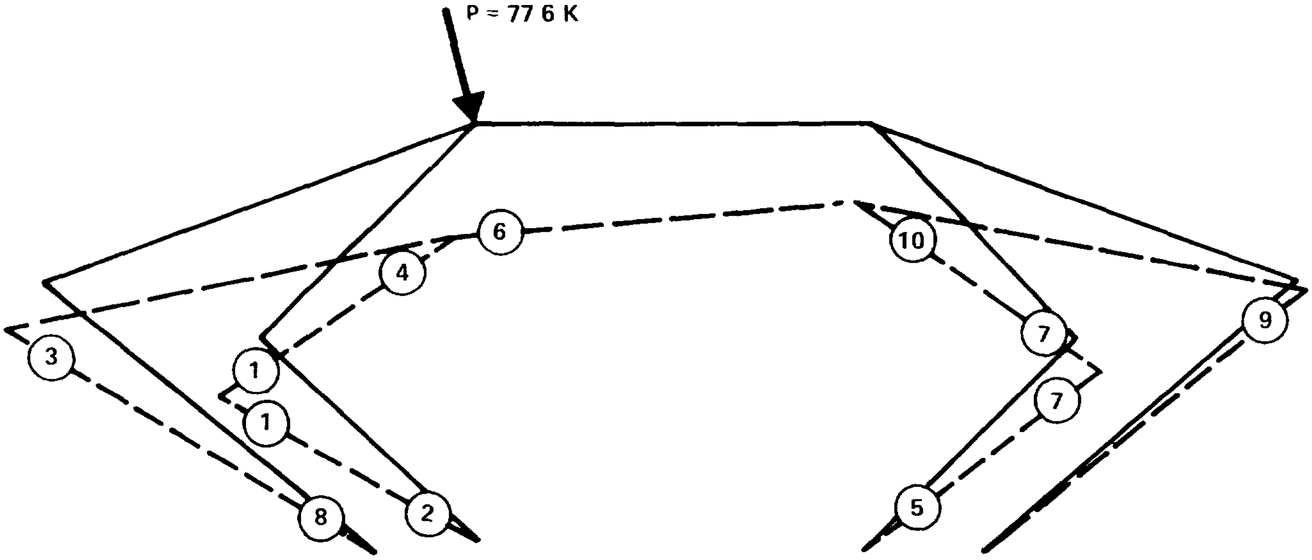
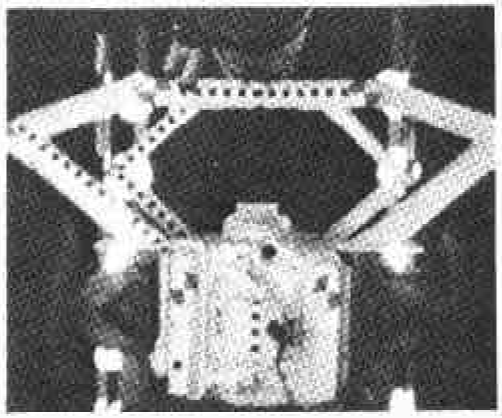
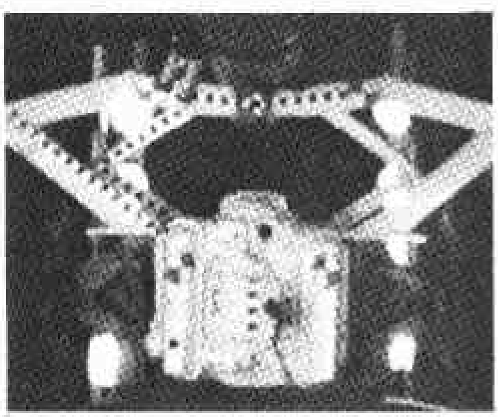


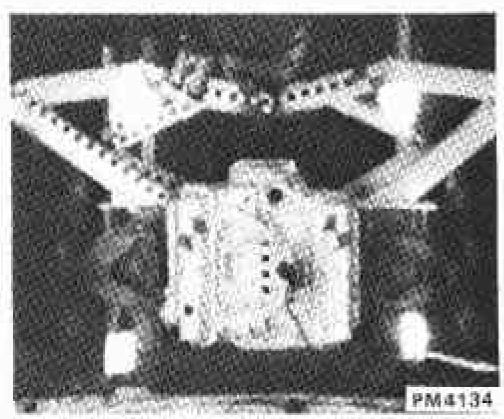
Figure I-5 OBLIQUE COLLISION COLLAPSE MECHANISM AND SEQUENCE OF HINGE FORMATION



t = 0 SEC



t = .0074 SEC



t = .0148 SEC

Figure I-6 HEAD-ON COLLISION COLLAPSE MECHANISM FROM HIGH SPEED MOVIE FILM for the oblique collision. The sequence of hinge formation in the oblique case, Figure I-5, shows an alternating pattern from impact to non-impact side. Therefore, the configuration should have the desirable ability to distribute the load away from the impact point and increase energy absorption. The results of the crash test verified this ability.

The rotation capacity of each joint at incipient collapse is also determined by the program. The technique involves the use of an elastic analysis of the structure prior to the formation of the last hinge. Figure I-7 gives some partial results of the hinge rotations for the oblique condition. If the rotations cannot be sustained (e.g., brittle failure) then the energy absorbed by the structure will be significantly decreased. The required rotation at each joint is highly dependent on the selected configuration.

3. CONCLUSIONS

The limitations of the structural model which is the basis for the program and the efforts required to reduce those limitations have been discussed at length in a previous study, Reference 4. An example of the information that such efforts can provide has been given for a structure for two different directions of impact. Although, some hindsight is present since the crash tests preceded the analysis, the program is useful in comparing optimal behavior of competing designs. The program could also be useful in studies to reduce the weight of the modified vehicle. In its present form, different loading conditions (corresponding to different impact directions) may be easily varied. To vary the structure, such as adding or deleting members, is cumbersome. The most immediate upgrading of the program for the uses envisioned would be to reduce the effort required to input a new configuration.

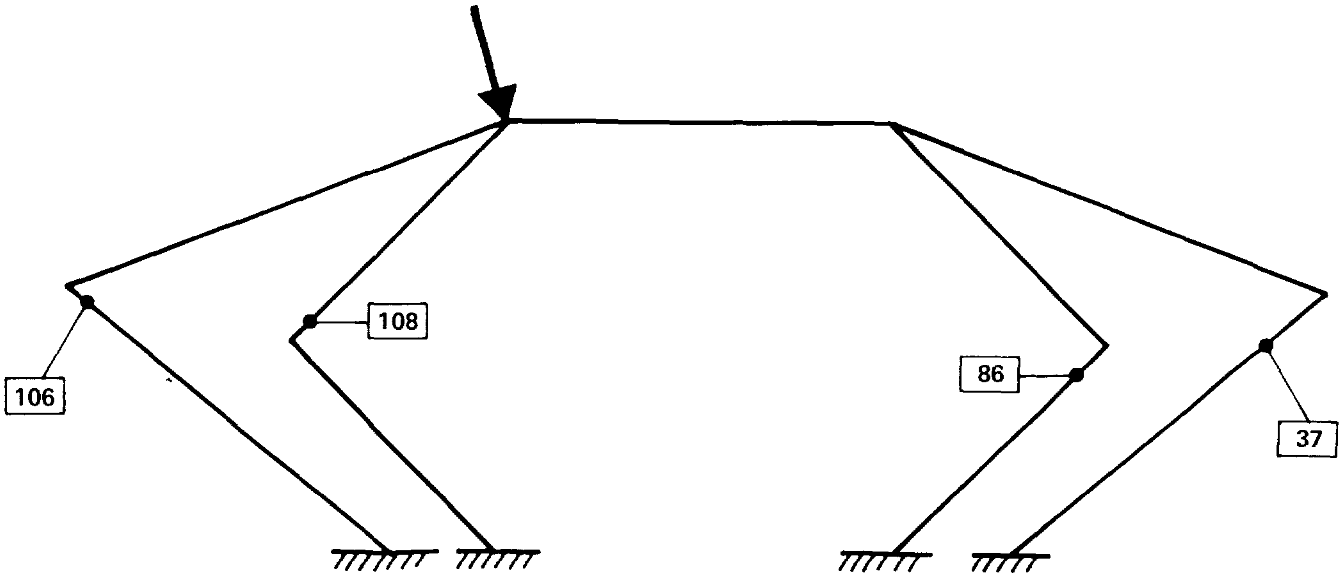


Figure I-7 OBLIQUE COLLISION REPRESENTATIVE ROTATIONS (MILLIRADS) AT INCIPIENT COLLAPSE

4. REFERENCES

1. Wang, C. K., "Matrix Methods of Structural Analysis", International, 1966.

2. Harrison, H. B., "The Elastic-Plastic Analysis of Plane Flexural Frames", Lehigh Univ. Rpt. No. 297.16, July, 1965.

3. I. D. Neilson, "Simple Representations of Car and Unrestrained Occupant Impacts in Road Accidents", Road Research Lab., Ministry of Transport, Report LR249, 1969.

4. Mayor, R. P. (Editor), "Basic Research Into Crashworthiness of Vehicle Structures", CAL Report No. YB-2498-V-1, July, 1968.

5. Miller, P. M. and Naab, K. N., "Basic Research in Automobile Crashworthiness - Testing and Evaluation of the Forward Structure Modification Concept", CAL Report No. YB-2684-V-1, September 1969.

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LUMPED MASS, NONLINEAR SPRING PROGRAM

by

Richard P. Mayor

November 1969

CAL REPORT YB-2684-V-5 PART II

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LIST OF FIGURES

LIST OF FIGURES (continued)

1. INTRODUCTION

The following describes a computer program developed as a part of the Basic Research in Crashworthiness project. The program enables the computation of time histories of responses of lumped masses connected by nonlinear springs, up to 30 degrees of freedom and up to five springs per mass can be treated. In applications of the program, it is necessary to represent the motion of each mass by a single coordinate; however, such simplified models have frequently been used to study a wide variety of impact problems.

Possible applications of the program to the experimental work are: (1) to aid in data interpretation and evaluation, especially where multiple data channels have been used and a simple single mass representation of the vehicle is inadequate, (2) parametric studies to determine and investigate vehicle properties in collisions involving combinations of modified and unmodified vehicles. Examples are included to indicate the type of problems that can be solved by the program.

2. FEATURES

- The equations of motion are assembled by the program using input information on how the masses and springs are connected. In this sense, a new program is developed for each specific problem without the necessity of driving the equations.

- Rapid turnaround time -- the program is written in BASIC+ for use on an on-line terminal.

- A variety of nonlinear springs, including hysteresis effects, can be accommodated.

- Rigid body motions are automatically treated by the program if they occur.

- Provision has been incorporated to determine response when the excitation is due to transient forces.

3. INPUT

Units

All input must be in compatible units which will also control the units of the output. In order to keep the program general, no conversion of units is performed within the program (e.g., ft/sec to mph). The following table illustrates compatible units that have been used in the sample problems.

MASS	FORCE	DISPLACEMENT	VELOCITY	ACCELERATION
$\frac{LB\text{-}SEC^2}{FT}$	LBS	FT	FT/SEC	FT/SEC^2
$\frac{LB\text{-}SEC^2}{IN}$	LBS	IN.	IN./SEC	IN/SEC^2
$\frac{KG\text{-}SEC^2}{METER}$	KILOGRAMS	METER	METER/SEC	$METER/SEC^2$

Control

Five pieces of input data are required to control the output and to enable the program to read in the necessary mass and spring data. These are:

Quantity	Program Variable	Description
Δt	T	Step size in numerical integration routine
t_s	T2	Stopping time, program halts when cumulative time = t_s
t_p	R2	Prints results every R2th step
--	J1	Number of masses
--	I1	Number of springs

· see Section 6 for selection of step size

Mass Data

Each mass is assigned a number in sequence. This assignment is arbitrary except that the number zero is reserved for an infinite mass, e.g., the barrier in a vehicle-barrier collision. The absence of a zero numbered mass indicates that rigid body motions are to be computed by the program. The mass data, then, consists of:

1. mass number,
2. magnitude in appropriate units,
3. initial displacement,
4. initial velocity.

A non-zero initial displacement implies that a force is applied on the mass at t = 0 (i.e., preset of the spring). If this is not the case, the initial displacement should be zero since the dynamic displacement of each mass is measured with reference to its initial position taken as zero (see Figure II-1). According to the sign convention adopted, a positive velocity denotes motion of a higher numbered mass away from the lower numbered mass.

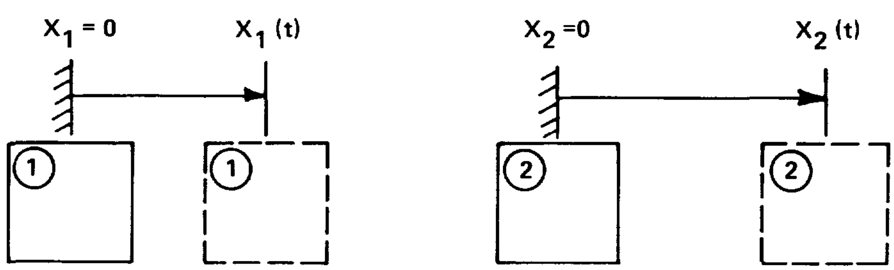


Figure II-1 COORDINATE SYSTEM FOR DISPLACEMENTS

Spring Representation

Each spring is represented as elasto- perfectly plastic, with unloading taking place parallel to the initial slope (Figure II-2). Pure elastic behavior is obtained by setting an arbitrary large value for the yield force. Figure II-3 shows how the spring will behave for various combinations of loading and unloading. An additional feature of the spring representation permits the specification of a delay, or null band, in which no force is exerted by the spring. Many impact problems can be modeled by a null band, or specified travel, that occurs before significant forces develop. This feature, in combination with the ability to use more than one spring between the same two masses, enables a wide variety of nonlinear springs to be simulated. Figure II-4 illustrates a spring which yields, followed by bottoming. The additional specification of a null band (or delay) for each spring requires that three quantities are needed as input for each spring. These are shown in terms of the program variables in Figure II-5. Note that symmetrical behavior is assumed for tension and compression.

Connectivity

In addition to the three spring properties for each spring, information on how the masses and springs are interconnected is required. This is done by sequentially numbering each spring beginning with 1. The order is arbitrary. Next, connections are specified by giving the previously assigned mass numbers at either end of the spring. Up to five different springs can be connected between the same two masses. The positive force direction (tension) is specified by the convention that the spring connects the lower numbered mass (first input) to the higher numbered mass (second input).

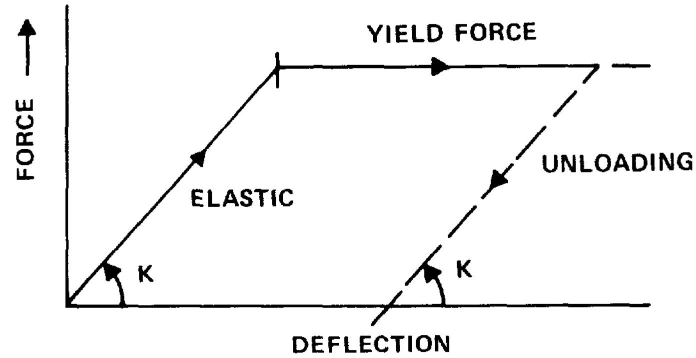
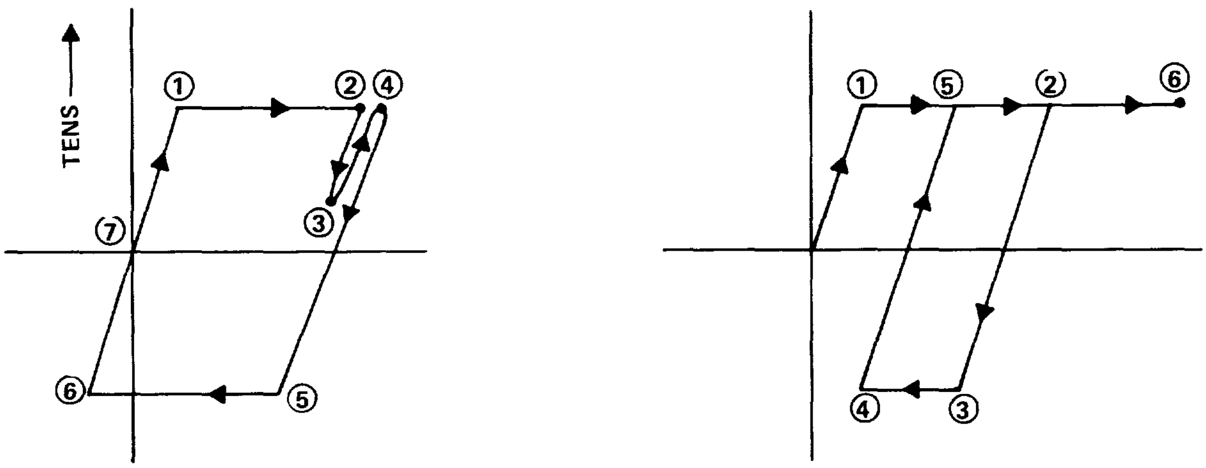


Figure II-2 ELASTO-PLASTIC SPRING



(a)

① TENSION YIELD
②-③ UNLOADING
③-④ RELOADING
④ ⑤ UNLOADING
⑤-⑥ COMPR. YIELD
⑥-⑦ RELOADING

(b)

① TENSION YIELD
②-③ UNLOADING
③-④ COMPR. YIELD
④-⑤ RELOADING
⑤-⑥ TENSION YIELD

Figure II-3 SPRING BEHAVIOR FOR VARIOUS LOADING & UNLOADING CYCLES

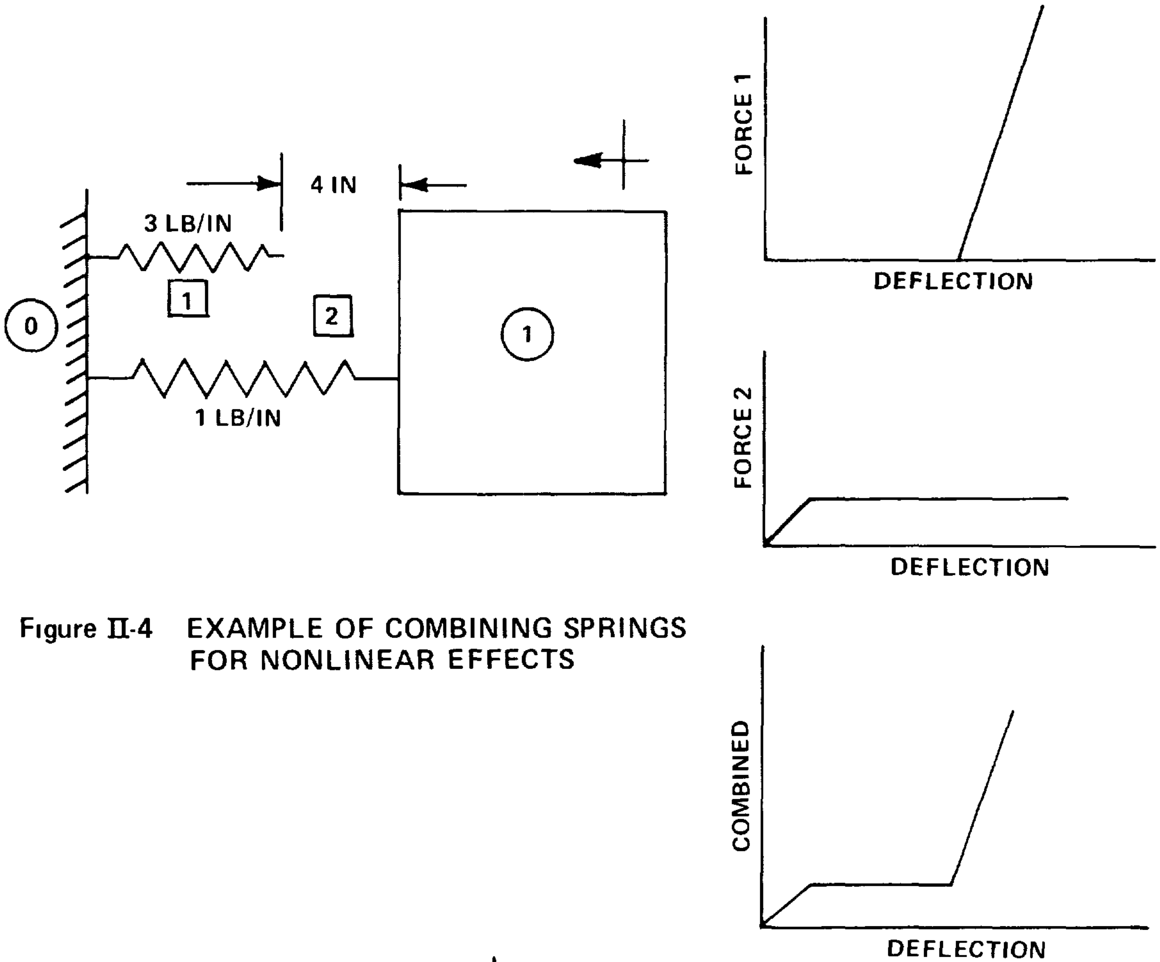


Figure II-4 EXAMPLE OF COMBINING SPRINGS FOR NONLINEAR EFFECTS

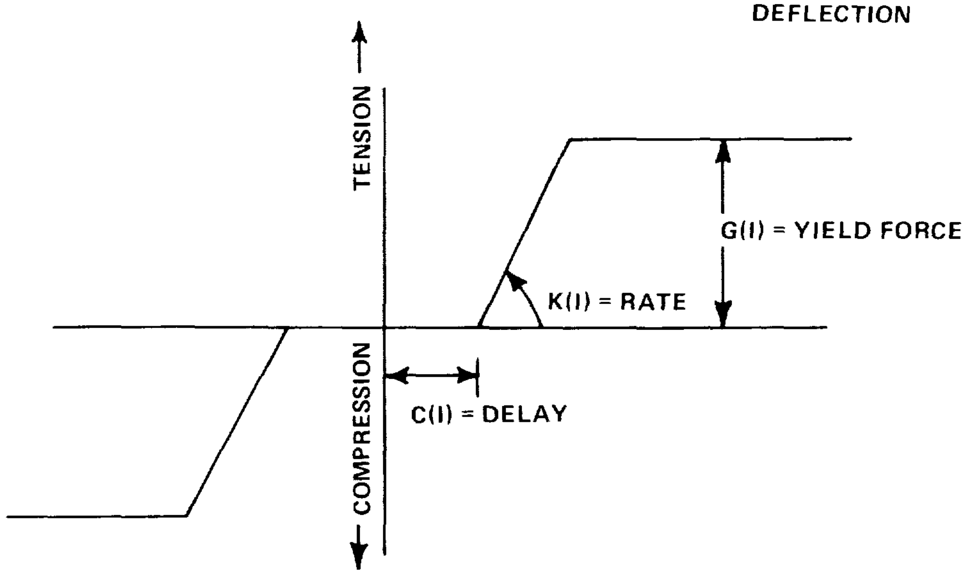


Figure II-5 PROPERTIES OF I TH SPRING

Summary of Required Input

Control: Step size (T), stopping time (T2), print interval (R2); number of masses (J1), number of springs (I1).

Mass: J = 1 to J1. Mass number (J), mass (M); initial displacement (X(J)); initial velocity (V(J)).

Spring: I = 1 to I1. Spring number (I), from mass number (N(I, 1)), to mass number (N(I, 2)); spring rate (K(I)), yield force (G(I)), delay (C(I)).

4. EXAMPLES

The sample problems consist of two types. First, checkout problems which involve a direct comparison with closed form solutions or alternative numerical solutions; the second type, demonstration problems, are to demonstrate the types of problems solved by the program. These latter types have been checked for reasonableness and internal consistency of results.

Checkout Problem 1: Rear End Collision

Model:

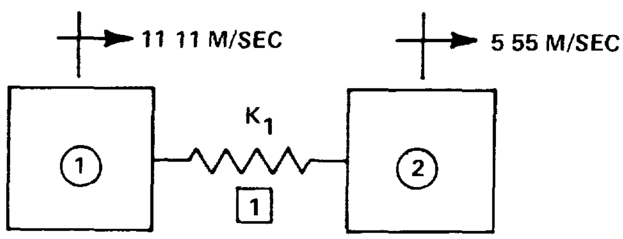


Figure II-6 REAR END COLLISION; PROB. 1

Data:

$M_1 = 200 \frac{kg\text{-}sec^2}{m}$ $\quad$ $V_1(0) = 11.11$ m/sec

$M_2 = 200 \frac{kg\text{-}sec^2}{m}$ $\quad$ $V_2(0) = 5.55$ m/sec

$K_1 = 70,000$ kg/m

Control: t = .0025, t_s .06 (stop time), print interval = 4

Spring: Because the spring is to remain elastic, an arbitrarily high value of one million is selected as the yield force for the spring. The delay is zero.

Connection:

Spring	From Mass	To Mass
1	1	2

Computer Input:

```
1490 DATA  .0025, .06, 4, 2, 1
1500 DATA  1,200, 0, 11.11
1510 DATA  2,200, 0, 5.55
1520 DATA  1, 1, 2, 70000, 1000000, 0
```

Results: The results of this problem are plotted in Figure II-7. The equations for the exact solution are given by Marquard, Reference 1. The velocities and displacements are the same for both the computer solution and the exact, but slight differences (.1 m/sec^2) are detectable in the accelerations. A sample of the computer output is given in Figure II-8. The X-REL column is the spring deflection, i.e., the difference in displacements between the two masses to which the spring is connected. The remaining values are intended to be self-explanatory.

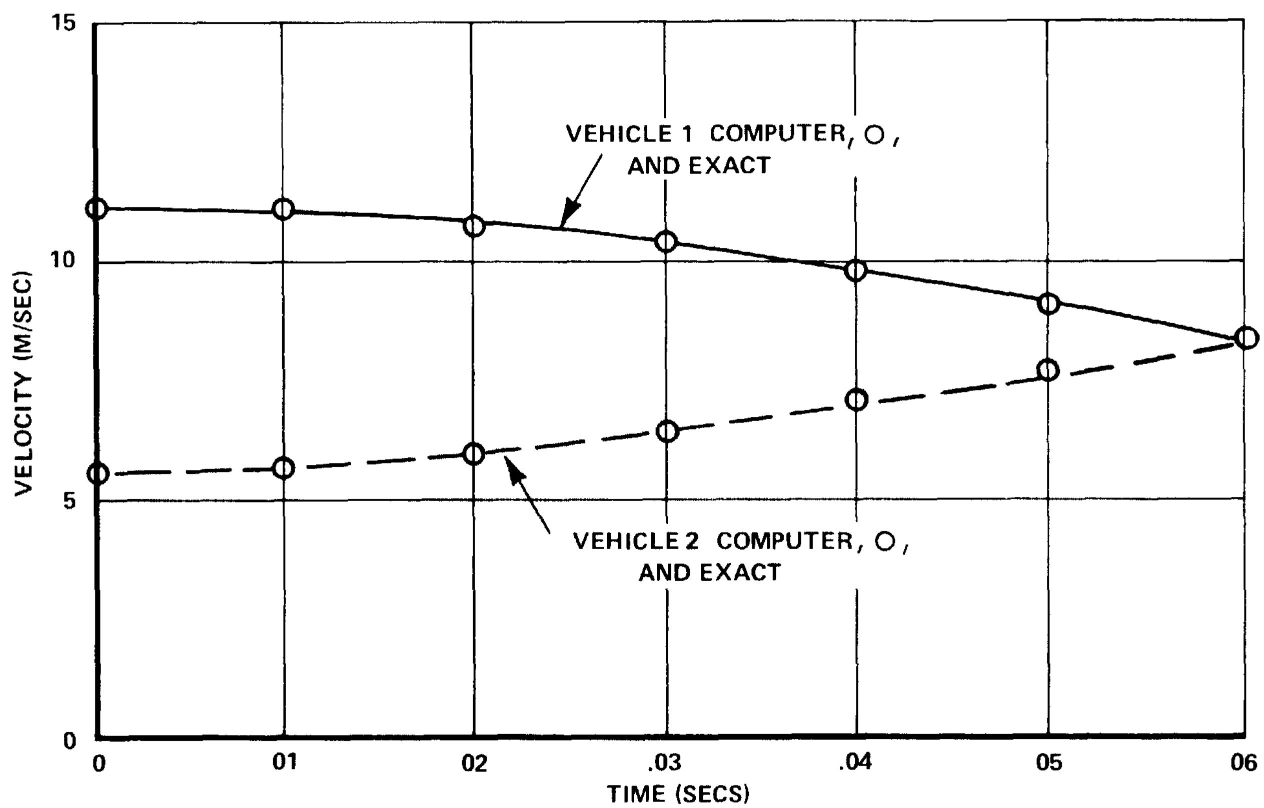


Figure II 7 COMPARISON OF EXACT AND COMPUTER SOLUTION FOR REAR END COLLISION, PROB. 1

DELTA T= .00250 LAST T= .08000 ELEMENTS= 4 NO MASSES= 2 NO. [illegible] = 1

MASS NO	MASS	X(0)	V(0)
1	200	.00	11.11
2	200	.00	5.55

NO	M1	M2	K	YIELD	DELAY
1	1	2	70000	100000	.01

RIGID BODY VELOCITY= 8.33

TIME	NO	FORCE	X-DIST	MASS	X-ABS	VELOC	ACCEL
.000	1	0	.000	1	.00	11.11	.00
					.00	5.55	.00
.010	1	-3650	-.05	1	.03	11.01	-18.25
				2	-.03	5.65	18.25
.020	1	-7431	-.11	1	.05	10.73	-37.16
				2	-.05	5.93	37.16
.030	1	-10495	-.15	1	.07	10.20	-52.48
				2	-.07	6.33	52.48
.040	1	-12828	-.18	1	.09	9.62	-64.14
				2	-.09	6.77	64.14
.050	1	-14270	-.20	1	.10	9.01	-71.35
				2	-.10	7.65	71.35
.060	1	-14716	-.21	1	.11	8.28	-73.58
				2	-.11	8.38	73.58
.070	1	-14138	-.20	1	.10	7.56	-70.69
				2	-.10	9.10	70.69

[illegible]

Figure II-8 SAMPLE OUTPUT, PROB. 1

Checkout Problem 2: Multi-Connected Two Degree System

Model:

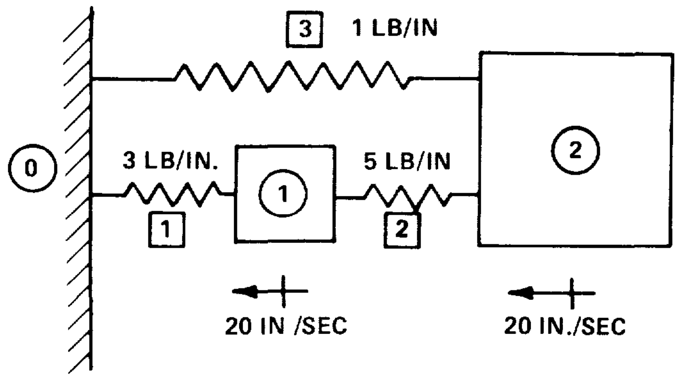


Figure II-9 MULTI-CONNECTED 2 DEGREE SYSTEM; PROB. 2

Data:

$M_1 = 1.0 \text{ lb-sec}^2/\text{in.}$ $\quad V_1(0) = -20 \text{ in/sec}$

$M_2 = 2.0 \text{ lbs-sec}^2/\text{in.}$ $\quad V_2(0) = -20 \text{ in/sec}$

$K_1 = 3.0 \text{ lbs/in.}$ $\quad K_2 = .5 \text{ lb/in.}$ $\quad K_3 = 1 \text{ lb/in.}$

Control: $t = .025$, $t_s = 2.0$, print interval $= 4$

Spring: Since the springs are to remain elastic, all yield forces are given an arbitrarily high value of 1000.

Connection:

Spring	From Mass	To Mass
1	0	1
2	1	2
3	0	2

Computer Input:

```
1490 DATA  .025, 2, 4, 2, 3
1500 DATA  1, 1.0, 0.0, -20
1510 DATA  2, 2.0, 0.0, -20
1520 DATA  1, 0, 1, 3, 1000, 0
1530 DATA  2, 1, 2, .5, 1000, 0
1540 DATA  3, 0, 2, 1.0, 1000, 0
```

Results: The difference between the computer results and the exact results (Reference 2) for peak values of displacement are indicated in Figure II-10. Use of a smaller time step in the integration routine would reduce the difference.

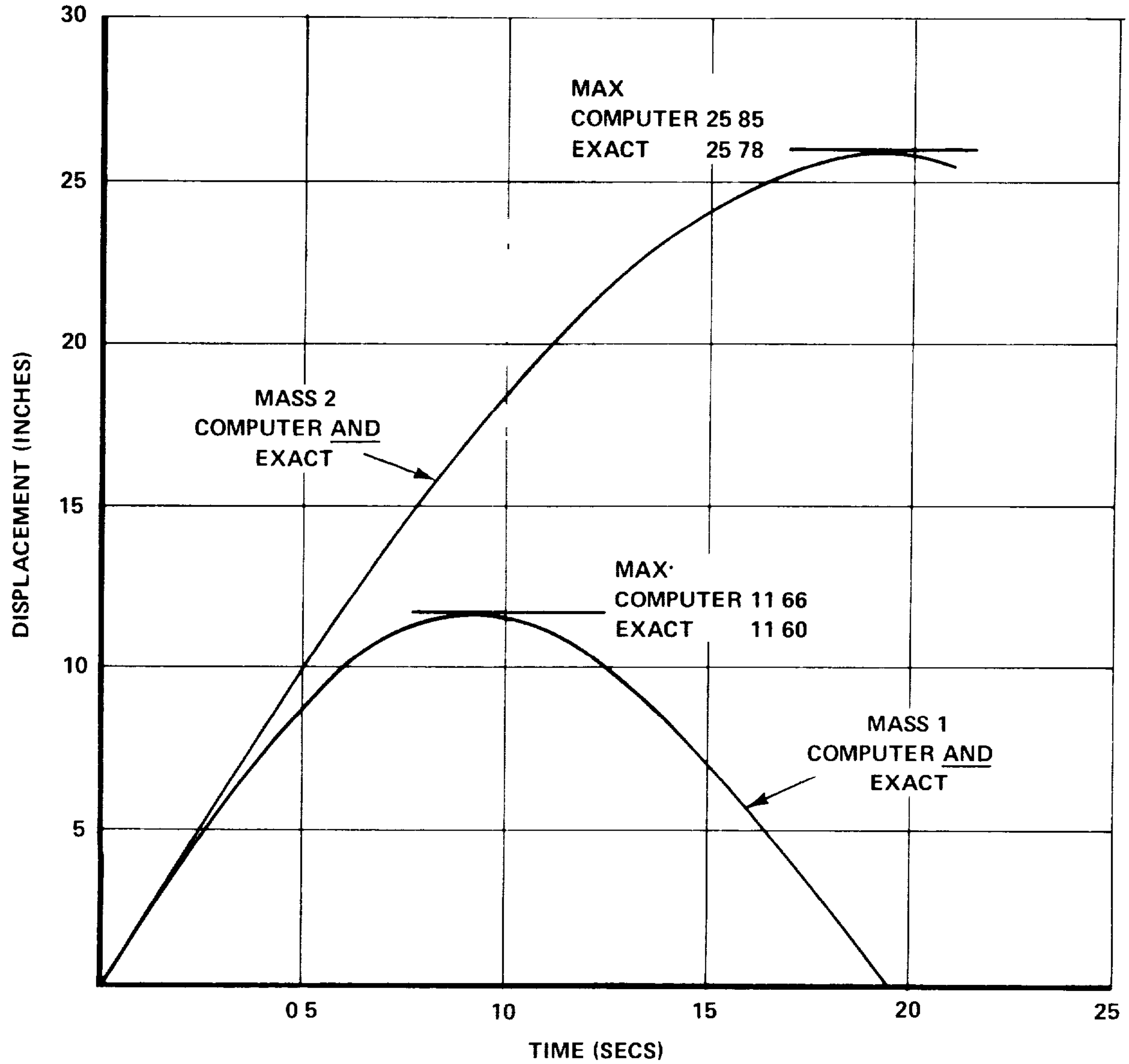


Figure II-10 COMPARISON OF EXACT AND COMPUTER SOLUTION FOR IMPACT OF MULTI-CONNECTED TWO DEGREE SYSTEM,PROB. 2

Checkout Problem 3: Forced Response of Two Degree System

Model:

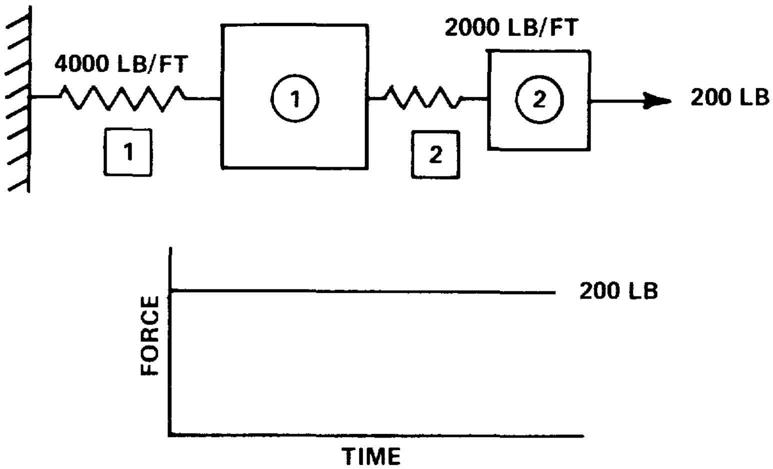


Figure II-11 FORCED RESPONSE OF 2 DEGREE SYSTEM; PROB. 3

Data:

M_1 = 2.0 lbs-sec^2/ft		$V_1(0)$ = 0	
M_2 = 1.0 lb-sec^2/ft		$V_2(0)$ = 0	
K_1 = 4000 lbs/ft		K_2 = 2000 lbs/ft	

Computer Input: To account for the applied force on mass 2, it is necessary to include the time variation of the force in the applied force subroutine. In this case, the force is constant with time, so the following statement is inserted in the program (see listing):

```
1465    E(2) = 200.
```

Additional forces and more complex time variations could be similarly treated. The remaining computer input follows that of the preceding examples:

```
1490 DATA  .0025, .21, 4, 2, 2
1500 DATA  1, 2.0, 0, 0
1510 DATA  2, 1.0, 0, 0
1520 DATA  1, 0, 1, 4000, 1000000, 0
1530 DATA  2, 1, 2, 2000, 1000000, 0
```

Results: In Figure II-12 the results are compared not to the exact solution but to those of a alternative numerical method given in Reference 3. It can be seen that the major difference between the two methods is due to a slight phase shift, while good agreement exists at peak values.

Demonstration Problem 4: Rear Collision with Various Springs

This problem illustrates the use of the program in parametric studies. Specifically, the effects of three different force-deflection characteristics on the deceleration response are investigated.

Model: See Figure II-13.

Computer Input: The computer input for the three cases is given below. Note that the elastic spring is modeled by using a very large yield force and also the use of two springs with a delay to simulate the bottoming effect.

```
Elastic  1490 DATA  .05, 1.2, 4, 2, 1
         1500 DATA  1, 5.0, 0, 20
         1510 DATA  2, 20.0, 0, 10
         1520 DATA  1, 1, 2, 36, 3600, 0

Plastic  1490 DATA  .05, 1.2, 4, 2, 1
         1500 DATA  1, 5.0, 0, 20
         1510 DATA  2, 20.0, 0, 10
         1520 DATA  1, 1, 2, 36, 72, 0
```

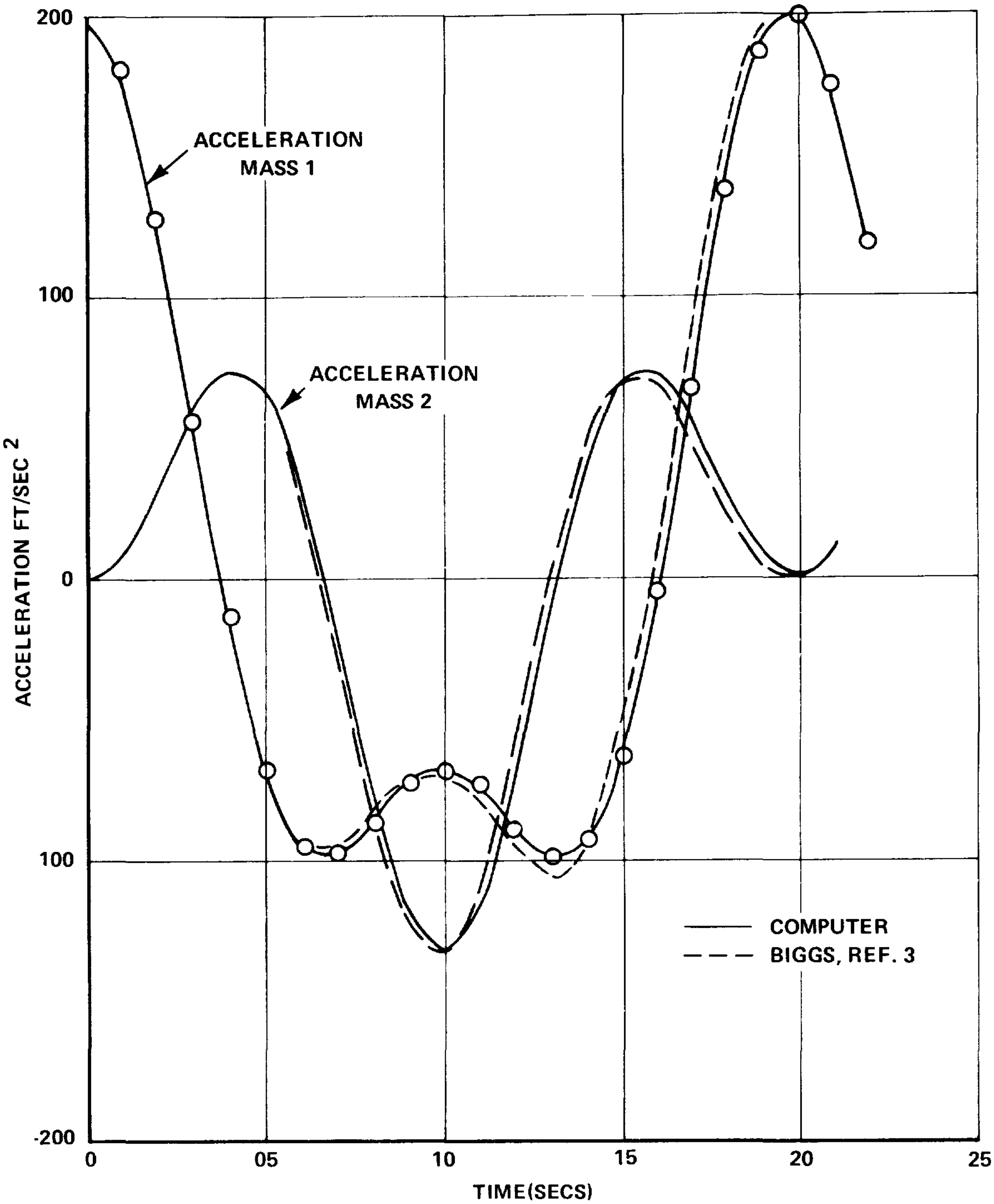
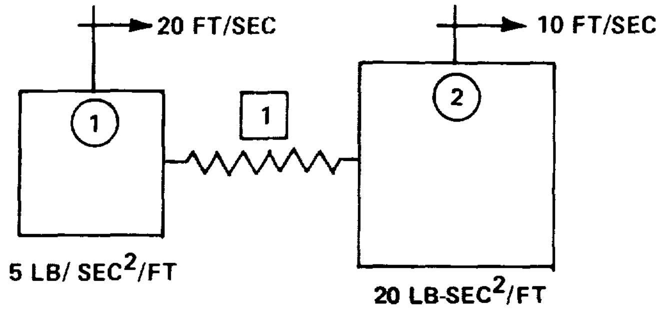
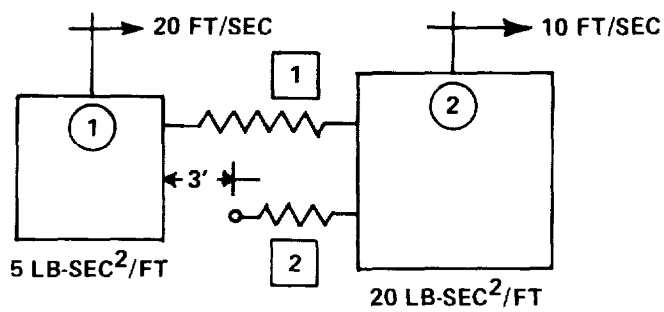


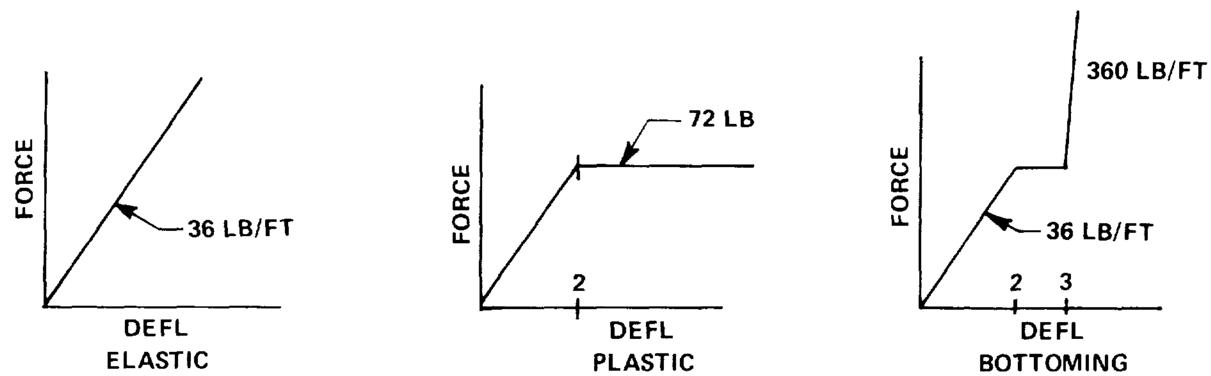
Figure II-12 COMPARISON OF TWO NUMERICAL SOLUTIONS: FORCED RESPONSE OF TWO DEGREE SYSTEM; PROB. D-3



(a) ELASTIC AND ELASTO-PLASTIC SPRING



(b) BOTTOMING SPRING



(c) SPRING MODELS

Figure II-13 MODEL AND SPRINGS: REAR COLLISION, PROB. 4

Bottoming 1490 DATA .05, 1.2, 1, 2, 2

1500 DATA 1, 5.0, 0, 20

1510 DATA 2, 20.0, 0, 10

1520 DATA 1, 1, 2, 36, 72, 0

1530 DATA 2, 1, 2, 360, 72000, 3

Results: The deceleration time history of the small mass for all three cases is shown in Figure II-14. As expected, the elastic spring results in a half-sine wave. The effects of the plastic and bottoming springs are also readily apparent.

Demonstration Problem 5: Experimental Matching Head-On Collision

The problem illustrates an iterative use of the program to match experimental data, Reference 4. By this method it is possible to determine the effective spring parameters in a collision. The effective spring can then be used to predict response for collisions other than that of the experiment. Only the computer solution giving the best match to the experimental data is given.

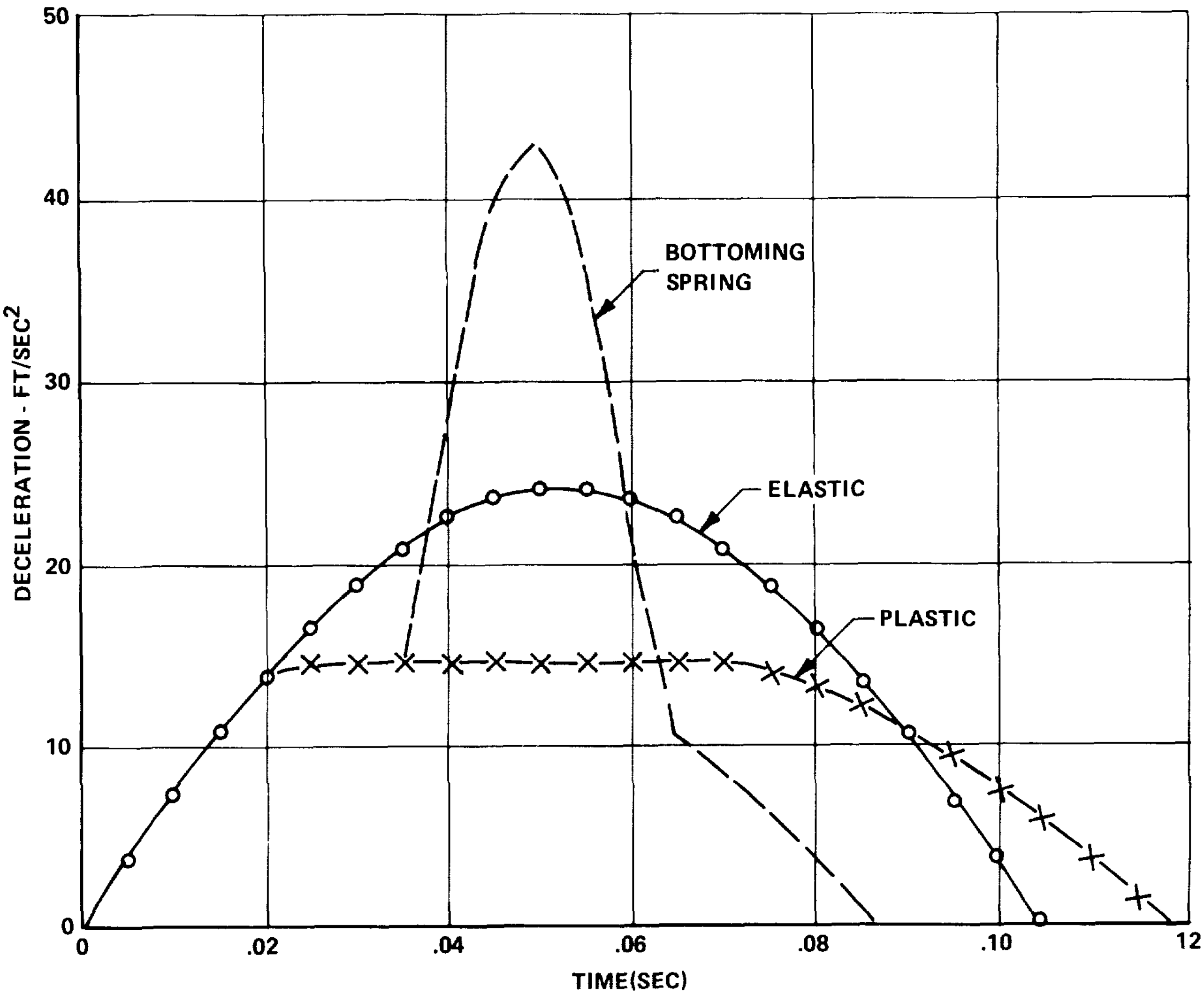


Figure II-14 EFFECT OF DIFFERENT SPRINGS; REAR COLLISION, PROB. 4

Model:

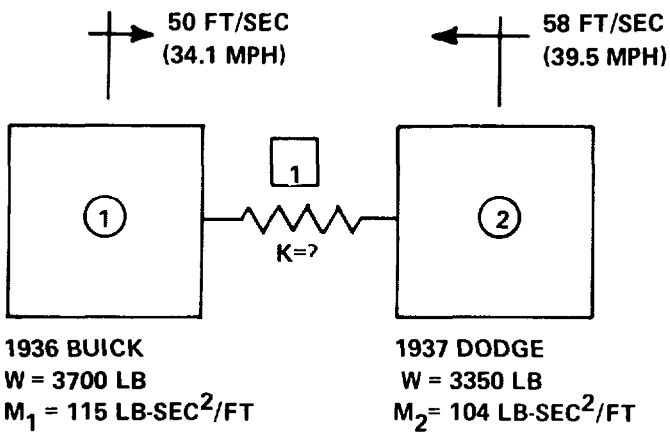


Figure II-15 HEAD-ON COLLISION, PROB. 5

Computer Input:

```
1490 DATA   .0025, .140, 8, 2, 1
1500 DATA   1, 115, 0, 50.0
1510 DATA   2, 104, 0, -58.0
1520 DATA   1, 1, 2, 20000, 53624
```

Results: Figure II-16 compares the best match computer solution to the experimental results. The corresponding elasto-plastic spring is shown in Figure II-17. The spring approximates the combined effects of the crush characteristic of both vehicles. There is insufficient data to determine springs for the individual vehicles.

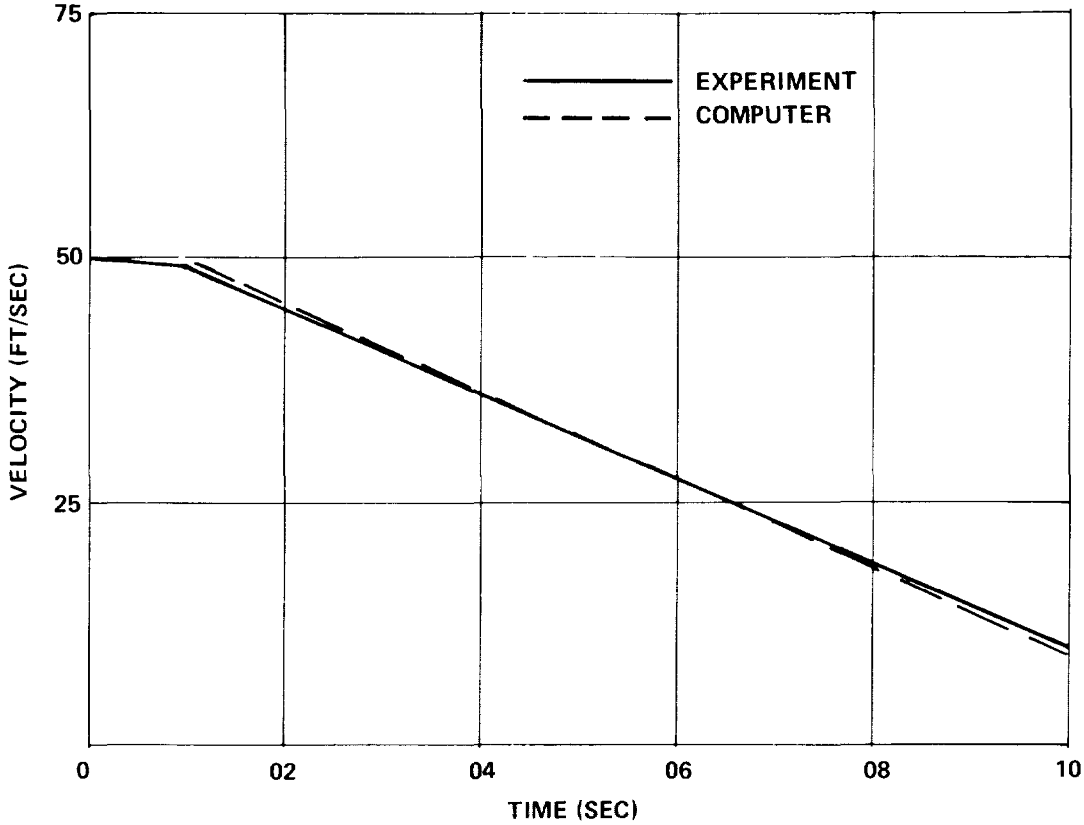


Figure II-16 COMPARISON OF EXPERIMENTAL AND COMPUTER SOLUTION HEAD-ON COLLISION, PROB. 5

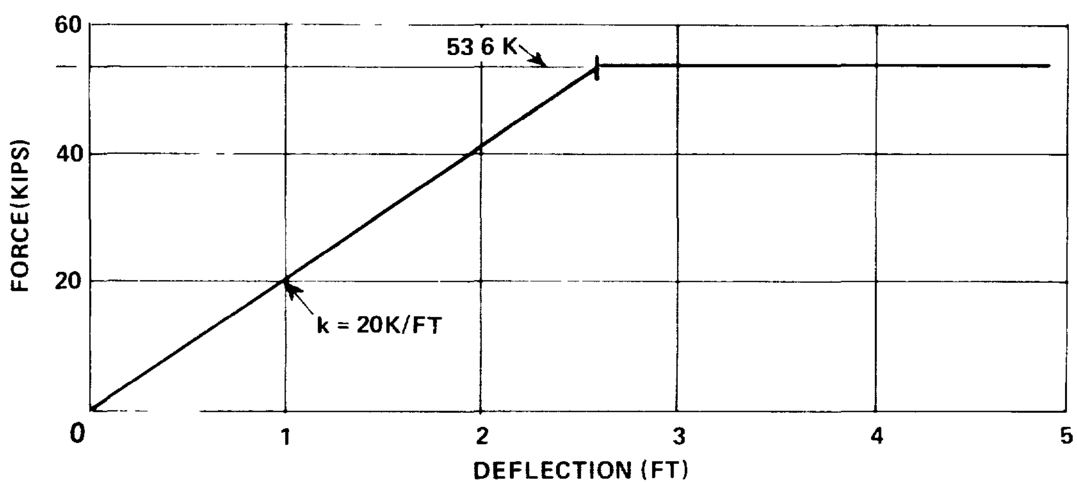


Figure II-17 EFFECTIVE SPRING FOR BOTH VEHICLES

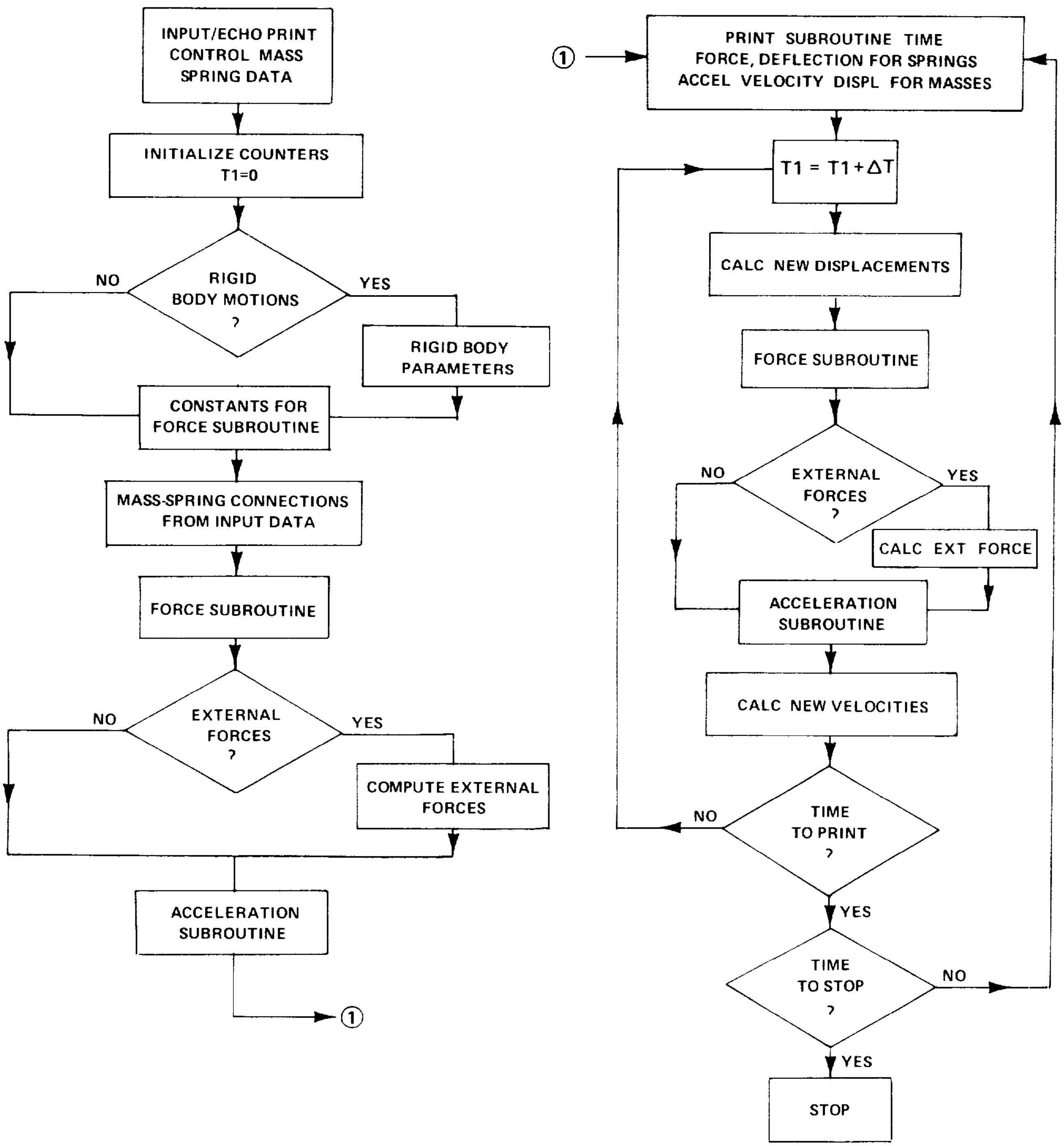


Figure II-18 MACRO FLOW CHART

Subroutines

As shown in the flow chart of Figure II-18, three subroutines are used in the program. The following gives a brief description of their functions.

Subroutine Print: This subroutine outputs the data in the format shown in the sample listing of Figure II-8.

Subroutine Force: Once the displacements of all masses are known, this routine calculates the spring deflections (i.e., the difference in displacements of the masses which are connected to the ends of the spring). From the deflections, the corresponding spring forces can be calculated.

Subroutine Acceleration: This subroutine computes the acceleration of each mass according to the formula:

$$a_i = \frac{\sum F_s + F_e}{M_i}$$

where

$\sum F_s$ = the algebraic summation of the forces of all springs connected to the mass,

F_e = is the external force, if any,

M_i = the mass.

Once the accelerations have been determined, the numerical integration scheme is used to calculate displacements and velocities for the next time interval.

RMN12

```
10 REM "RMN12" SEP 12,1969
20 DIM A(30),B(30),C(50),F(30),G(50),H(6,30),K(50),L(50)
30 DIM M(30),N(50,2),P(30),S(50,4),V(30),X(30)
40 READ T,T2,R2,J1,I1
50 SET T=1.5\SET P=6.0\SET S=X=C=3.2\SET V=A=5.2
60 PRINT
70 PRINT"DELTA T=";T;" TSTØP=";T2;" PRNT INT=";R2;" NØ MASS=";J1:
80 " NO. SPR=";I1
90 REM T1=CUM TIME:R1=CUM CØUNT:T1,R1 TIME CØUNT
100 REM R3=CUM MØMENTUM:R4=CUM MASS:R5=R/B VELØC
110 REM INIT:R6=0,NØ WALL
120 R1=T1=R3=R4=R6=X(0)=V(0)=0
130 LET R7=R2
140 PRINT
150 PRINT "MASS NO","MASS",TAB(32);"X(0)";TAB(48);"V(0)"
160 FØR Q=1 TØ J1
170 READ J,M(J),X(J),V(J)
180 PRINT J,M(J),X(J),V(J)
190 NEXT Q
200 PRINT
210 REM:     SPRING DATA
220 PRINT"NO ";" M1";" M2","K","YIELD","DELAY"
230 FØR Q=1 TØ I1
240 SET K=G=6.0\SET T=2.3
250 READ I,N(I,1),N(I,2),K(I),G(I),C(I)
260 IF R6<>0 THEN 310
270 IF N(I,1)=0 THEN 300
280 IF N(I,2)=0 THEN 300
290 GØ TØ 310
300 LET R6=1
310 PRINT I;N(I,1);N(I,2);K(I),G(I),C(I)
320 NEXT Q
330 IF R6<>0 THEN 430
340 FØR J=1 TØ J1
350 LET R4=R4+M(J)
360 LET R3=R3+M(J)*V(J)
370 NEXT J
380 LET R5=R3/R4
390 FØR J=1 TØ J1
400 LET V(J)=V(J)-R5
410 NEXT J
420 PRINT "RIGID BØDY VELØCITY=";R5\GØ TØ 440
430 R5=0
440 PRINT "  TIME";" SPR";"  FØRCE":
450 TAB(12);"X-REL";TAB(25);"M";TAB(30);"X-ABS";TAB(37);"VELØC":
460 TAB(47);"ACCEL"
470 REM* CØNST FØR FØRCE SUBR;S(I,1)=INIT ELAS DEFL LIMIT;G(I)=YLD
480 REM K(I)=SPR RATE;C(I)=DELAY
490 FØR I=1 TØ I1
500 LET S(I,1)=G(I)/K(I)
```

RMN12 CØNTINUED

```
510 LET S(I,2)=0
520 LET S(I,3)=S(I,2)-2*S(I,1)
530 NEXT I
540 REM            TØPØLØGY
550 FØR J=0 TØ J1
560 LET L(J)=0
570 REM L(J) WILL CØNTAIN ØT NØ ØF SPR TØ JTH MASS
580 NEXT J
590 FØR I=1 TØ I1
600 FØR Q=1 TØ 2
610 LET J=N(I,Q)
620 LET L(J)=L(J)+1
630 LET H(L(J),J)=I
640 REM H(1,NØ SPRGS CØNN TØ M(J);MASS NAME) = SPR NAME
650 NEXT Q
660 NEXT I
670 REM       TØ CALC A(Q),F(Q)
680 GØ SUB 1060
690 GØ SUB 1310
700 GØ SUB 730
710 GØ TØ 880
720 REM: PRNT SUBR; S(I,3) SAVES REL DISPL FRØM FØRCE SUBR
730 Q=1\V1=V(Q)+R5
740 PRINT T1;Q;P(Q);S(Q,3);Q;X(Q);V1;A(Q)
750 FØR Q=2 TØ I1
760 IF Q>J1 THEN 800
770 V1=V(Q)+R5
780 PRINT TAB(7);Q;P(Q);S(Q,3);Q;X(Q);V1;A(Q)
790 GØ TØ 810
800 PRINT TAB(7);Q;P(Q);S(Q,3)
810 NEXT Q
820 IF J1<=I1 THEN 870
830 FØR Q=I1+1 TØ J1
840 V1=V(Q)+R5
850 PRINT TAB(24);Q;X(Q);V1;A(Q)
860 NEXT Q
870 RETURN
880 LET R1=R1+1
890 LET T1=R1*T
900 FØR J=1 TØ J1
910 LET B(J)=A(J)
920 LET X(J)=X(J)+V(J)*T+B(J)*T*T/2
930 NEXT J
940 GØ SUB 1060
950 GØ SUB 1310
960 FØR J=1 TØ J1
970 LET V(J)=V(J)+(A(J)+B(J))*(T/2)
980 NEXT J
990 REM TEST FØR PRINT
1000 IF R1<R2 THEN 880
```

RMN12 CONTINUED

```
1010 LET R2=R1+R7
1020 GO SUB 730
1030 IF T1<T2 THEN 880
1040 STOP
1050 REM          SUB FORCE: CALC ALL SPRNG FORCES
1060 FOR I=1 TO I1
1070 LET U1=N(I,1)
1080 LET U2=N(I,2)
1090 LET U=X(U2)-X(U1)
1100 LET S(I,3)=U
1110 IF U<S(I,2)-C(I)-S(I,1) THEN 1180
1120 IF U<S(I,2)-C(I) THEN 1210
1130 IF U<S(I,2)+C(I) THEN 1230
1140 IF U<S(I,2)+C(I)+S(I,1) THEN 1250
1150 LET F=G(I)
1160 LET S(I,2)=U-S(I,1)-C(I)
1170 GO TO 1260
1180 LET F=-G(I)
1190 LET S(I,2)=U+S(I,1)+C(I)
1200 GO TO 1260
1210 LET F=K(I)*(U-(S(I,2)-C(I)))
1220 GO TO 1260
1230 LET F=0
1240 GO TO 1260
1250 LET F=K(I)*(U-(S(I,2)+C(I)))
1260 LET P(I)=F
1270 NEXT I
1280 RETURN
1290 REM SUB ACCEL:E(J) PROV. FOR APPL FORCE
1300 REM J=0: WALL
1310 FOR J=1 TO J1
1320 LET D=0
1330 FOR Q=1 TO L(J)
1340 LET I=H(Q,J)
1350 REM IF J IS M1 END,+ IN MOTION EQ
1360 IF J=N(I,1) THEN 1390
1370 LET D=D-P(I)
1380 GO TO 1400
1390 LET D=D+P(I)
1400 NEXT Q
1410 GO SUB 1450
1420 LET A(J)=(D+E(J))/M(J)
1430 NEXT J
1440 REM          SUB APPL FORCE
1450 FOR J=1 TO J1
1460 LET E(J)=0
1470 NEXT J
1480 RETURN
1490 DATA
1500 END
```

6. METHOD OF NUMERICAL INTEGRATION

The method of integration used in the program is the constant acceleration method. The necessary relationships can be derived by assuming that the acceleration between two times separated by Δt is constant. If the acceleration is indeed constant, then exact results are obtained, otherwise results are approximate. In general, an approximate method introduces errors in amplitude as well as phase (e.g., time of occurrence of peaks). The constant acceleration method provides a good compromise between these two types of errors. This method is one of those studied by Newmark, Reference 5. Figure II-19, plotted from that study, shows how phase and amplitude relative errors vary as the step size is increased. An often quoted rule-of-thumb is to use ten samples per the smallest period present in the model. Unfortunately, the smallest period (highest frequency) is usually not known. Also, for nonlinear problems, particularly where bottoming can occur, the higher frequency can change abruptly. Consequently, additional rules for choice of the time step are in order. For the masses and stiffnesses typical of automotive type collisions, a step size of .001 seconds produces results of acceptable accuracy. For problems which involve a limited number of computer runs, a very small interval is the safest approach. Where total computer time can be large, parametric studies, for example, the following approach is suggested. Run a typical problem at successively smaller step sizes until convergence is demonstrated. Then, select the largest step size consistent with the accuracy required and consistent with the accuracy to which the input quantities are known.

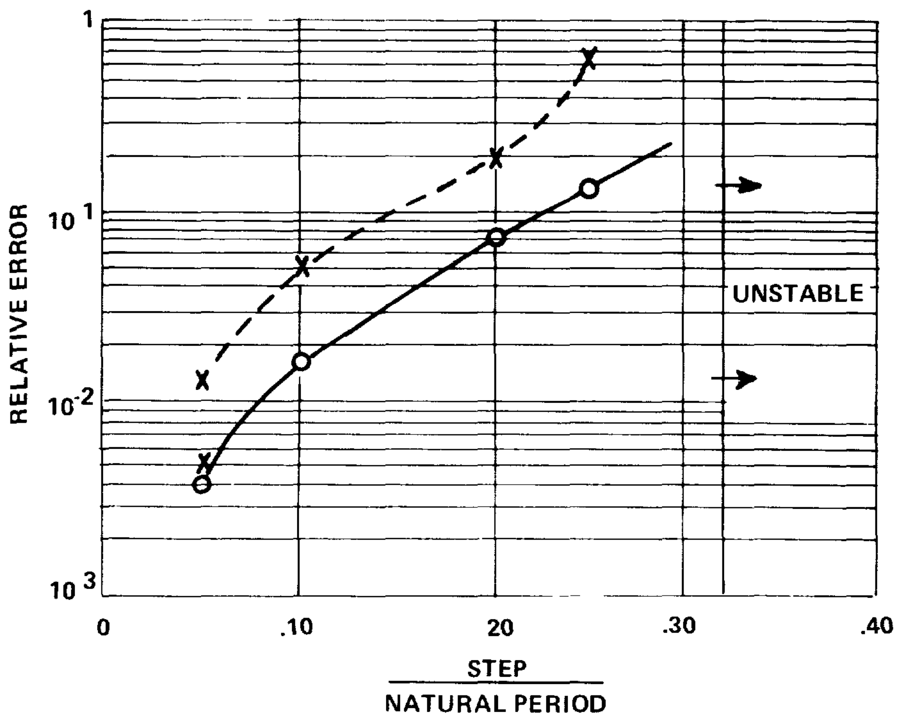


Figure II-19 CONSTANT ACCELERATION METHOD: RELATIVE ERROR FOR INITIAL VELOCITY RESPONSE FOR PERIOD (———) AND MAX. ALTITUDE (– – –)

7. REFERENCES

1. Marquard, E., "Zur Mechanik Des Zussammenstobes von Fahrzeugen", Automobiltechnische Zeitschrift, Volume 64 (1962), Number 5, pages 141-148.

2. Crandall, S. H., "Numerical Methods of Analysis", Chapter 28 in Shock and Vibration Handbook, Harris and Crede, editors, McGraw-Hill, 1961.

3. Biggs, J. M., "Introduction to Structural Dynamics", McGraw-Hill, 1964.

4. Roth, H. P., "Physical Factors Involved in Head-On Collisions of Automobiles", Proceedings - HRB, Volume 31, 1952, pages 349-356.

5. Newmark, N. M., "A Method of Computation for Structural Dynamics", ASCE, Journal of Engineering Mechanics, July, 1959.

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ANALYSIS OF SIDE IMPACT DYNAMICS

by

Calvin M Theiss
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November 1969

CAL REPORT YB-2684-V-5 PART III

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1. INTRODUCTION

During its first phase, the Basic Research in Crashworthiness Program has been primarily concerned with modifying full size automobiles with the objective of demonstrating that shorter rise times of deceleration and nearly uniform decelerations are possible and feasible in collisions with fixed objects. If such concepts can be developed, their implications in car-to-car collisions (instead of car-to-fixed object collisions) must be intensively investigated before any related performance standards can be specified.

Certainly, the most severe car-to-car collision, from the standpoint of occupant response, occurs when the passenger compartment itself is directly impacted, such as when a vehicle is impacted in its side. The present analytical investigation was undertaken in the anticipation of eventual side collision experiments related to automotive crashworthiness. The purpose of the study has been to provide a simplified mathematical model that will simulate important phenomena of side collisions, yet, that would also be simplified to the extent that it could be programmed on a time sharing computer, permitting immediate access by an engineer during the vehicle design stages. Thus, results of the study should provide further insight into the side collision phenomena and also provide inputs to the specification of the vehicle characteristics.

A simple two-dimensional collision model (i.e., all motions are restricted to the horizontal plane), which should be taken as a first-order approximation to the collision process, has been developed. This mathematical model has been programmed in the BASIC-plus computer language for use on a GC-5 time-share computer. The program permits the variation of 28 variables that relate to the vehicles, to the test configuration, and to the occupant of the struck vehicle. Consequently, the program provides considerable flexibility, and various hypotheses may be analytically tested.

The program has been exercised in a limited side collision analysis to determine the probable effects on occupant response, that would result from changes in the frontal and side compartmental force-deflection characteristics of full size automobiles and from changes of the restraint system's stiffness.

2. CONCLUSIONS AND RECOMMENDATIONS

2.1 Conclusions

From the few tests run on the computer, three conclusions were reached.

2.1.1 The restraint systems currently used in production automobiles are probably causing higher peak decelerations than restraint systems with either more or less stiffness.

Variation of seat belt stiffness between 300 lb/in. and 4800 lb/in. indicates that the highest peak acceleration of the passenger in a production car hit by another production car occurs at 1200 lb/in. restraint system stiffness. Since this value is near that for the lap belts presently installed in production vehicles, there is some question as to whether or not the compatibility between restraint stiffness and vehicle stiffness has ever been fully explored. This finding is not new, but rather tends to confirm that of R. I. Emori in his simplified frontal collision study*.

2.1.2 The introduction of vehicles with more nearly constant deceleration during impact would probably require different, i.e., softer or more yielding, restraint systems.

* Emori, Richard I., "Analytical Approach to Automobile Collisions", Society of Automotive Engineers Paper No. 680016, Detroit, January 1968.

According to the present study, a 600 lb/in. restraint system combined with the proposed vehicle changes would have about the same deceleration response as a 1200 lb/in. restraint system has with current production vehicles.

As an alternative, deceleration may also be limited to desired levels by using a yielding restraint system. Here, the stiffness governs only the rate-of-onset of the deceleration response.

2.1.3 The more hazardous conditions that are generally anticipated to be produced by a vehicle with stronger front end structure impacting the side of a production vehicle are probably not as severe as would normally be expected.

The simulation indicates, for example, that if the front stiffness of the impacting vehicle is increased (see Figure III-8), occupant peak deceleration would increase from 30 g's to 37 g's and vehicle penetration from 17 in. to 23 in. Of course, no claims can be made relating these changes to occupant injury potential.

2.2 Recommendations

2.2.1 An exhaustive computer study of the developed side-crash model should be made.

The few tests run so far indicate only the potential of the model. The next logical step would be to study all important car-restraint system combinations. This would be very helpful in refining the model and in setting up the experimental program for side-crash tests.

2.2.2 The program results should be verified through comparisons with side collision experiments.

It is expected that the continuing activities of the Basic Research in Crashworthiness Program will involve side impact studies of both production and prototype vehicles. These experiments will provide a unique opportunity to verify the prediction capabilities of this model.

2.2.3 Changes should be made in the model as indicated by the verification study.

Certain over-simplifications in the model may be uncovered when its predictions are compared to the results of experiments. It is quite probable, for example, that better representation of the occupant is necessary or that the distributed forces in the contact regions must be considered.

3. MATHEMATICAL MODEL

A mathematical model of side collision was developed to study the influence of vehicle collapse characteristics and occupant restraint systems on occupant safety. The model consists of a vehicle with an occupant (vehicle 1) struck on the right side by a second vehicle (vehicle 2). The mathematical model has been kept simple so that first-order approximations of vehicle and occupant motions can be obtained on a digital time-share computer.

3.1 Assumptions

The following assumptions are made:

1. The vehicles have no height; all movements and forces are confined to the street plane.

2. The vehicles are neither braked nor steered; wheels are always freely rolling, rolling resistance is ignored. Until impact, then, both vehicles move as free bodies with no external and inertia forces.

3. The striking vehicle (vehicle 2) has no width; it is approximated by a single line.

4. Side-skid forces of vehicle 2 are ignored.

5. The front wheels and the rear wheels of vehicle 1 are lumped together on the center line (bicycle).

6. The occupant is reduced to a point mass.

7. Two orthogonal coordinate systems (frames) are employed. One (X-Y) is fixed in inertial space; the second (x-y) is fixed to vehicle 1 with its origin at the vehicle's CG, the x-axis pointing forward and the y-axis to the right. At the beginning of impact, the two frames coincide.

8. Figure III-1 shows the initial conditions of the model. All initial dimensions can be computed from those shown.

9. The resultant interface force between colliding vehicles is assumed to act near the front end of vehicle 2 (collision point).

10. The resultant interface force is resolved into two components -- a force, N , in line with vehicle 2 (crush force), and a force F parallel to x-axis of vehicle 1 (side force), Figure III-2.

11. The crush force, N , acts only during the short forward penetration. During this time, the relative yaw angle, $\psi_2 - \psi_1$, remains practically constant. Hence, the direction of the crush force, N , relative to vehicle 1 does not change. Consequently, the force-deflection characteristic of vehicle 2 does not change either, which simplifies computation of crush force, N .

12. Forces the occupant exerts onto vehicle 2 are neglected.

13. All impact energy is dissipated (except for small deflections of the seat belt).

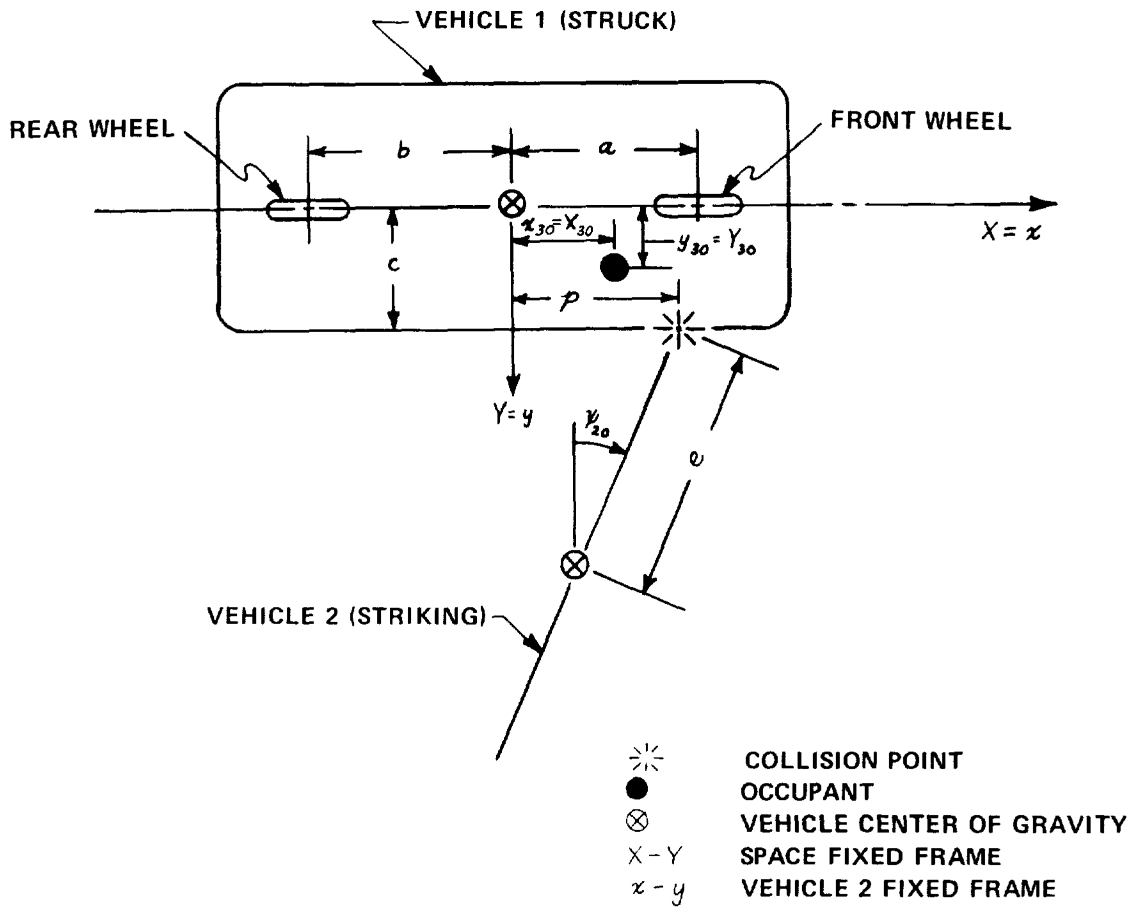
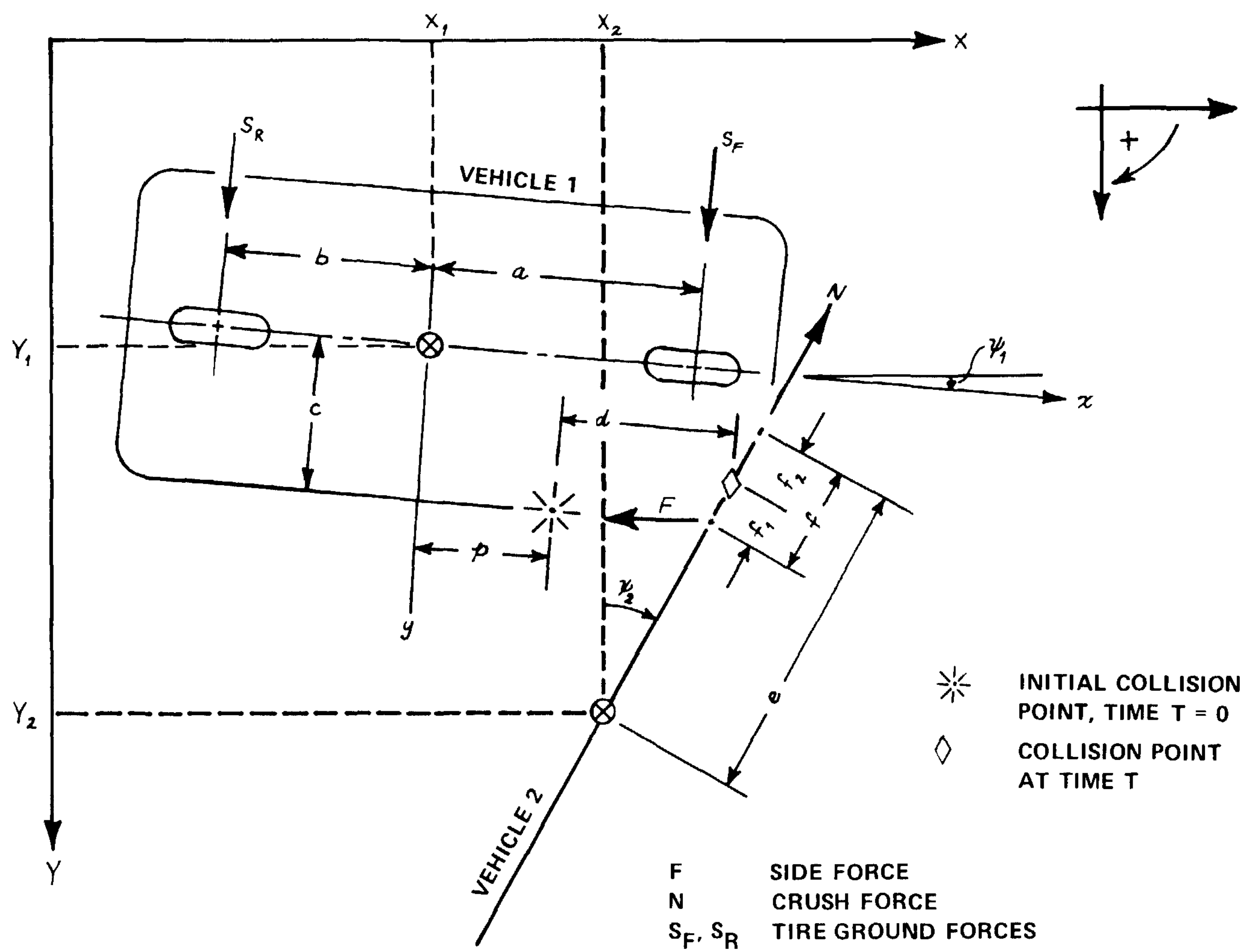


Figure III-1 INITIAL POSITIONS OF VEHICLES AND OCCUPANT



F	SIDE FORCE
N	CRUSH FORCE
S_F, S_R	TIRE GROUND FORCES
d	LONGITUDINAL DEFLECTION OF VEHICLE 1
f_1	LATERAL DEFLECTION OF VEHICLE 1 IN IMPACT DIRECTION
f_2	LONGITUDINAL DEFLECTION OF VEHICLE 2

Figure III 2 FREE-BODY DIAGRAM OF VEHICLE 1 (STRUCK)

3.2 Equations of Motion

Of interest are the motions of three bodies: vehicle 1, vehicle 2, and the occupant. (For mathematical notations see Notations 3.7).

Vehicle 1 -- forces in X-direction, Figure III-2

$$-F \cos \psi_1 + N \sin \psi_2 - (S_F + S_R) \sin \psi_1 = \ddot{X}_1 M_1$$

Vehicle 1 -- forces in Y-direction, Figure III-2

$$-F \sin \psi_1 - N \cos \psi_2 + (S_F + S_R) \cos \psi_1 = \ddot{Y}_1 M_1$$

Vehicle 1 -- moment around CG, Figure III-2

$$Fc - N\left[(Y_2 - Y_1) \sin \psi_2 + (X_2 - X_1) \cos \psi_2\right] + S_F a - S_R b = \dot{\psi}_1 I_1$$

Vehicle 2 -- forces in X-direction, Figure III-2

$$F \cos \psi_1 - N \sin \psi_2 = X_2 M_2$$

Vehicle 2 -- forces in Y-direction, Figure III-2

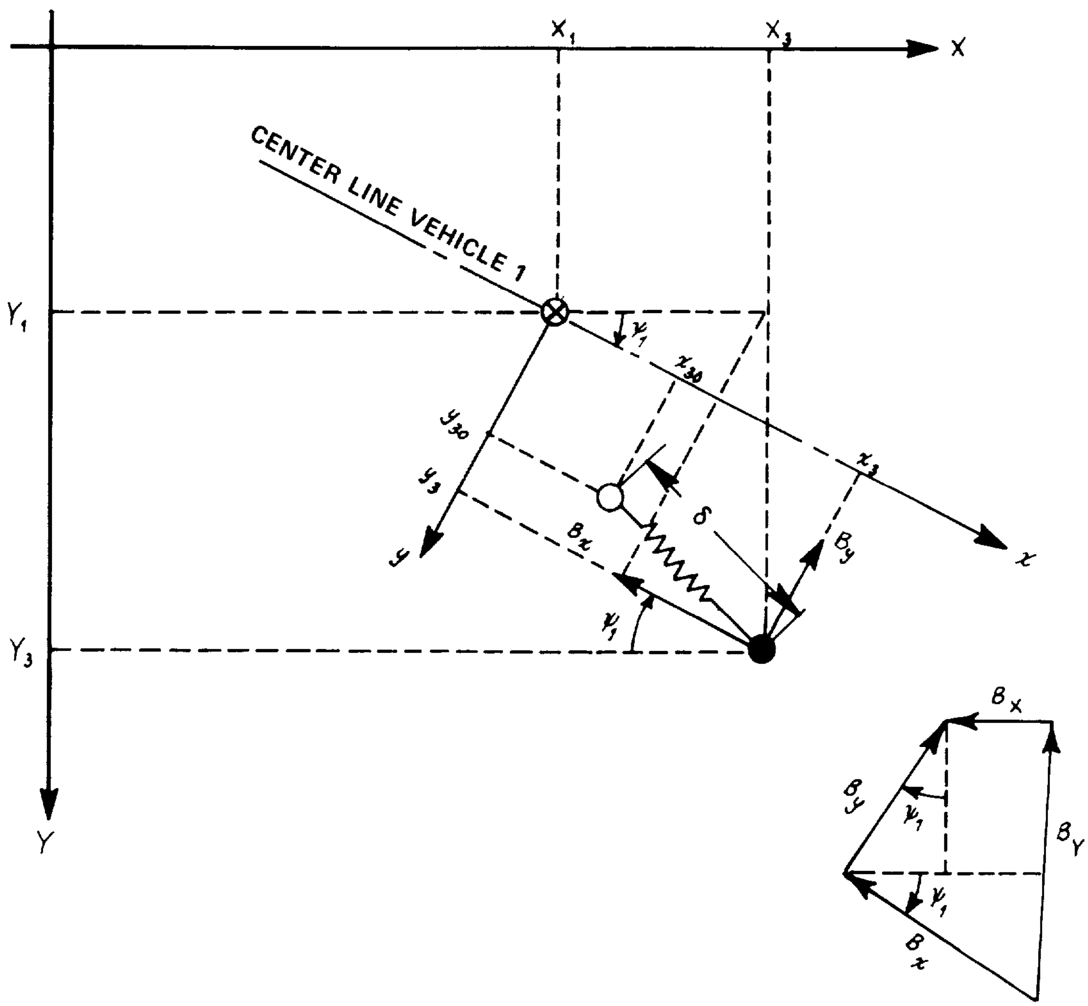
$$F \sin \psi_1 + N \cos \psi_2 = \ddot{Y}_2 M_2$$

Vehicle 2 -- moment around CG, Figure III-2

$$F\left[(Y_2 - Y_1) \cos \psi_1 - (X_2 - X_1) \sin \psi_1 - c\right] = \ddot{\psi}_2 I_2$$

Occupant -- forces in X-direction, Figure III-3

$$-B_X = \dot{X}_3 M_3$$



⊗ CG OF VEHICLE 1
○ INITIAL POSITION OF OCCUPANT
● POSITION OF OCCUPANT
/\/\/\/\ RESTRAINT SYSTEM

Figure III-3 FREE-BODY DIAGRAM OF OCCUPANT

Occupant -- forces in Y-direction, Figure III-3

$$-B_Y = \ddot{Y}_3 M_3$$

Occupant -- no moment (mass point)

<u>Velocity and position of each body at time T in space frame X-Y</u>

CG-Velocities of vehicle 1, vehicle 2, and occupant

$$\dot{X}_i = \int_0^T \ddot{X}_i \, dt + \dot{X}_{i0}$$

$$\dot{Y}_i = \int_0^T \ddot{Y}_i \, dt + \dot{Y}_{i0}$$

i = 1, vehicle 1; i = 2, vehicle 2, i = 3, occupant

Angular velocities of vehicles 1 and 2 ($\dot{\psi}_{10} = \dot{\psi}_{20} = 0$)

$$\dot{\psi}_1 = \int_0^T \ddot{\psi}_1 \, dt$$

$$\dot{\psi}_2 = \int_0^T \ddot{\psi}_2 \, dt$$

CG-Positions of vehicle 1, vehicle 2, and occupant ($X_{10} = Y_{10} = 0$)

$$X_i = \int_0^T \dot{X}_i \, dt + X_{i0}$$

$$Y_i = \int_0^T \dot{Y}_i \, dt + Y_{i0}$$

Yaw angles of vehicles 1 and 2 ($\psi_{10} = 0$)

$$\psi_1 = \int_0^T \dot{\psi}_1 \, dt$$

$$\psi_2 = \int_0^T \dot{\psi}_2 \, dt + \psi_{10}$$

3.3 Crush Force

The force, N , to crush the front end of the impacting vehicle 2 equals the crush force of the side-impacted vehicle 1. The deformations of both vehicles depend on their force-deformation characteristics. For both vehicles, we assume the crush force to be linearly proportional to the deformation up to a maximum value, from then on the crush force remains constant, as shown in Figure III-4.

According to Figure III-2, the total penetration, $f = f_1 + f_2$, is

$$f = e - \frac{(Y_2 - Y_1)\cos\psi_1 - (X_2 - X_1)\sin\psi_1 - c}{\cos(\psi_2 - \psi_1)}$$

If this value is smaller than a previous penetration, contact between the vehicles in the crush direction is lost and the crush force is zero (in the computer program, the previous total penetration is called D5). However, if the total penetration is larger than the previous value, then the two vehicles act like two plastic springs in series: the deformation of vehicle 1 is

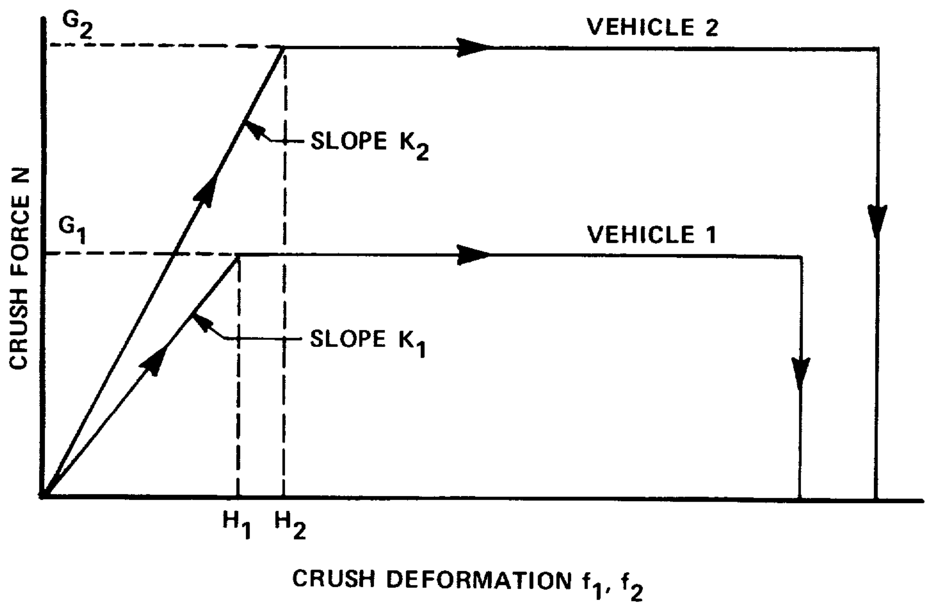
$$f_1 = \frac{K_2 f}{K_1 + K_2}$$

The deformation of the second vehicle is, of course

$$f_2 = f - f_1$$

and the crush force is

$$N = \frac{K_1 K_2}{K_1 + K_2} f$$



VEHICLES 1 AND 2 WITH $G_2 > G_1$

f_1, f_2	CRUSH DEFORMATION OF VEHICLE 1,2
G_1, G_2	LIMIT OF CRUSH FORCE FOR VEHICLE 1,2
H_1, H_2	BREAK POINT FOR VEHICLE 1,2

Figure III-4 CRUSH FORCE-DEFORMATION CHARACTERISTIC

The computed values of f_1 , f_2 , and N are used only if (see Figure III-4)

$$N \leq G_1 \quad \text{and} \quad N < G_2$$

i.e., as long as the total deformation, f , increases linearly with the crush force and neither the limit G_1 nor G_2 is reached. However, if $G_1 < G_2$ and $N > G_1$, then N is limited to G_1 , that is (see Figure III-4).

$$\left.\begin{aligned} N &= G_1 \\ f_2 &= \frac{G_1}{K_2} \end{aligned}\right\} \text{ if } N > G_1 < G_2$$

Analogously,

$$\left.\begin{aligned} N &= G_2 \\ f_1 &= \frac{G_2}{K_1} \end{aligned}\right\} \text{ if } N > G_2 < G_1$$

For $N > G_1 = G_2$ deformations are assumed to follow the equation

$$f_1 = \frac{K_2 f}{K_1 + K_2}$$

$$\text{and} \quad f_2 = f - f_1$$

3.4 Side Force, F

According to Figure III-2, the side deformation, d , of vehicle 1 can be computed as

$$d = (X_2 - X_1)\cos\psi_1 + (Y_2 - Y_1)\sin\psi_1 - p + \left[(Y_2 - Y_1)\cos\psi_1 - (X_2 - X_1)\sin\psi_1 - c\right]\tan(\psi_2 - \psi_1)$$

If the absolute value of side deflection, d , is smaller than that of a previous deflection, contact between vehicles in the side direction is lost and the side force is zero. Otherwise, the side force, F , is computed in accordance with the assumed side force-deformation characteristic, Figure III-5. (In the computer program, the previous deformation in forward direction as called D6, and the previous deformation in the rearward direction -D7.)

3.5 Tire Forces, S_F and S_R

After contact between the two vehicles is made, the struck vehicle 1 starts moving sideward, thereby developing side slip forces at tire-ground contact points. The CG-velocity components of vehicle 1 in vehicle coordinates are

CG-velocity component in x-direction, Figure III-6a

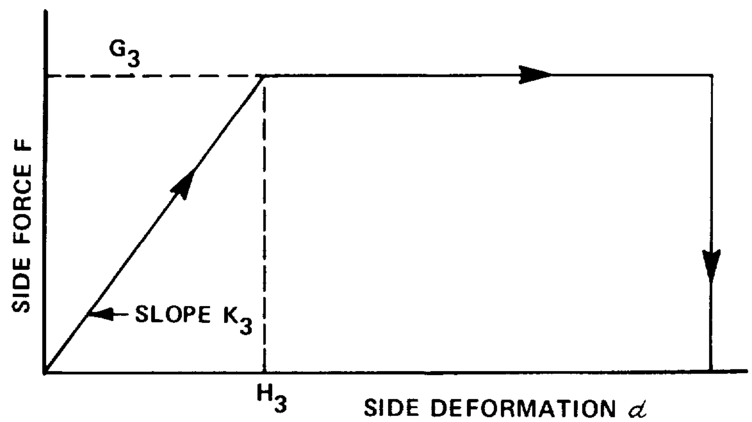
$$v_x = \dot{X}_1 \cos\psi_1 + \dot{Y}_1 \sin\psi_1$$

CG-velocity component in y-direction, Figure III-6a

$$v_y = -\dot{X}_1 \sin\psi_1 + \dot{Y}_1 \cos\psi_1$$

The side velocity of the front tire in y-direction is, Figure III-6b

$$v_F = v_y + a\dot{\psi}_1$$

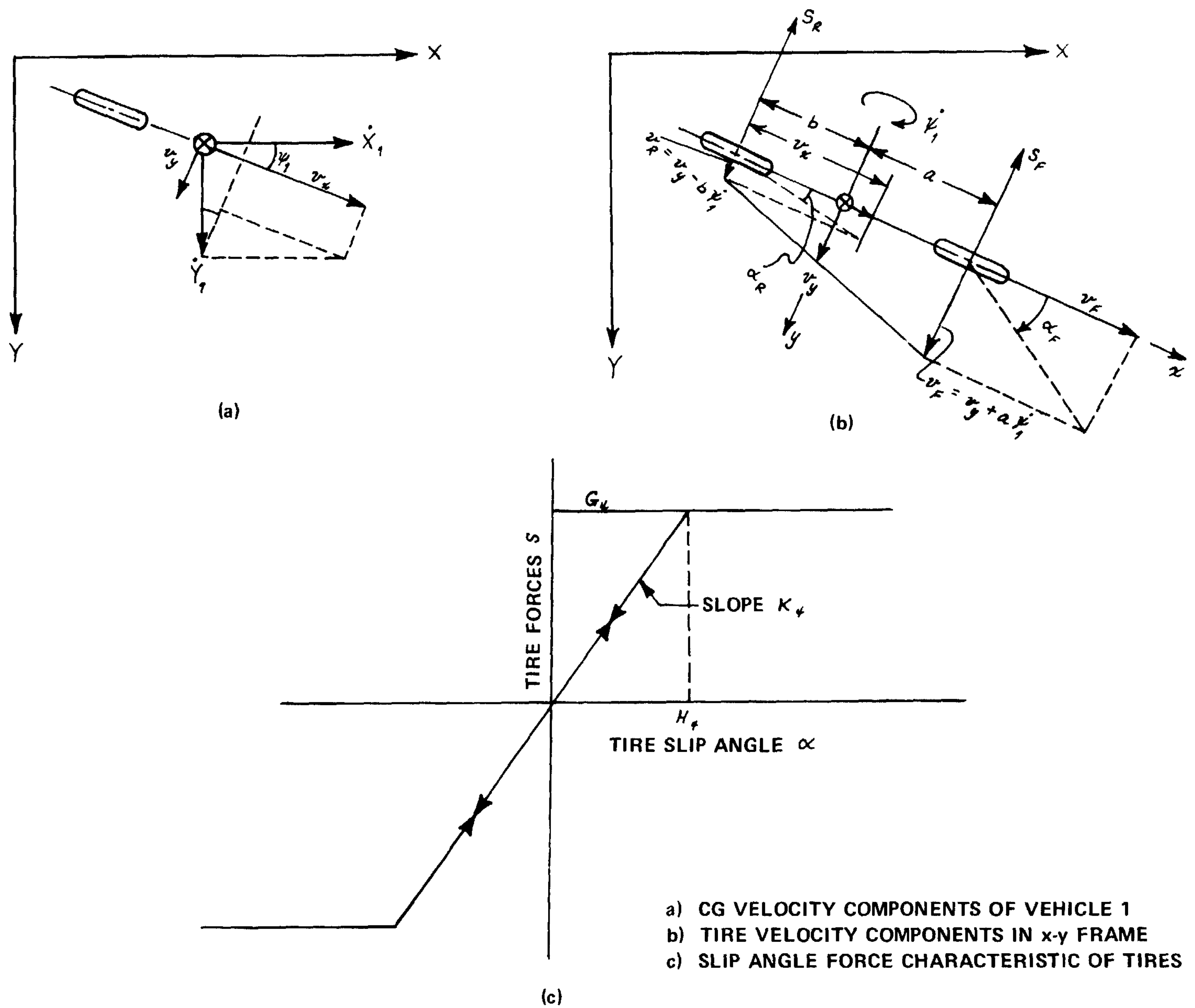


VEHICLES 1 AND 2 COMBINED

G_3 LIMIT OF SIDE FORCE

H_3 BREAK POINT

Figure III-5 SIDE FORCE-DEFORMATION CHARACTERISTIC OF VEHICLES 1 AND 2 COMBINED



a) CG VELOCITY COMPONENTS OF VEHICLE 1
b) TIRE VELOCITY COMPONENTS IN x-y FRAME
c) SLIP ANGLE FORCE CHARACTERISTIC OF TIRES

Figure III-6 TIRE FORCES AND SLIP ANGLES

The side velocity of the rear tire in y-direction is, Figure III-6b

$$v_R = v_y - b\,\dot{\psi}_1$$

The slip angle, α_F , of the front tire is

$$\alpha_F \approx \frac{v_F}{v_x}$$

and the slip angle, α_R , of the rear tire

$$\alpha_R \approx \frac{v_R}{v_x}$$

The two slip angles are used to compute the tire forces, S_F , and S_R , in accordance with the assumed slip angle-force diagram, Figure III-6c. The tire forces are directed opposite to the velocities v_R and v_F .

It is assumed that if the slip angle is so large that the approximation of the two slip-angle formulas is not valid any more, the slip angle has already passed the break point, H_4 , in Figure III-6c.

3.6 Forces on Occupant

The occupant is held in vehicle 1 by a restraint system that is purely elastic up to a maximum force, G_5 (restraint-displacement break point H_5), Figure III-7. At larger displacements, the restraint system is plastic. The occupant's coordinates with respect to vehicle 1 are, Figure III-3.

$$x_3 = (X_3 - X_1)\cos\psi_1 + (Y_3 - Y_1)\sin\psi_1$$

$$y_3 = -(X_3 - X_1)\sin\psi_1 + (Y_3 - Y_1)\cos\psi_1$$

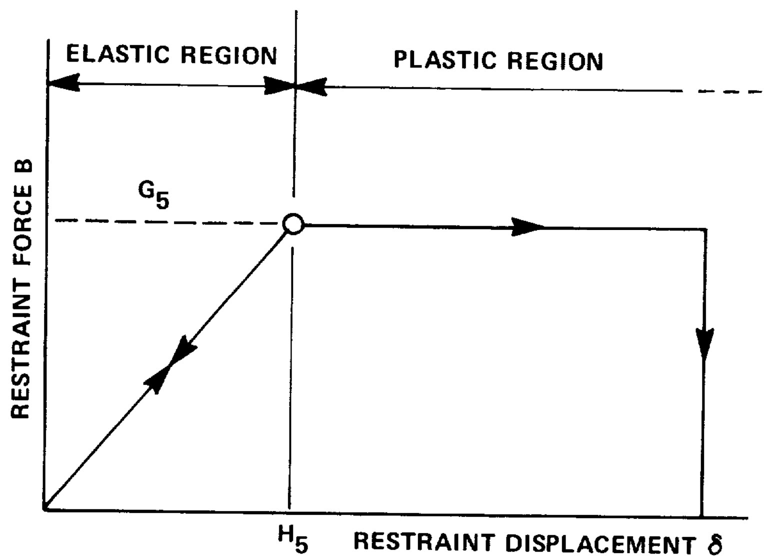


Figure III-7 FORCE-DEFORMATION CHARACTERISTIC OF RESTRAINT SYSTEM

The total displacement of the occupant with respect to his initial position is

$$\delta^2 = (x_3 - x_{30})^2 + (y_3 - y_{30})^2$$

If the total displacement is smaller than the break point displacement, H_5 , and has never been larger before, the force components exerted on the occupant are

$$B_x = -(x_3 - x_{30}) K_5$$

$$B_y = -(y_3 - y_{30}) K_5$$

If, however, $\delta > H_5$

then

$$B_x = -G_5 \frac{x_3 - x_{30}}{\delta}$$

$$B_y = -G_5 \frac{y_3 - y_{30}}{\delta}$$

The force components on the occupant in space-frame coordinates are

$$B_X = B_x \cos \psi_1 - B_y \sin \psi_1$$

$$B_Y = B_x \sin \psi_1 + B_y \cos \psi_1$$

3.7 Notation

Table III-1 Listing of Equation and Computer Symbols

Constants (Dimensions in the computer program are lbs, inches, seconds, and rad. Dimensions in the print-out as stated.)

Equation Symbol	Computer Symbol	Purpose
a	B(1)	distance between CG and front axle of vehicle 1 (struck)
b	-B(2)	distance between CG and rear axle of vehicle 1
c	C	half width of vehicle 1
e	E	distance between CG and front end of vehicle 2 (striking)
M_1	M(1, 1) = M(2, 1)	mass of vehicle 1
M_2	M(1, 2) = M(2, 2)	mass of vehicle 2
M_3	M(1, 3) = M(2, 3)	mass of occupant
I_1	M(3, 1)	moment of inertia around CG of vehicle 1
I_2	M(3, 2)	moment of inertia around CG of vehicle 2
G_1	G(1)	crush force limit of vehicle 1 (side deformation)
G_2	G(2)	crush force limit of vehicle 2
G_3	G(3)	side force limit - vehicles 1 and 2 combined
G_4	G(4)	Tire side-force limit
G_5	G 9	Restraint force limit of occupant restraint system

Table III-1 (continued) (Constants)

Equation Symbol	Computer Symbol	Purpose
H_1	D(1)	crush-deformation break point of vehicle 1 (side deformation)
H_2	D(2)	crush-deformation break point of vehicle 2 (front deformation)
H_3	D(3)	side-deformation break point of vehicles 1 and 2 combined
H_4	D(4)	tire slip-angle break point
H_5	D 9	restraint displacement break point of occupant restraint system
k_1	K(1)	slope of crush force-deformation characteristic of vehicle 1
k_2	K(2)	slope of crush force-deformation characteristic of vehicle 2
k_3	K(3)	slope of side force-deformation characteristic of both vehicles
k_4	K(4)	slope of tire slip angle-side force characteristic
k_5	K 9	slope of restraint displacement-force characteristic of restraint system
p	P	distance in x-direction from vehicle 1 CG to initial collision point
x_{30}	X3	initial position of occupant in vehicle frame
y_{30}	Y3	

Table III-1 (continued)

<u>Variables</u> (i = 1 = vehicle 1, i = 2 = vehicle 2; i = 3 = occupant)

Equation Symbol	Computer Symbol	Purpose
X_i	S(1,i)	space-frame coordinates of CG positions
Y_i	S(2,i)	
ψ_1	S(3, 1)	yaw angle of vehicle 1
ψ_2	S(3, 2)	yaw angle of vehicle 2
$\dot{X}_i$	V(1,i)	space-frame coordinates of CG velocities
$\dot{Y}_i$	V(2,i)	
$\dot{\psi}_1$	V(3, 1)	yaw velocity of vehicle 1
ψ_2	V(3, 2)	yaw velocity of vehicle 2
$\ddot{X}_i$	A(1,i)	space-frame coordinates of CG accelerations
$\ddot{Y}_i$	A(2,i)	
$\ddot{\psi}_1$	A(3, 1)	yaw acceleration of vehicle 1
$\ddot{\psi}_2$	A(3, 2)	yaw acceleration of vehicle 2
x_3	X2	vehicle-frame coordinates of occupant
y_3	Y2	
f_1	D1	side deformation of vehicle 1 due to crush force
f_2	D2	front deformation of vehicle 2 due to crush force
f	D3	$f_1 + f_2$
d	D	combined side deformation of vehicles 1 and 2 due to side force
Y_2-Y_1	Y1	
X_2-X_1	X1	
Y_3-Y_1	Y1	
X_3-X_1	X1	

Table III-1 (continued) (Variables)

Equation Symbol	Computer Symbol	Purpose
$x_3 - x_{30}$	S1	
$y_3 - y_{30}$	S2	
δ	$\sqrt{X1}$	deformation of restraint system
F	F1	side force
N	F2	crush force
S_F	F4	front tire side force
S_R	F5	rear tire side force
B_x	F1	restraint force components (vehicle frame) on occupant
B_y	F2	
B_X	F(1, 3)	restraint force component (space frame) on occupant
B_Y	F(2, 3)	
v_x	X1	CG-speed components (vehicle frame) of vehicle 1
v_y	Y1	
v_F	B1	front tire slip speed -- vehicle frame
v_R	B1	rear tire slip speed -- vehicle frame
α_F	B	slip angle of front tires
α_R	B	slip angle of rear tires

Table III-1 (continued)

$-F \cos\psi_1 + N \sin\psi_2 - (S_F + S_R)\sin\psi_1 \quad F$	F(1, 1)
$-F \sin\psi_1 - N \cos\psi_2 + (S_F + S_R)\cos\psi_1$	F(2, 1)
$F c - N\big((Y_2 - Y_1)\sin\psi_2 + (X_2 - X_1)\cos\psi_2\big) + S_F a - S_R b$	F(3, 1)
$F \cos\psi_1 - N \sin\psi_2$	F(1, 2)
$F \sin\psi_1 + N \cos\psi_2$	F(2, 2)
$F\big((Y_2 - Y_1)\cos\psi_1 - (X_2 - X_1)\sin\psi_1 - c\big)$	F(3, 2)

4. COMPUTER PROGRAM

The simple mathematical model described in Section 3 has been programmed for time history solutions on the GC-5 (Graphic Controls Corporation) remote time share computing system in the BASIC-plus programming language. It has been completely checked out and used a number of times. Sample outputs are shown in Appendices A and B. A complete listing of the program is given in Appendix C.

4.1 Symbology

Due to the simplicity of the BASIC-plus programming language used, the choice of symbols to represent numerical parameters is limited to single capital Latin letters with a possible suffix of a single digit. The development of the equations in Section 3 was also restricted to this symbology to retain correspondence between the equations there and the computer program. It should be remembered that the symbols in a digital computer program serve dual roles.

(1) They are parameter representations of variables and constants normally used in algebraic equations.

(2) They are addresses to storage cells in the computer. Therefore, a given symbol may represent one parameter in one portion of the program and something entirely different elsewhere in the program. This is desirable for efficient use of computer storage, it is imperative in small computers such as the GC-5.

The following table lists the symbols and their definitions as used in this program. Symbols consisting of a single letter (i. e., A), a letter and a digit (A3) or a letter with subscripts (i. e., A(2, 3)) represent computer storage cells containing numeric values. Symbols consisting of double letters, such as AA and BB(6), represent storage cells containing alpha-numeric characters for purposes of printing notes, definitions, headings, etc. These latter do not appear in this symbology table; however, they do in the program lists of Appendix C.

Symbol	Purpose
A	Scratch (used for temporary storage only, in one or more places)
A(j, k)	Acceleration in direction j for object k (see note 1 below)
B, B1	Scratch. (Temporary storage of intermediate value)
B(j)	Distance from vehicle 1 c.g. to wheel j, j = 1 ≡ front, j = 2 ≡ rear
C(j, k)	Cosine (j = 1) or sine (j = 2) of heading angle for vehicle k
C	Distance of vehicle 1 right hand side from its c.g.
C1, C2, C3	Scratch
D	Scratch
D1	Crush distance (penetration) for vehicle 1
D2	Crush distance (penetration) for vehicle 2
D3	Total depth of vehicle interference D1 + D2
D5	Current maximum interference between vehicles
D6	Maximum side displacement
D7	Minimum side displacement
D9	Maximum elastic point for occupant restraint
D(j)	Displacement of break point in spring j (see note 2)

Symbol	Purpose
E	Distance from vehicle 2 c.g. to its undeformed front end
F(j, k)	Force in direction j exerted on object k (see note 1)
F1	Friction force applied to vehicle 2 (striking)
F2	Crush force applied to vehicle 2 (striking)
F3	Total tire slip force applied to vehicle 1 (struck)
G(j)	Maximum force exerted by spring j (see note 2)
G9	Maximum occupant restraint force
H(j)	Input data identification numbers, j = 1, 5
J, K	Scratch (loop counters, etc.)
K9	Occupant restraint spring constant
K1	Switch to indicate which maximum crush value prevails
K8	Intermediate value K(2)/K(1) + K(2)
K(j)	Spring constant of spring j (see note 2)
L1	Scratch
M(j, k)	Mass in dimension j for object k (see note 1)
Ø1	Time to next regular data print-out
Ø2	Line counter for paging print-out
Ø3	Intermediate line counter
Ø4	Occupant condition switch
Ø5	Vehicle interference condition switch
Ø6	Event code print switch
Ø7	Data condition switch
Ø8	Carriage control switch
P	Distance in X direction from vehicle 1 c.g. to initial strike point on vehicle 1
P(j)	Distance of compartment walls from occupant initial position; j = 1 ≡ rear, 2 ≡ forward, 3 ≡ left, 4 ≡ right

Symbol	Purpose
Q1	File indicator switch for input data
S(j, k)	Position in direction j for object k (see note 1)
S1, S2	Scratch
T	Elapsed time since start of test
T1	Termination time for test
T2	Period of computation cycle
T3	Period of print cycle
T4	Initial yaw angle of vehicle 2
T9	Scratch
V(j, k)	Velocity in direction j for object k (see note 1)
V1, V2, V3, V5, V6, V7, W1, W2, X1, X2	Scratch (temporary storage for intermediate values)
X3	Initial X position of occupant in vehicle 1
Y1, Y2	Scratch
Y3	Initial Y position of occupant in vehicle 1
Z9	Program file identification switch

Note 1: Direction j = 1 ≡ inertial direction X
= 2 ≡ inertial direction Y
= 3 ≡ angular direction about Z axis
Object k = 1 ≡ vehicle 1 (struck vehicle)
2 ≡ vehicle 2 (striking vehicle)
3 ≡ occupant of vehicle 1

Note 2: Spring j = 1 ≡ vehicle 1 crush spring equivalent
2 ≡ vehicle 2 crush spring equivalent
3 ≡ friction equivalent between vehicles
4 ≡ tire slip angle force
5 ≡ springs 1 + 2 in series

4.2 Program Operation

The results of a sample test are shown in Appendix A. The first page lists all the input parameters as read from computer input data file CTDVF1. The first column is the file line number, and the second column is the initial value for the test followed by a brief description of the datum. The dimensions have been added to the computer print-out.

The table at the bottom of the page is a list of the computer program files used and their most recent revision dates. These are for future references for comparing tests when the program is being changed.

To use the program, the input data are placed in file CTDVF1, one per line, in the order shown. Four more values are included after the 31 shown, namely, (1) the clock time, (2) day, (3) month and (4) year in which the data set is compiled. All pages of data print-out are identified by these four values.

The remaining three pages give the position, velocity and acceleration of the occupant and the two vehicles, respectively, as functions of elapsed time. All vehicle data are given with respect to inertial space. The penetration or crush distance of the vehicles is also listed. Data for the occupant are given with respect to the occupant initial position and the struck vehicle coordinate axes.

Note that the time between printed entries in the output data is greater than the time increment for computation. When some special event occurs, data are also printed at the time the event is detected and the event identified by a code at the end of the respective data line. The meaning of these event codes are as follows.

D Disengagement between vehicles commences.

S Separation between vehicles is complete.

OC The occupant has struck a compartment wall.

R Reengagement between vehicles has begun.

Running computer program file CTDV01 will produce the results shown in Appendix A. In case a test is desired without using the computer time to print the input values, computer program file CTDV05 is run to produce identical results minus the first page.

In case a test is desired with only a few changes in the data, as stored in file CTDVF1, computer program file CTDV04 is run. The results of such a test are shown in Appendix B. After printing the identification now including the revision time, a question mark is printed, and the computer pauses. The user then types in two values: (1) the line number of the parameter to be changed, and (2) its new value. The computer will then identify the value and ask for the next change with another question mark. This is continued until all desired changes have been entered, then the user types 0, 0 to indicate the test is ready to proceed.

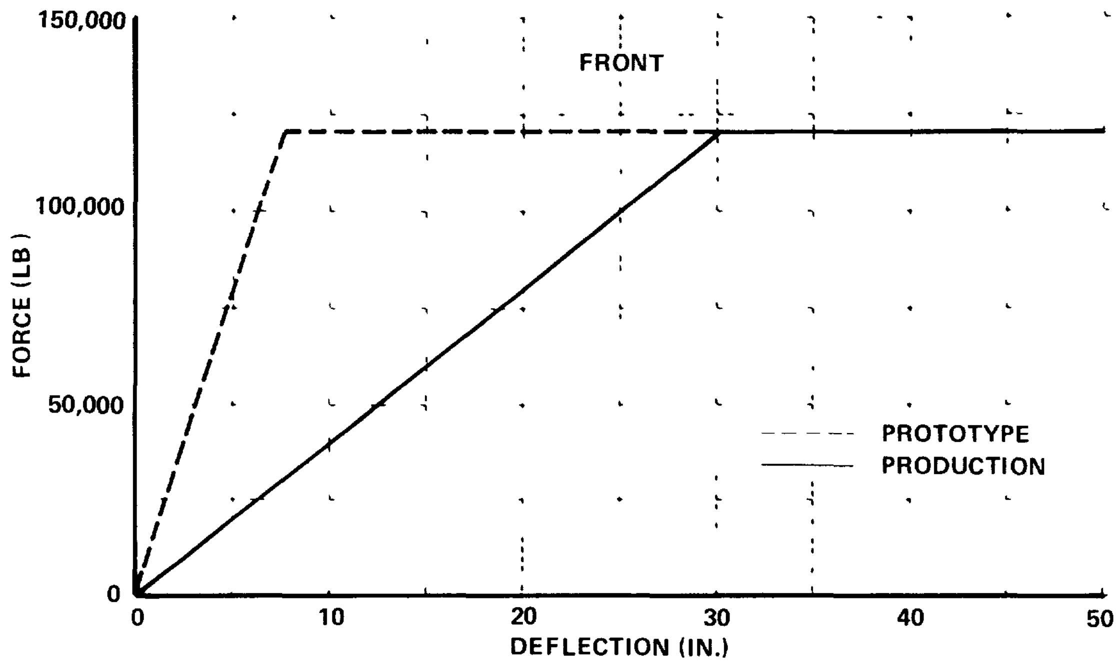
Upon completing a test and printing all the output data, the computer pauses with a question mark at the end of the final page delineation line. At this time the user, after viewing the data as it had been printed, may choose to continue the test beyond the specified termination time by typing in a new value for the termination time; otherwise, he types in a zero and the program terminates. In case he wants to make this decision after printing the last line of occupant data, he may indicate this reservation before starting the test by eliminating line 330 of computer program file CTDV03 (see Appendix C). This line should be restored to the file, however, after the test is completed for possible future use.

5. SIDE COLLISION ANALYSIS

The computer program has been aimed at the provision of a basis for sound engineering judgment in specifying the characteristics for improved automotive crashworthiness. Particular emphasis must be placed on the effects any proposed changes would have on the occupants when a vehicle compartment is hit by another vehicle. The program was developed so that it is compatible with a time-share computer, permitting an engineer to explore questions as they arise during the vehicle design stage. Consequently, rather than investigate all collision possibilities, we have elected to explore the variations in occupant response that could be expected if the vehicle structural properties and the restraint characteristics are varied. In addition, this demonstration serves as an example of the way the program might be employed.

An immediate concern of the Crashworthiness Program is the effect a structure having a more uniform deceleration response would have on the occupant behavior during side collisions. Present production vehicles may be modeled, to a first-degree approximation, by a linear force-deflection relationship. If structural changes are made so that a more constant deceleration results, then the prototype vehicle would have front and side deceleration responses with short rise times followed by a nearly constant deceleration level. Shown in Figure III-8 are the assumed force-deflection characteristics for the front and side of these vehicles, designated production and prototype, respectively. If such a prototype vehicle were introduced to the highways, four collision cases would be possible, Table III-2.

Vehicle structural changes should not be made independently of the stiffness properties of the occupant restraint system. Shown in Figure III-9 are first-order approximations of typical lap-belt force-deflection curves. The nominal stiffness for lap belts currently used in production automobiles is 1200 lb/in. For this investigation, a family of linear elastic restraint systems having a stiffness range from 300 lb/in. to 4800 lb/in. was considered.



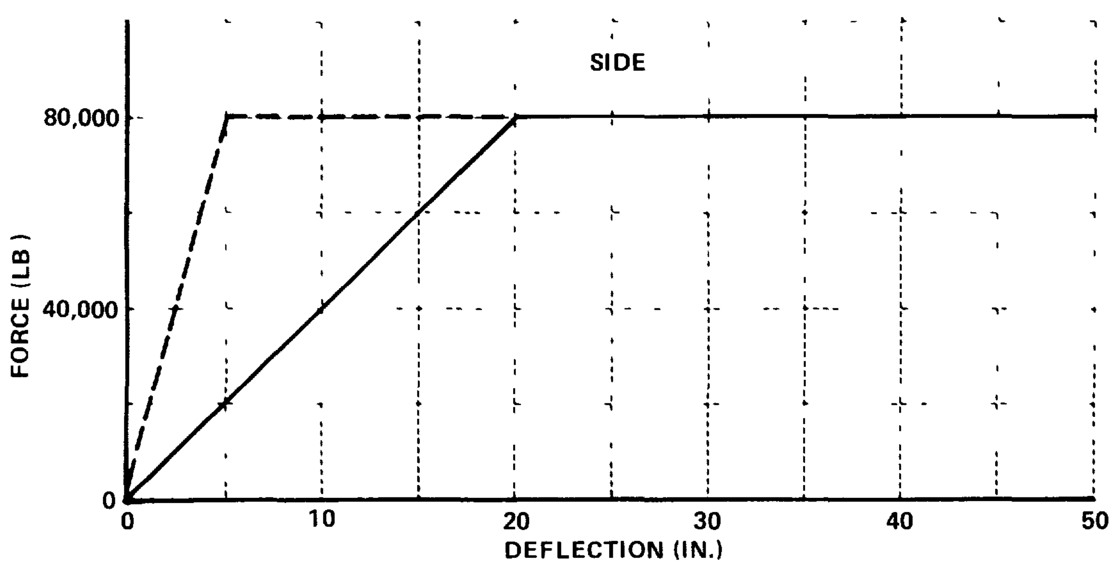


Figure III-8 ASSUMED VEHICLE FORCE-DEFLECTION CHARACTERISTICS

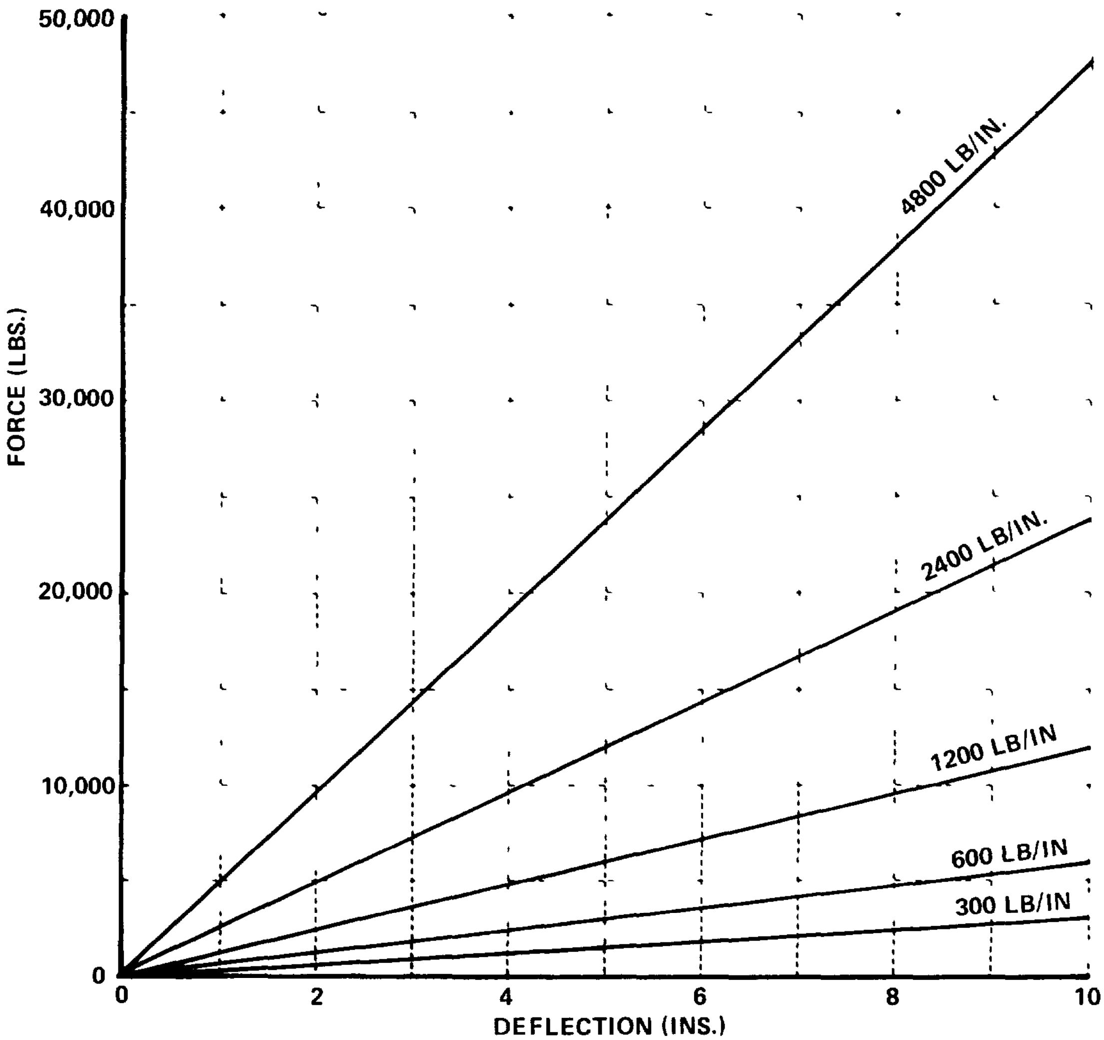


Figure III-9 ASSUMED RESTRAINT FORCE-DECELERATION CHARACTERISTICS-ELASTIC RESTRAINT SYSTEM

In an intensive investigation, different impact angles, impact velocities and vehicle sizes would be considered. However, this limited study was restricted to full size vehicles of 3700 lbs., where a vehicle at rest* is struck perpendicularly on the passenger side, at 12 in. in front of its c.g. by a vehicle having an intial velocity of 40 mph. This configuration is thought to produce the most severe lateral decelerations of the occupant, for a given impact velocity. On the other hand, longitudinal decelerations, although computed, are not significant.

5.1 Elastic Restraint Systems

Four collision cases, as indicated in Table III-2, were run along with five elastic restraint systems, yielding a total of 20 different computer runs. For each case, the lateral decelerations of the occupant, in the driver position, are presented along with the struck vehicle's decelerations** and the penetrations for both vehicles. Although the deceleration was calculated for an occupant in the driver position, it is felt that in this test configuration a similar response would occur for the front passenger occupant, if non-contact with the compartment was assumed.

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* A few computer runs were made with the struck vehicle moving with an initial velocity of 40 mph. The results are indicated in the figures by dotted lines.

** With this collision configuration, the absolute values of the lateral decelerations are about the same for both vehicles.

Shown in Figure III-10 are the occupant's lateral decelerations for Case I (production vehicle impacting production vehicle). Peak decelerations of 30 g's occur with the normal 1200 lb/in. restraint system. Similar curves are presented for the 4800, 2400, 600 and 300 lb/in. systems. It is apparent from these results that the production restraint system yields the highest deceleration values. This observation generally agrees with results obtained by R. I. Emori with a simplified model for frontal collisions[*].

Table III-2 Collision Possibilities Considered in Investigation

Striking Vehicle	Struck Vehicle: Production	Struck Vehicle: Prototype
Production	Case I	Case IV
Prototype	Case III	Case II

Emori, Richard I., "Analytical Approach to Automobile Collisions", Society of Automotive Engineers Paper No. 680016, Detroit, January 1968.

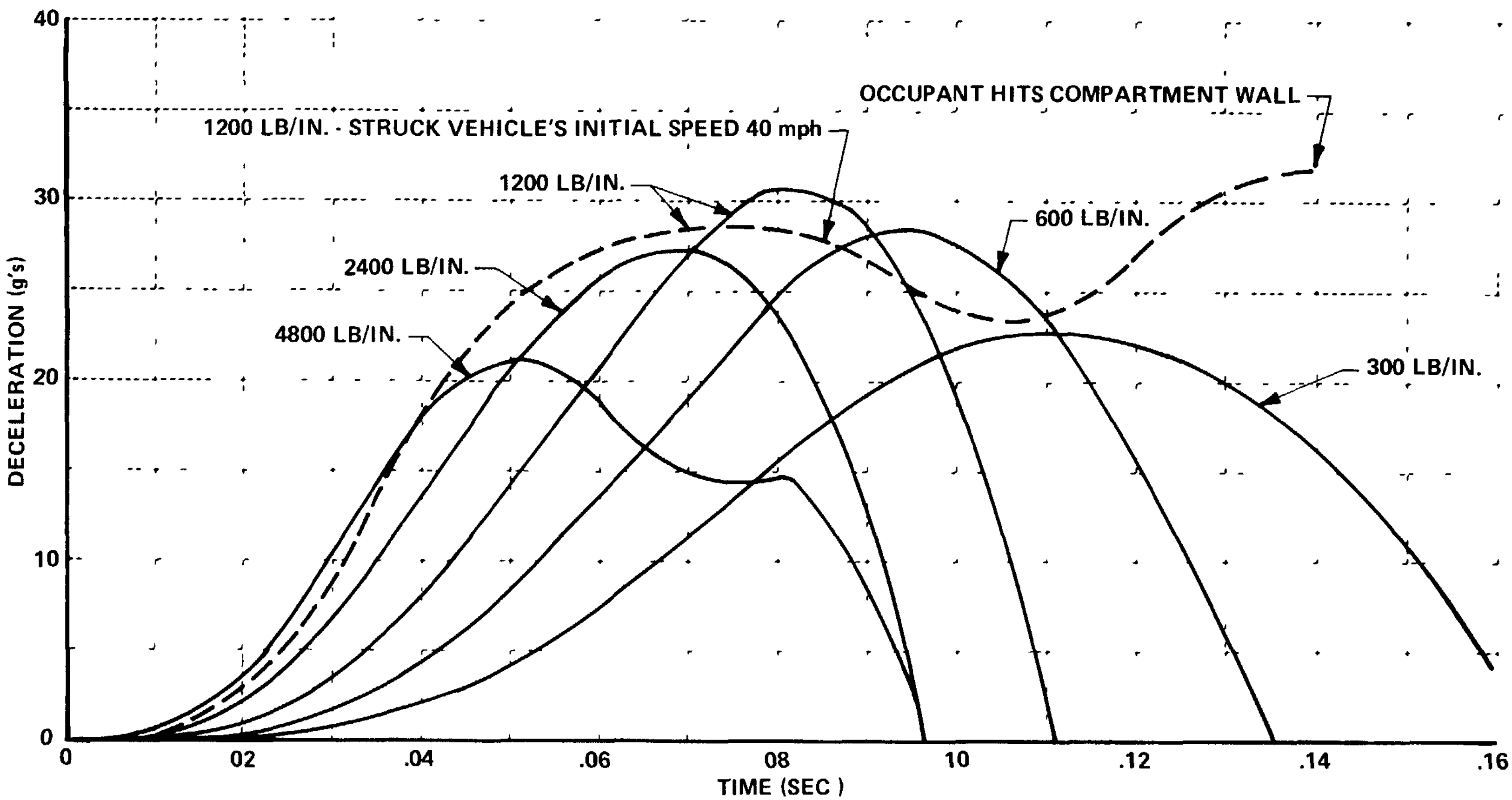


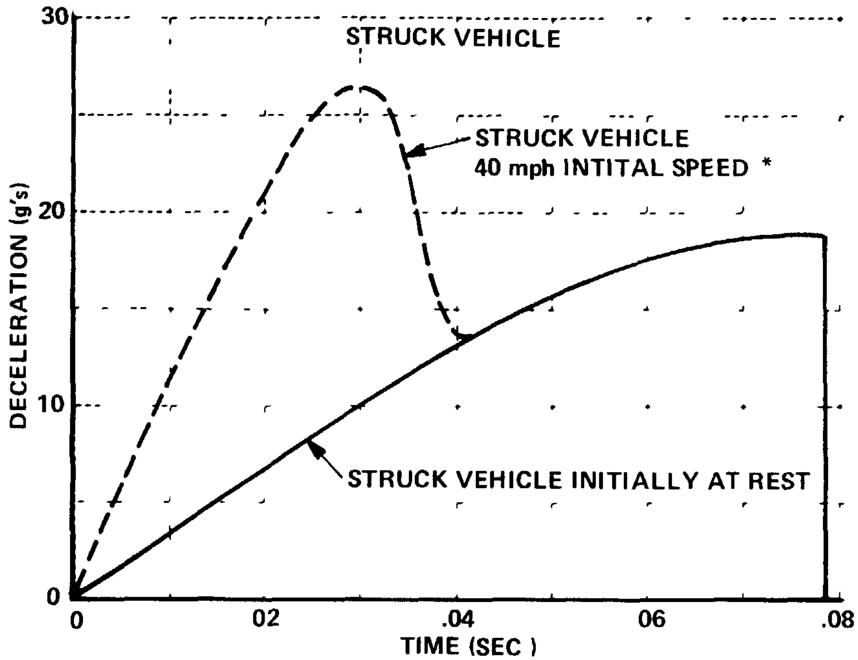
Figure III-10 OCCUPANT RESPONSE PRODUCTION VEHICLE INTO PRODUCTION VEHICLE - TOTAL DECELERATIONS FOR DIFFERENT RESTRAINT CONSTANTS

The vehicle lateral decelerations and penetration are shown in Figure III-11. With Case I, the same penetration results with both vehicles.

Occupant lateral decelerations for Case II (prototype vehicle into prototype vehicle) are shown in Figure III-12. As expected this system has a higher natural frequency, and consequently, the peak decelerations occur with the stiffer restraint systems. For a 40 g threshold limit the restraint stiffness must be slightly less than 1200 lb/in., while for a 30 g level it must be less than 600 lb/in. Significantly, the prototype vehicle will probably require a restraint system which is not as stiff as that currently installed in production automobiles.

In addition, the struck vehicle's penetration (see Figure III-13) is not significantly different from that of Case I, i.e., 15 in. compared to 17 in.

Shown in Figure III-14 are the same deceleration responses for Case III (production vehicle impacted by prototype vehicle). Assuming that the production vehicle is equipped with the 1200 lb/in. restraint system, the peak deceleration of 37 g's represents a 7 g increase over that for impact by a production vehicle (see Case I). In addition, the penetration of the struck vehicle is increased to 22 in. from 17 in. (see Figure III-15). Thus, as expected, the introduction of a prototype vehicle with a more uniform collapse load will produce a somewhat more hazardous condition for the occupants of the production vehicle. However, injury thresholds have not yet been established so that a quantitative assessment of this increase cannot be made in terms of injury potential.



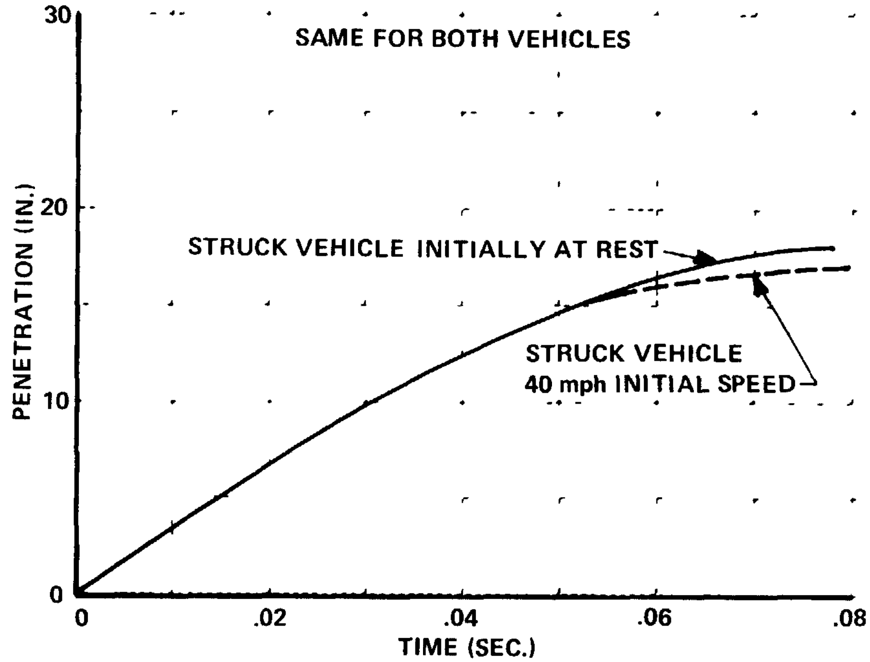


Figure III-11 VEHICLE RESPONSE PRODUCTION VEHICLE INTO PRODUCTION VEHICLE

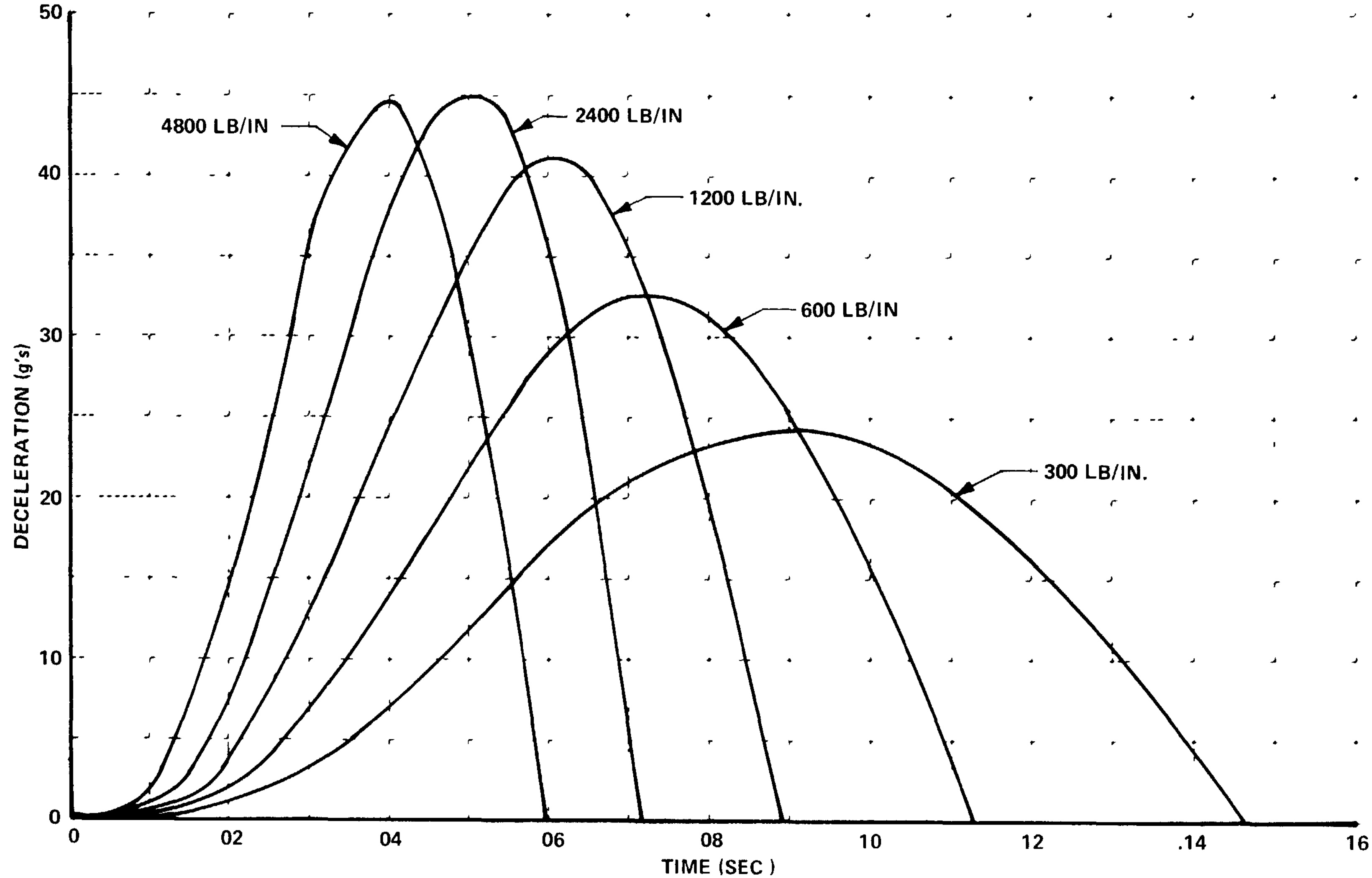


Figure III-12 OCCUPANT RESPONSE PROTOTYPE VEHICLE INTO PROTOTYPE VEHICLE LATERAL DECELERATION FOR DIFFERENT RESTRAINT CONSTANTS

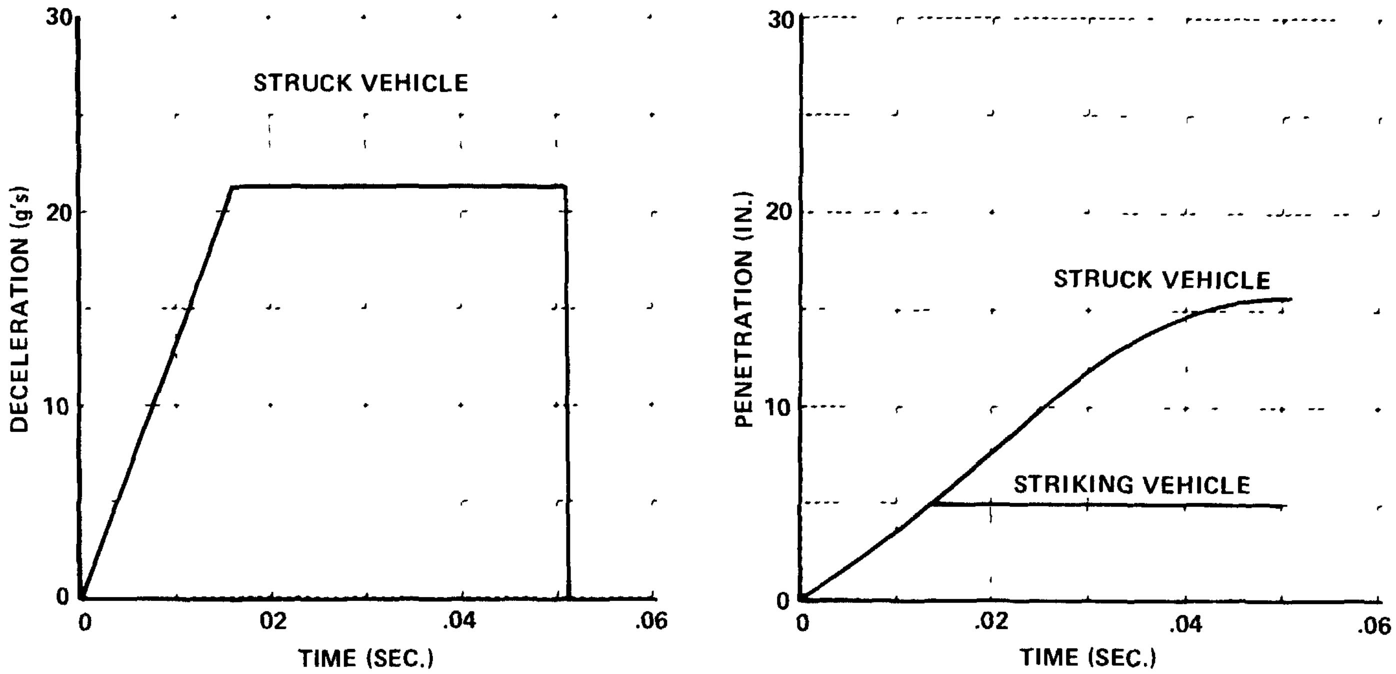


Figure III-13 VEHICLE RESPONSE PROTOTYPE VEHICLE INTO PROTOTYPE VEHICLE

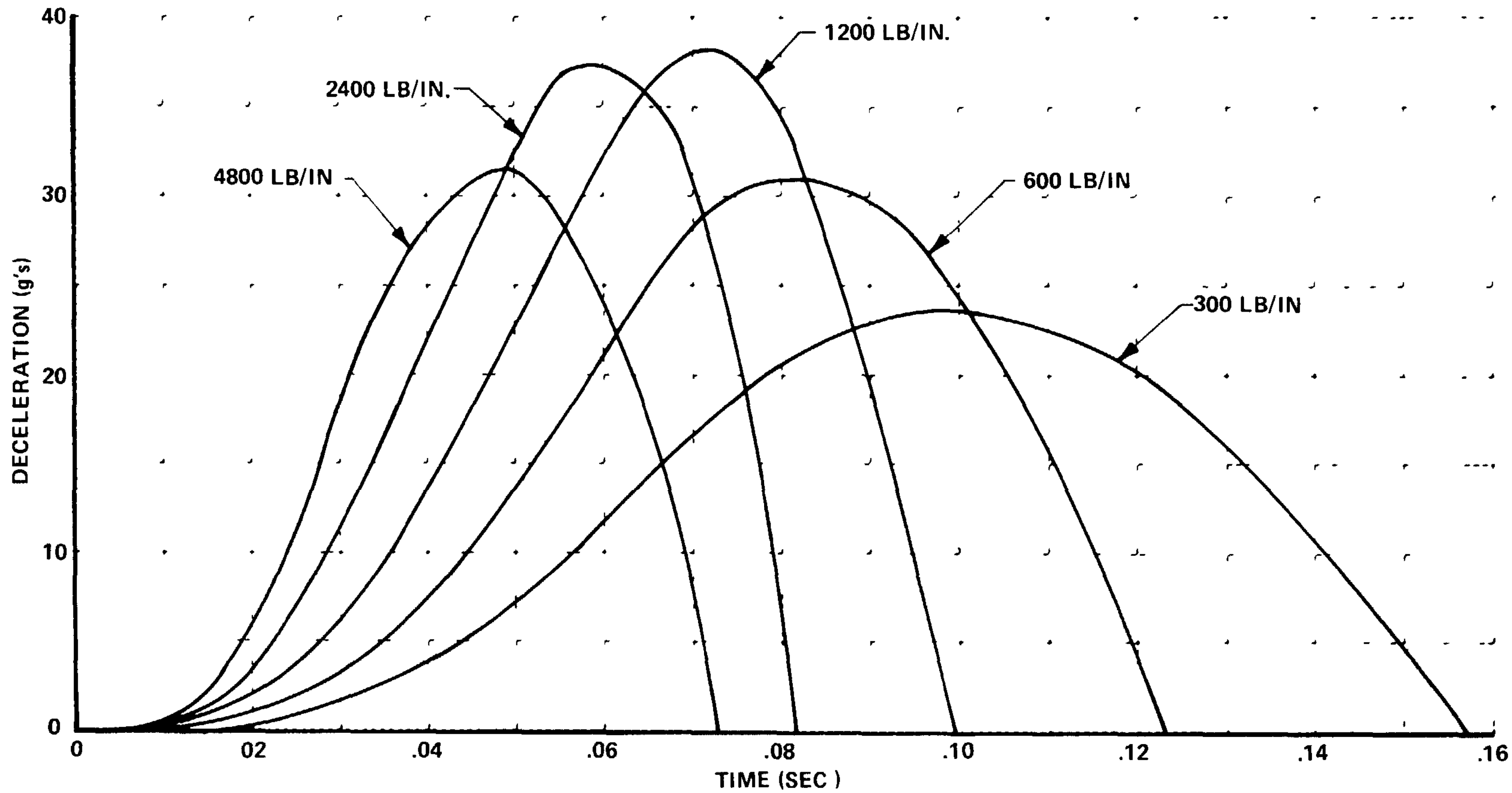


Figure III-14 OCCUPANT RESPONSE PROTOTYPE VEHICLE INTO PRODUCTION VEHICLE LATERAL DECELERATIONS FOR DIFFERENT RESTRAINT CONSTANTS

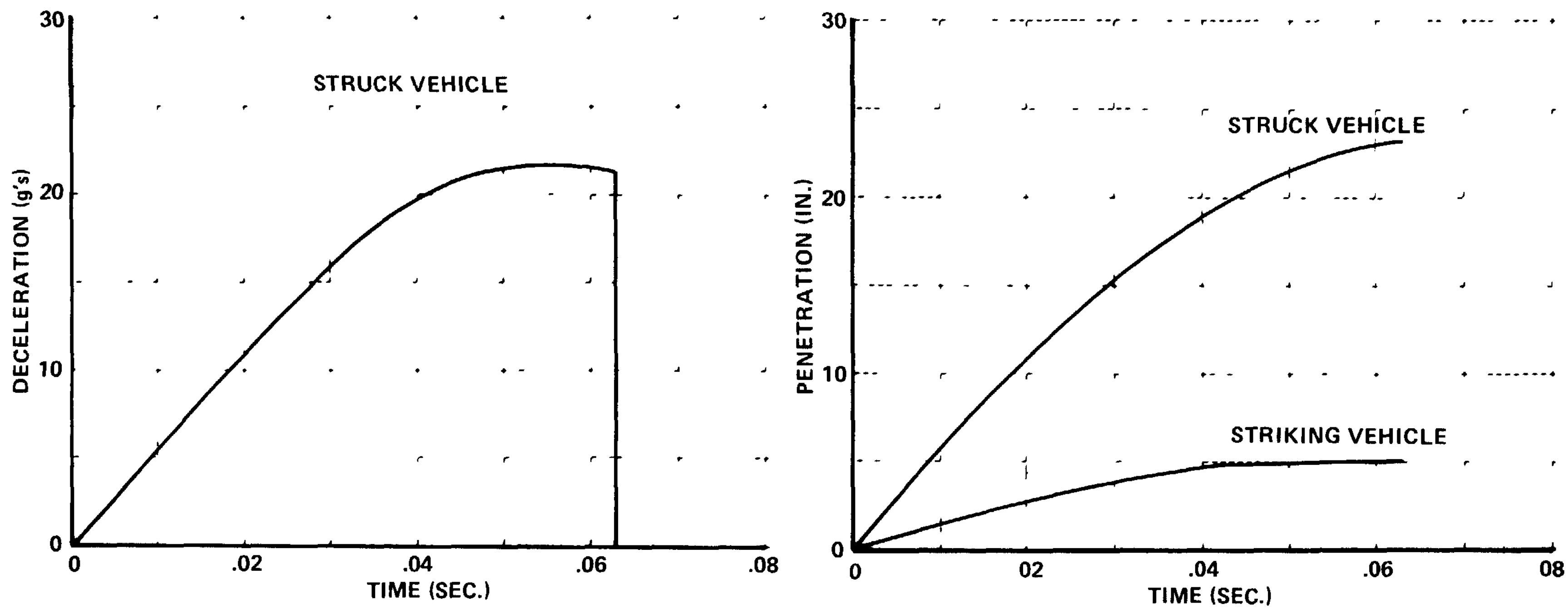


Figure III-15 VEHICLE RESPONSE PROTOTYPE VEHICLE INTO PRODUCTION VEHICLE

Finally, the results for the Case IV (production vehicle impacting prototype vehicle) are presented in Figures III-16 and III-17. Certainly any restraint system which would be acceptable for the prototype vehicle in Case II (see Figure III-12) will be acceptable for this situation.

For example, a 600 lb/in. restraint system yields a peak deceleration of 33 g's in Case II while it is reduced to 30 g's for this latter case. Significantly, the absolute deceleration of the struck vehicle is increased slightly, but the penetration is greatly reduced.

The maximum values for the decelerations and vehicle penetrations for all four cases are summarized in Table III-3. Naturally, before a design criterion is finalized, other conditions such as different force-deflection relationships, vehicle sizes, etc. should be investigated. Nevertheless, this example should serve to demonstrate quantitatively the trends that could be expected if structural modifications of this type are pursued.

5.2 Yielding Restraint System

It is quite likely that vehicles having a more constant deceleration response will require a non-elastic, yielding restraint system in order to remain below a prescribed threshold deceleration. Thus, a yielding type restraint system was simulated for the collision configuration identical to Case II, a prototype vehicle impacting a prototype vehicle. For this simulation, restraint systems having an initial stiffness of 4800 lb/in. with yield limits of 6000 lbs, 4000 lbs, 3000 lbs and 2000 lbs were considered. The occupant deceleration responses for each system are shown in Figure III-18 while the corresponding occupant displacements are shown in Figure III-19.

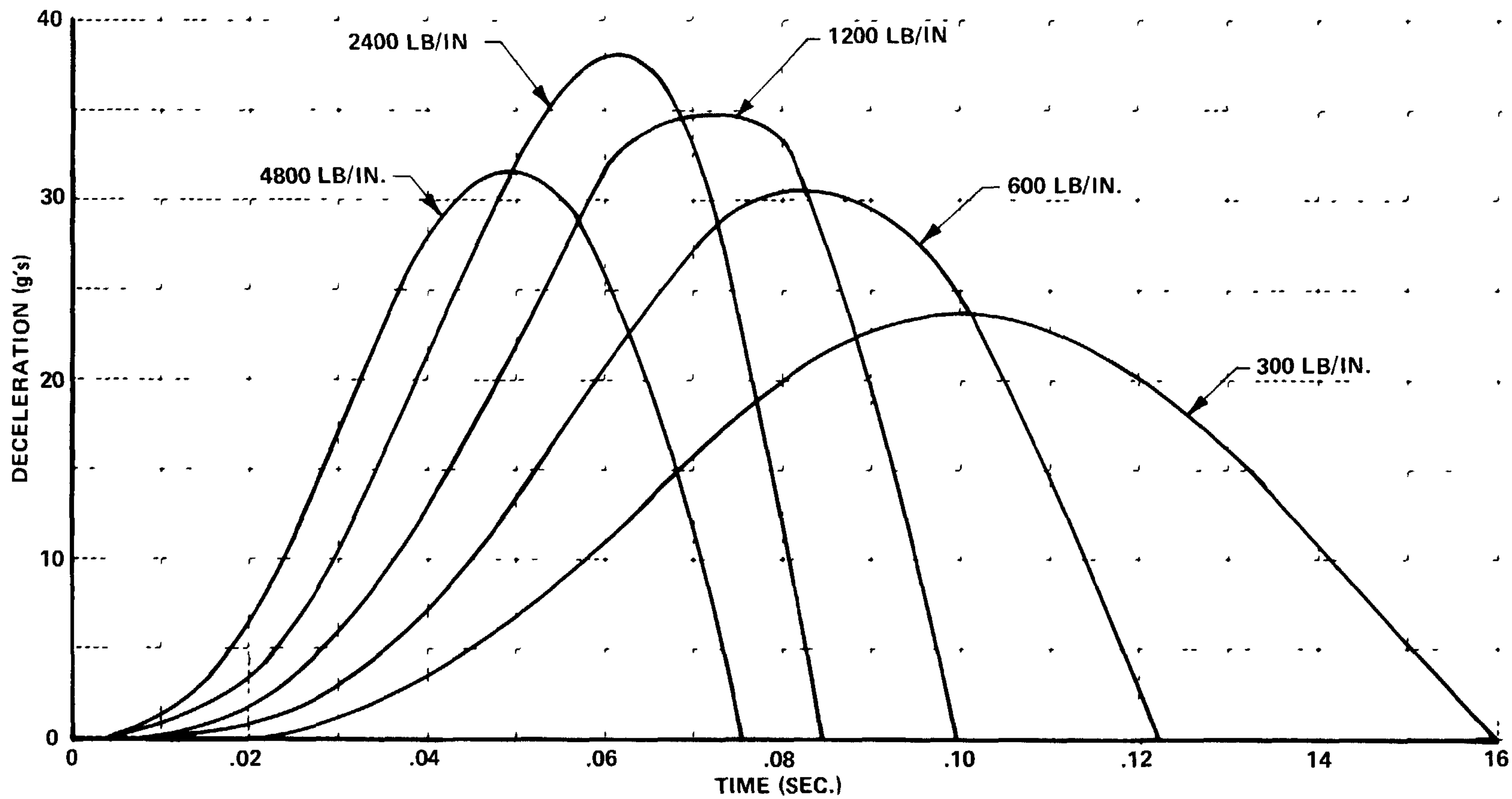


Figure III-16 OCCUPANT RESPONSE PRODUCTION VEHICLE INTO PROTOTYPE VEHICLE LATERAL DECELERATION FOR DIFFERENT RESTRAINT CONSTANTS

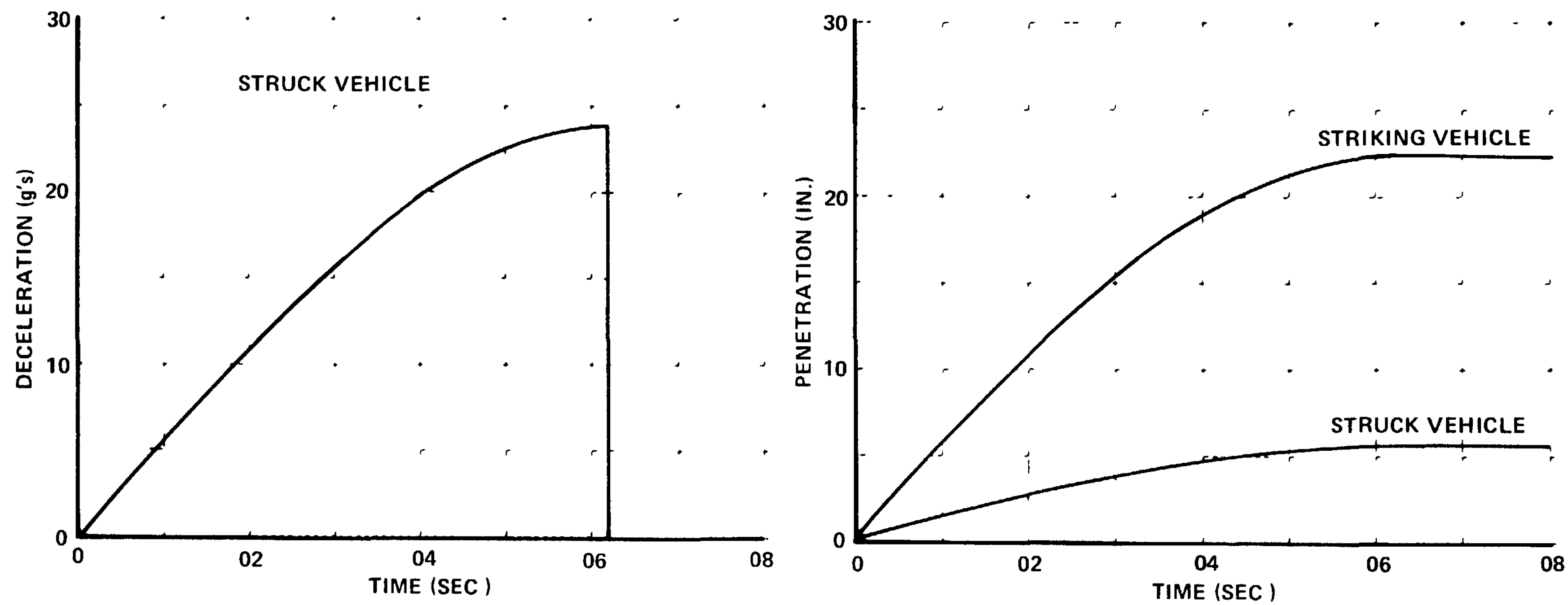


Figure III-17 VEHICLE RESPONSE PRODUCTION VEHICLE INTO PROTOTYPE VEHICLE

Table III-3 Summary of Side Collision Results

Case*	Peak Occupant Lateral Deceleration (g's) Restraint System (lb/in)					Collision Time (sec.)	Max. Vehicle Penetration		Peak Vehicle CG Deceleration (g's)
	300	600	1200	2400	4800		Struck (in.)	Striking (in.)	
I	22.5	28.0	30.5	26.0	20.8	.078	17.5	17.5	18.6
II	24.2	32.4	41.1	45.0	44.6	.051	15.3	5.0	21.3
III	23.6	30.9	37.5	37.2	31.5	.063	22.8	5.0	21.3
IV	23.8	31.0	34.0	37.8	31.5	.062	5.5	22.2	23.6

* See Table III-2 for details

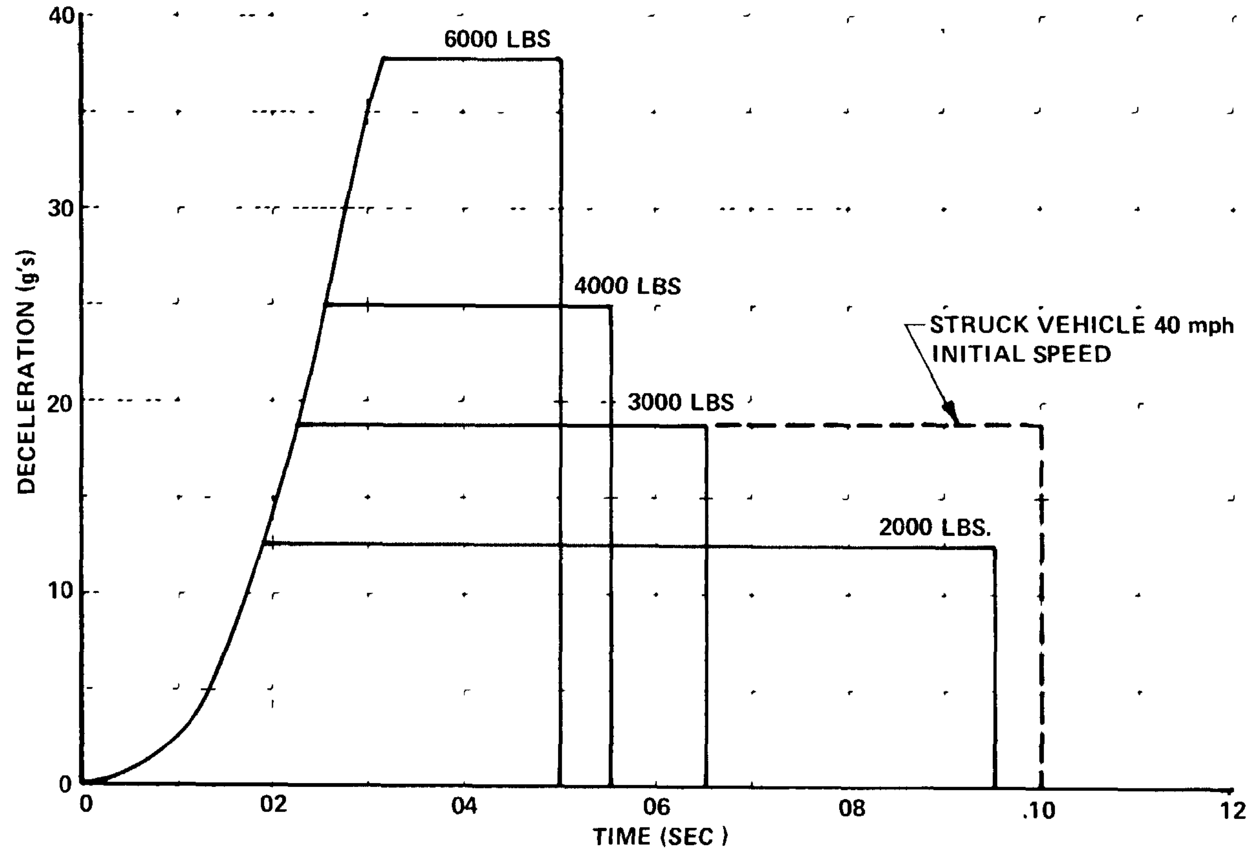


Figure III-18 OCCUPANT DECELERATION RESPONSE FOR YIELDING RESTRAINT SYSTEM WITH DIFFERENT YIELD LIMITS

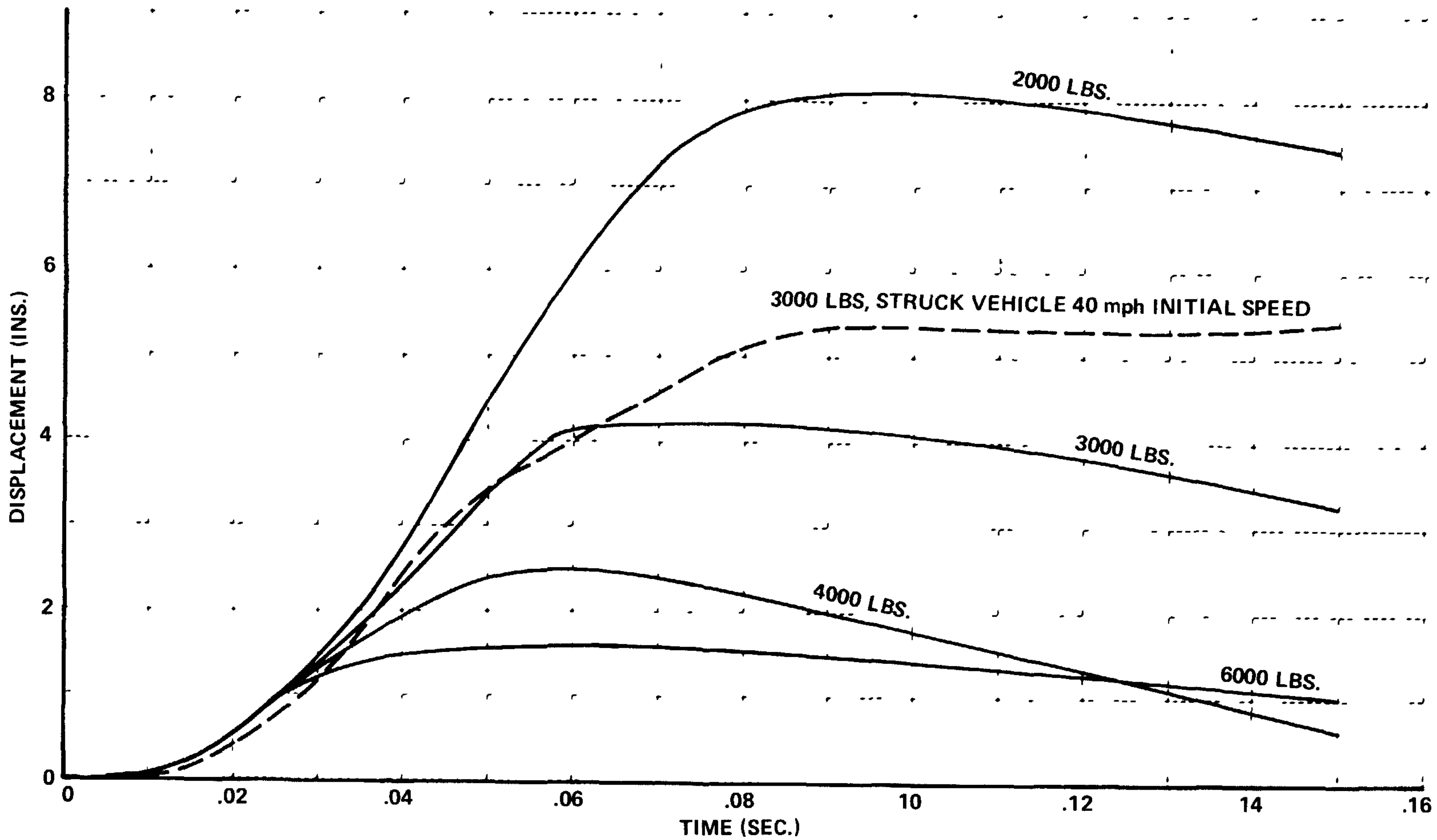


Figure III-19 DISPLACEMENT - TIME RELATIONSHIP FOR YIELDING RESTRAINT SYSTEM WITH DIFFERENT YIELD LIMITS

Since the occupant is treated as a single mass, occupant displacements are meaningful only in a relative sense, i.e., in a qualitative manner. Figure III-19 indicates the relative changes in displacements that could be expected if the yield force was varied. A more precise simulation would require some articulation to represent the motion of the occupant's head and upper torso. Nevertheless, this analysis should provide a reasonable estimate of the expected decelerations and also an indication of the probable need for yielding type restraint systems.

APPENDIX A: FIRST OUTPUT SAMPLE

VEHICLE CØLLISIØN SIMULATIØN

INPUT DATA 1300 4 8 69

1	.2	STØP TIME	sec	
2	.001	CALCULATIØN PERIØD	sec	
3	.01	PRINT PERIØD	sec	
4	.0	YAW, STRIKING VEHICLE	deg	γ_{20}
5	48.	STRUCK VEH. CG TØ FRONT WHEEL	in.	a
6	72.	STRUCK VEH. CG TØ REAR WHEEL	in.	b
7	39.	STRUCK VEH. CG TØ RIGHT SIDE	in.	c
8	80.	STRIKING VEH. CG TØ FRONT END	in.	e
9	1200.	SPRING CON., OCC. RESTRAINT	lbs/in.	k_s
10	3700.	WEIGHT, STRUCK VEHICLE	lbs	$M_1 g$
11	33800.	MØMENT ØF INERTIA, STRUCK VEH.	lb-sec^2-in.	I_1
12	3700.	WEIGHT, STRIKING VEHICLE	lbs	$M_2 g$
13	33800.	MØMENT ØF INERTIA, STRIKE VEH.	lb-sec^2-in.	I_2
14	160.	WEIGHT, OCC.	lbs	$M_3 g$
15	.0	SPEED, STRUCK VEH.	mph	$\dot{\lambda}_{10}$
16	40.	SPEED, STRIKING VEHICLE	mph	$\dot{Y}_{20}$
17	6.	INITIAL X POSITION OF OCC.	in.	x_{30}
18	-22.	Y	in.	y_{30}
19	12.	CG TO STRIKE PT., STRUCK VEH.	in.	p
20	4000.	SPRING CON., STRUCK VEH CR.	lb/in.	k_1
21	80000.	MAX. CRUSH, STRUCK VEH.	lb	G_1
22	4000.	SPRING CON., STRIKE VEH. CR.	lb/in.	k_2
23	120000.	MAX. CRUSH, STRIKE VEH.	lb	G_2
24	6000.	SPRING CON., TAN. FRICTION	lb/in.	k_3
25	120000.	MAX. FR. FORCE	lb	G_3
26	100.	TIRE SLIP COEFF.	lb/deg	k_4
27	500.	MAX. TIRE SLIP FORCE	lb	G_4
28	6.	REAR LIMIT, OCC. COMP.	in.	
29	18.	FWD.	in.	
30	4.	LEFT	in.	
31	48.	RIGHT	in.	
32	1000000.	MAX. OCC. RES. FORCE	lbs	

PROC. FILES USED

FILE	TIME	DY	MO	YR
CTDV01	1515	17	7	69
CTDVAR	1600	7	8	69
CTDVIR	1400	7	8	69
CTDV02	1430	17	7	69
CTDVIN	1330	7	8	69
CTDATS	1345	7	8	69
CTDV03	1230	16	7	69
CTBUF	0830	18	7	69
CTDATB	1345	7	8	69
CTVDYN	1700	11	7	69
CTFORC	0950	7	8	69
CTOCC	1330	7	8	69
CTDVOU	1330	7	8	69

1300 4 8 69

OCCUPANT

TIME SEC.	X POS IN	Y POS IN	X VEL FPS	Y VEL FPS	SPEED FPS	X ACC G-S	Y ACC G-S	ACC G-S	
.000	.0	.0	.00	.00	.00	.00	.00	.00	
.010	.0	.0	-.00	.42	.42	-.01	-.08	.08	
.020	.0	.1	-.01	1.90	1.90	-.08	-.95	.96	
.030	.0	.5	-.05	3.99	3.99	-.29	-3.46	3.47	
.040	.1	1.1	-.17	5.98	5.98	-.64	-7.92	7.95	
.050	.2	2.0	-.39	7.13	7.14	-1.08	-14.00	14.04	
.060	.2	2.8	-.74	6.85	6.89	-1.52	-20.72	20.77	
.070	.3	3.6	-1.19	4.93	5.07	-1.86	-26.69	26.76	
.078	.3	4.0	-1.59	2.35	2.84	-2.01	-29.97	30.04	D
.080	.3	4.1	-1.65	.69	1.79	-2.03	-30.49	30.55	
.090	.3	3.6	-1.85	-9.08	9.27	-1.97	-28.05	28.12	
.100	.2	2.1	-2.30	-16.70	16.86	-1.67	-17.41	17.49	
.110	.2	-.1	-2.56	-19.93	20.09	-1.20	-1.36	1.81	
.120	.1	-2.3	-2.66	-17.70	17.90	-.75	15.66	15.68	
.130	.0	-4.0	-2.69	-10.50	10.84	-.41	28.79	28.79	
.131	.0	-4.1	-2.69	-9.57	9.94	-.38	29.73	29.73	OC
.140	.0	-4.6	-2.72	-.23	2.73	-.15	34.17	34.17	
.150	-.0	-3.9	-2.78	10.24	10.61	.14	30.02	30.02	
.160	-.1	-2.1	-2.83	17.92	18.15	.58	17.26	17.27	
.170	-.2	.3	-2.76	20.55	20.73	1.21	-.71	1.40	
.180	-.3	2.7	-2.48	17.21	17.38	1.91	-18.90	18.99	
.190	-.3	4.4	-1.92	8.69	8.90	2.42	-32.09	32.18	
.200	-.3	4.8	-1.16	-2.71	2.94	2.43	-36.38	36.46	

1300 4 8 69

STRUCK VEHICLE

TIME SEC.	X PØS IN	Y PØS IN	YAW DEG	X VEL MPH	Y VEL MPH	X ACC G-S	Y ACC G-S	PEN. IN.
.000	.0	.0	.00	.00	.00	.00	.00	.00
.010	-.0	-.0	-.00	-.00	-.29	-.00	-3.14	3.50
.020	-.0	-.2	-.04	-.00	-1.39	-.03	-6.81	6.[illegible]
.030	-.0	-.5	-.12	-.02	-3.26	-.11	-10.20	9.96
.040	-.0	-1.3	-.28	-.06	-5.84	-.25	-13.16	12.67
.050	-.0	-2.6	-.54	-.13	-9.00	-.46	-15.57	14.65
.060	-.1	-4.5	-.92	-.26	-12.62	-.70	-17.32	16.40
.070	-.1	-7.1	-1.40	-.44	-16.55	-.96	-18.32	17.2
.078	-.2	-9.7	-1.88	-.63	-19.80	-1.14	-18.56	17.46
.080	-.2	-10.4	-2.01	-.68	-19.99	-1.24	.31	17.46
.090	-.4	-13.9	-2.65	-.95	-19.92	.01	.27	17.46
.100	-.5	-17.4	-3.27	-.95	-19.86	.02	.27	17.46
.110	-.7	-20.9	-3.89	-.94	-19.80	.02	.27	17.46
.120	-.9	-24.3	-4.52	-.94	-19.74	.02	.27	17.46
.130	-1.0	-27.8	-5.15	-.94	-19.68	.02	.27	17.46
.131	-1.0	-28.2	-5.21	-.93	-19.68	.02	.27	17.[illegible]
.140	-1.2	-31.3	-5.78	-.93	-19.62	.03	.27	17.[illegible]
.150	-1.4	-34.7	-6.41	-.92	-19.56	.03	.27	17.46
.160	-1.5	-38.2	-7.04	-.92	-19.51	.03	.27	17.46
.170	-1.7	-41.6	-7.68	-.91	-19.45	.04	.27	17.46
.180	-1.8	-45.0	-8.31	-.90	-19.39	.04	.27	17.46
.190	-2.0	-48.4	-8.95	-.89	-19.33	.04	.27	17.46
.200	-2.1	-51.8	-9.60	-.86	-19.27	.04	.27	17.46

1300 4 8 69

STRIKING VEHICLE

TIME SEC.	X PØS IN	Y PØS IN	YAW DEG	X VEL MPH	YVEL MPH	X ACC G-S	Y ACC G-S	PEN. IN.	
.000	12.0	119.0	.00	.00	-40.00	.00	.00	.00	
.010	12.0	112.0	.00	.00	-39.66	.00	3.41	3.50	
.020	12.0	105.1	.00	.00	-38.51	.03	7.08	6.88	
.030	12.0	98.5	.00	.02	-36.57	.11	10.47	9.98	
.040	12.0	92.3	.01	.06	-33.94	.26	13.43	12.67	
.050	12.0	86.6	.02	.13	-30.72	.46	15.84	14.85	
.060	12.1	81.5	.05	.26	-27.04	.70	17.59	16.40	
.070	12.1	77.1	.10	.45	-23.05	.96	18.59	17.28	
.078	12.2	74.0	.17	.63	-19.76	1.15	18.83	17.46	D
.080	12.2	73.4	.18	.68	-19.55	1.25	-.04	17.46	
.090	12.4	69.9	.30	.96	-19.56	.00	.00	17.46	
.100	12.5	66.5	.43	.96	-19.56	.00	.00	17.46	
.110	12.7	63.0	.56	.96	-19.56	.00	.00	17.46	
.120	12.9	59.6	.69	.96	-19.56	.00	.00	17.46	
.130	13.0	56.1	.83	.96	-19.56	.00	.00	17.46	
.131	13.1	55.8	.84	.96	-19.56	.00	.00	17.46	ØC
.140	13.2	52.7	.96	.96	-19.56	.00	.00	17.46	
.150	13.4	49.3	1.09	.96	-19.56	.00	.00	17.46	
.160	13.5	45.8	1.22	.96	-19.56	.00	.00	17.46	
.170	13.7	42.4	1.35	.96	-19.56	.00	.00	17.46	
.180	13.9	38.9	1.49	.96	-19.56	.00	.00	17.46	
.190	14.0	35.5	1.62	.96	-19.56	.00	.00	17.46	
.200	14.2	32.0	1.75	.96	-19.56	.00	.00	17.46	

APPENDIX B: SECOND SAMPLE OUTPUT

```
VEHICLE CØLLISIØN SIMULATIØN

INPUT DATA  1300   4  8 69 REV.    856
? 32,4000
   10000000.        TØ          4000.       MAX. OCC. RES. FORCE
? 0,0
```

```
PRØC. FILES USED

FILE    TIME DY  MØ  YR

CTDV04  1515 17   7  69
CTDVAR  1600  7   8  69
CTDVCH  1400  7   8  69
CTDV02  1430 17   7  69
CTDVIN  1330  7   8  69
CTDATS  1345  7   8  69
CTDV03  1230 16   7  69
CTBUF   0830 18   7  69
CTDATR  1345  7   8  69
CTVDIN  1700 11   7  69
CTFØRC  0950  7   8  69
CTØCC   1330  7   8  69
CTDVØU  1330  7   8  69
```

1300 4 8 69 REV. 856

ØCCUPANT

TIME SEC.	X PØS IN	Y PØS IN	X VEL FPS	Y VEL FPS	SPEED FPS	X ACC G-S	Y ACC G-S	ACC G-S	
.000	.0	.0	.00	.00	.00	.00	.00	.00	
.010	.0	.0	-.00	.42	.42	-.01	-.08	.08	
.020	.0	.1	-.01	1.90	1.90	-.08	-.95	.96	
.030	.0	.5	-.05	3.99	3.99	-.29	-3.46	3.47	
.040	.1	1.1	-.17	5.98	5.98	-.64	-7.92	7.95	
.050	.2	2.0	-.39	7.13	7.14	-1.08	-14.00	14.04	
.060	.2	2.8	-.74	6.85	6.89	-1.52	-20.72	20.77	
.070	.3	3.6	-1.18	5.02	5.16	-1.74	-24.94	25.00	
.078	.3	4.1	-1.52	3.35	3.68	-1.67	-24.94	25.00	D
.080	.3	4.2	-1.57	2.03	2.57	-1.66	-24.95	25.00	
.090	.3	4.1	-1.40	-1.69	2.19	.00	.00	.00	
.100	.3	4.0	-1.38	-1.79	2.26	.00	.00	.00	
.110	.4	3.8	-1.36	-1.89	2.33	.00	.00	.00	
.120	.4	3.7	-1.34	-1.99	2.40	.00	.00	.00	
.130	.4	3.5	-1.31	-2.09	2.47	.00	.00	.00	
.140	.5	3.3	-1.29	-2.20	2.55	.00	.00	.00	
.150	.5	3.1	-1.27	-2.30	2.62	.00	.00	.00	
.160	.6	2.9	-1.24	-2.40	2.70	.00	.00	.00	
.170	.7	2.7	-1.21	-2.50	2.78	.00	.00	.00	
.180	.7	2.5	-1.18	-2.60	2.86	.00	.00	.00	
.190	.8	2.2	-1.15	-2.70	2.94	.00	.00	.00	
.200	.9	2.0	-1.12	-2.80	3.02	.00	.00	.00	

1300 4 8 69 REV. 856

STRUCK VEHICLE

TIME SEC.	X PØS IN	Y PØS IN	YAW DEG	X VEL MPH	YVEL MPH	X ACC G-S	Y ACC G-S	PEN. IN.
.000	.0	.0	.00	.00	.00	.00	.00	.00
.010	-.0	-.0	-.00	-.00	-.29	-.00	-3.14	3.50
.020	-.0	-.2	-.04	-.00	-1.39	-.03	-6.61	6.86
.030	-.0	-.5	-.12	-.02	-3.26	-.11	-10.20	9.95
.040	-.0	-1.3	-.28	-.06	-5.84	-.25	-13.16	12.67
.050	-.0	-2.6	-.54	-.13	-9.00	-.46	-15.57	14.55
.060	-.1	-4.5	-.92	-.26	-12.62	-.70	-17.32	16.40
.070	-.1	-7.1	-1.40	-.44	-16.55	-.96	-18.32	17.25
.078	-.2	-9.7	-1.88	-.63	-19.60	-1.14	-16.56	17.46
.080	-.2	-10.4	-2.01	-.68	-19.99	-1.24	.31	17.46
.090	-.4	-13.9	-2.65	-.95	-19.92	.01	.27	17.46
.100	-.5	-17.4	-3.27	-.95	-19.86	.02	.27	17.46
.110	-.7	-20.9	-3.89	-.94	-19.80	.02	.27	17.46
.120	-.9	-24.3	-4.52	-.94	-19.74	.02	.27	17.46
.130	-1.0	-27.8	-5.15	-.94	-19.68	.02	.27	17.46
.140	-1.2	-31.3	-5.78	-.93	-19.62	.03	.27	17.46
.150	-1.4	-34.7	-6.41	-.92	-19.56	.03	.27	17.46
.160	-1.5	-38.2	-7.04	-.92	-19.51	.03	.27	17.46
.170	-1.7	-41.6	-7.68	-.91	-19.45	.04	.27	17.46
.180	-1.8	-45.0	-8.31	-.90	-19.39	.04	.27	17.46
.190	-2.0	-48.4	-8.95	-.89	-19.33	.04	.27	17.46
.200	-2.1	-51.8	-9.60	-.88	-19.27	.04	.27	17.46

1300 4 8 69 REV. 856

STRIKING VEHICLE

TIME SEC.	X PØS IN	Y PØS IN	YAW DEG	X VEL MPH	YVEL MPH	X ACC G-S	Y ACC G-S	PEN. IN.	
.000	12.0	119.0	.00	.00	-40.00	.00	.00	.00	
.010	12.0	112.0	.00	.00	-39.66	.00	3.41	3.50	
.020	12.0	105.1	.00	.00	-38.51	.03	7.08	6.88	
.030	12.0	98.5	.00	.02	-36.57	.11	10.47	9.96	
.040	12.0	92.3	.01	.06	-33.94	.26	13.43	12.67	
.050	12.0	86.6	.02	.13	-30.72	.46	15.84	14.85	
.060	12.1	81.5	.05	.26	-27.04	.70	17.59	16.40	
.070	12.1	77.1	.10	.45	-23.05	.96	18.59	17.28	
.078	12.2	74.0	.17	.63	-19.76	1.15	18.83	17.46	D
.080	12.2	73.4	.18	.68	-19.55	1.25	-.04	17.46	
.090	12.4	69.9	.30	.96	-19.56	.00	.00	17.46	
.100	12.5	66.5	.43	.96	-19.56	.00	.00	17.46	
.110	12.7	63.0	.56	.96	-19.56	.00	.00	17.46	
.120	12.9	59.6	.69	.96	-19.56	.00	.00	17.46	
.130	13.0	56.1	.83	.96	-19.56	.00	.00	17.46	
.140	13.2	52.7	.96	.96	-19.56	.00	.00	17.46	
.150	13.4	49.3	1.09	.96	-19.56	.00	.00	17.46	
.160	13.5	45.8	1.22	.96	-19.56	.00	.00	17.46	
.170	13.7	42.4	1.35	.96	-19.56	.00	.00	17.46	
.180	13.9	38.9	1.49	.96	-19.56	.00	.00	17.46	
.190	14.0	35.5	1.62	.96	-19.56	.00	.00	17.46	
.200	14.2	32.0	1.75	.96	-19.56	.00	.00	17.46	

CTDV01

```
100 FILE CTDVF1
110 FILE CTAUXF
120 REM                          PRINT INPUT AND RUN PRØGRAM
130 XX(1)="CTDV01 "
140 YY(1)="1515 17"
150 ZZ(1)="  7 69"
160 CALL CTDVAR
170 CALL CTDVIR
180 J=1
190 WRITE BINARY 2,J
200 CHAIN CTDV02
210 END
```

CTDV02

```
100 FILE CTDVF1
110 FILE CTAUXF,CTAUXG
120 FILE CTDATF
130 DIM S(3,3)
140 DIM V(3,3)
150 DIM A(3,3)
160 DIM M(3,3)
170 DIM F(3,3)
180 DIM C(2,2)
190 PRINT "CTDV02 ";
200 PRINT "1430 17";
210 PRINT "  7 69"
220 READ BINARY 2,Q1
230 CALL  CTDVIN
240 CC="---"
250 CALL CTDATS
260 CHAIN CTDV03
270 END
```

CTDV03

```
100 FILE CTDVF1
110 FILE CTAUXF,CTAUXG
120 FILE CTDATF
130 DIM S(3,3)
140 DIM V(3,3)
150 DIM A(3,3)
160 DIM M(3,3)
170 DIM F(3,3)
180 DIM C(2,2)
190 PRINT "CTDV03 ";
200 PRINT "1230 16";
210 PRINT "  7 69"
220 CALL CTBUF
230 CALL CTDATB
240 CALL CTVDYN
250 CALL CTFØRC
260 CALL CTØCC
270 CALL CTDVØU
280 T=T+T2
290 IF Ø2<55 THEN 320
300 PRINT
310 GØ TØ 490
320 IF T<=T1 THEN 240
330 GØ TØ 380
340 Ø8=0
350 PRINT BB(4);
360 INPUT T1
370 IF T<=T1 THEN 240
380 Ø2=58-Ø2
390 FØR J=1 TØ Ø2
400 PRINT
410 NEXT J
420 FØR J=2 TØ 3
430 WRITE BINARY J,-5,Ø2
440 WRITE BINARY J,-4
450 RESTØRE FILE J
460 NEXT J
470 CALL CTBUF
480 GØ TØ 240
490 PRINT
500 GØ TØ 380
510 END
```

CTDV04

```
100 FILE CTDVF1
110 FILE CTAUXF
120 FILE CTAUXG
130 REM                    MAKE INPUT CHANGES AND RUN PRØGRAM
140 XX(1)="CTDV04 "
150 YY(1)="1515 17"
160 ZZ(1)="  7 69"
170 CALL CTDVAR
180 CALL CTDVCH
190 J=3
200 WRITE BINARY 2,J
210 CHAIN CTDV02
220 END
```

CTDV05

```
100 FILE CTAUXF
110 REM                    RUN PRØGRAM ØNLY
120 PRINT "CTDV05 ";
130 PRINT "0930 18";
140 PRINT "  7 69"
150 WRITE BINARY 1,1
160 CHAIN CTDV02
170 END
```

CTDVAR

```
100 DIM P(32)
110 DIM AA(32)
120 DIM BB(32)
130 XX(2)="CTDVAR "
140 YY(2)="1600  7"
150 ZZ(2)="  8 69"
160 FØR J=1 TØ 32
170 BB(J)="   "
180 NEXT J
190 SET P=10.5,,Z
200 SET J=3.0,S
210 AA(1)="STØP TIME"
220 AA(2)="CALCULATIØN PER"
230 BB(2)="IØD"
240 AA(3)="PRINT PERIØD"
250 AA(4)="YAW, STRIKING "
260 AA(5)="STRUCK VEH. CG "
270 BB(5)="TØ FRØNT WHEEL"
280 AA(6)=AA(5)
290 BB(6)="TØ REAR WHEEL"
300 AA(7)=AA(5)
310 BB(7)="TØ RIGHT SIDE"
320 AA(8)="STRIKING VEH. "
330 BB(8)="CG TØ FRØNT END"
340 BB(9)="ØCC. RESTRAINT"
350 AA(9)="SPRING CØN., "
360 AA(10)="WEIGHT, STRUCK "
370 BB(10)="VEHICLE"
380 BB(4)=BB(10)
390 AA(11)="MØMEMT ØF INERT"
400 BB(11)="IA, STRUCK VEH."
410 AA(12)="WEIGHT, STRIKIN"
420 BB(12)="G VEHICLE"
430 AA(13)=AA(11)
440 BB(13)="IA, STRIKE VEH."
450 AA(14)="WEIGHT, ØCC."
460 AA(15)="SPEED, STRUCK V"
470 BB(15)="EH."
480 AA(16)="SPEED, STRIKING"
490 BB(16)=" VEHICLE"
500 AA(17)="INITIAL X PØSIT"
510 BB(17)="IØN ØF ØCC."
520 AA(18)="          Y"
530 AA(19)="CG TØ STRIKE PT"
540 BB(19)="., STRUCK VEH."
550 AA(20)=AA(9)
560 AA(21)="MAX. CRUSH, "
570 BB(21)="STRUCK VEH."
580 BB(20)="STRUCK VEH CR."
590 AA(22)=AA(20)
```

CTDVAR CØNTINUED

```
600 BB(22)="STRIKE VEH. CR."
610 AA(23)=AA(21)
620 BB(23)="STRIKE VEH."
630 AA(24)=AA(20)
640 BB(24)="TAN. FRICTIØN"
650 AA(25)="MAX. FR. FØRCE"
660 AA(26)="TIRE SLIP CØEF."
670 AA(27)="MAX. TIRE "
680 BB(27)="SLIP FØRCE"
690 AA(28)="REAR LIMIT, "
700 BB(28)="ØCC. CØMP."
710 AA(29)="FWD."
720 AA(30)="LEFT"
730 AA(31)="RIGHT"
740 AA(32)="MAX. ØCC. RES. "
750 BB(32)="FØRCE"
760 CC="---"
770 FØR J=1 TØ 10
780 PRINT CC;
790 NEXT J
800 FØR J=1 TØ 5
810 PRINT
820 NEXT J
830 PRINT "VEHICLE CØLLISIØN SIMULATIØN"
840 PRINT
850 FØR J=1 TØ 32
860 READ FILE 1,P(J)
870 NEXT J
880 READ FILE 1,B,C1,C2,C3
890 SET B=4.0
900 SET C=2.0
910 PRINT "INPUT DATA ";
920 PRINT B;C1;C2;C3;
930 RETURN
940 END
```

CTDVIR

```
100 PRINT
110 PRINT
120 FØR J=1 TØ 32
130 PRINT J;P(J);AA(J);BB(J)
140 NEXT J
150 PRINT
160 PRINT "PRØG. FILES USED"
170 PRINT
180 PRINT "FILE    TIME DY MØ YR"
190 PRINT
200 FØR K=1 TØ 2
210 PRINT XX(K);YY(K);ZZ(K)
220 NEXT K
230 PRINT "CTDVIR ";
240 PRINT "1400  7";
250 PRINT "  8 69"
260 RETURN
270 END
```

CTDVCH

```
100 B1=CLK(1)*100
110 PRINT " REV. ";B1
120 L1=32
130 INPUT J,A
140 IF J<=0 THEN 200
150 PRINT P(J);" TØ ";
160 P(J)=A
170 PRINT P(J);AA(J);BB(J)
180 L1=L1-2
190 GØ TØ 130
200 FØR J=1 TØ 32
210 WRITE FILE 3,P(J)
220 NEXT J
230 WRITE FILE 3,B,C1,C2,C3,B1
240 FØR J=1 TØ L1
250 PRINT
260 NEXT J
270 RESTØRE FILE 3
280 PRINT
290 PRINT "PRØG. FILES USED"
300 PRINT
310 PRINT "FILE   TIME DY MØ YR"
320 PRINT
330 FØR K=1 TØ 2
340 PRINT XX(K);YY(K);ZZ(K)
350 NEXT K
360 PRINT "CTDVCH ";
370 PRINT "1400  7";
380 PRINT "  8 69"
390 RETURN
400 END
```

CTDVIN

```
100 PRINT "CTDVIN ";
110 PRINT "1330  7";
120 PRINT "  8 69"
130 AA="---"
140 READ FILE Q1,:
150 T1,T2,T3,T4
160 S(3,2)=T4/57.3
170 READ FILE Q1 ,:
180 B(1),B(2),C,E,K9
190 B(2)=-B(2)
200 FØR J=1 TØ 2
210 READ FILE Q1,:
220 M(1,J),M(3,J)
230 NEXT J
240 READ FILE Q1,:
250 M(1,3)
260 FØR J=1 TØ 3
270 M(1,J)=M(1,J)/386.4
280 M(2,J)=M(1,J)
290 NEXT J
300 READ FILE Q1,:
310 V(1,1),V(2,2),S(1,3),S(2,3),P
320 X3=S(1,3)
330 Y3=S(2,3)
340 C(1,1)=1
350 C(1,2)=CØS(S(3,2))
360 C(2,2)=SIN(S(3,2))
370 S(1,2)=P-E*C(2,2)
380 S(2,2)=C+E*C(1,2)
390 V(1,1)=17.6*V(1,1)
400 V(2,2)=-17.6*V(2,2)
410 V(1,2)=-V(2,2)*C(2,2)
420 V(2,2)=V(2,2)*C(1,2)
430 V(1,3)=V(1,1)
440 FØR J=1 TØ 4
450 READ FILE Q1,:
460 K(J),G(J)
470 NEXT J
480 K(4)=K(4)*57.3
490 K8=K(2)/(K(1)+K(2))
500 K(5)=K8*K(1)
510 IF G(2)=0 THEN 540
520 IF G(1)<G(2) THEN 600
530 G(5)=G(2)
540 IF G(1)=G(2) THEN 580
550 G(1)=G(2)
560 K1=1
570 GØ TØ 620
580 K1=3
590 GØ TØ 620
```

CTDVIN CØNTINUED

```
600 G(5)=G(2)=G(1)
610 K1=2
620 FØR J=1 TØ 5
630 IF K(J)<=0 THEN 650
640 D(J)=G(J)/K(J)
650 NEXT J
660 FØR J=1 TØ 4
670 READ FILE Q1,:
680 P(J)
690 NEXT J
700 P(1)=-P(1)
710 P(3)=-P(3)
720 READ FILE Q1,G9
730 FØR J=1 TØ 4
740 READ FILE Q1,:
750 H(J)
760 NEXT J
770 IF Q1=1 THEN 790
780 READ FILE Q1,H(5)
790 BB(1)=" ØC"
800 BB(2)=" D"
810 BB(3)=" S"
820 BB(4)="   "
830 BB(5)=" R"
840 Ø4=Ø5=1
850 RETURN
860 END
```

CTFØRC

```
100 IF Z9<>0 THEN 140
110 PRINT "CTFØRC ";
120 PRINT "0950  7";
130 PRINT "  8 69"
140 X1=S(1,2)-S(1,1)
150 Y1=S(2,2)-S(2,1)
160 V2=C(2,2)*C(2,1)+C(1,1)*C(1,2)
170 V3=C-Y1*C(1,1)+X1*C(2,1)
180 V1=V3/V2
190 D=E+V1
200 D3=D
210 IF D>0 THEN 280
220 F2=0
230 IF Ø5>3 THEN 690
240 IF T<0.05 THEN 690
250 Ø5=4
260 GØ TØ 690
270 REM             FØRCES BETWEEN VEHICLES
280 IF D>D5 THEN 340
290 F2=0
300 IF D=D5 THEN 490
310 IF Ø5<>1 THEN 490
320 Ø5=2
330 GØ TØ 490
340 D5=D
350 IF Ø5=1 THEN 370
360 Ø5=6
370 IF D<D(5) THEN 460
380 IF D(5)=0 THEN 460
390 F2=G(5)
400 GØ TØ (410,430,470),K1
410 D1=D(1)
420 GØ TØ 480
430 D2=D(2)
440 D1=D-D2
450 GØ TØ 490
460 F2=K(5)*D
470 D1=K8*D
480 D2=D-D1
490 X2=X1*C(1,1)+Y1*C(2,1)-V1*(C(2,2)*C(1,1)-C(1,2)*C(2,1))
500 D=X2-P
510 IF D>=D6 THEN 550
520 IF D<=D7 THEN 570
530 F1=0
540 GØ TØ 620
550 D6=D
560 GØ TØ 580
570 D7 = D
580 IF ABS(D)<D(3) THEN 610
590 F1=-SGN(D)*G(3)
```

CTFØRC CØNTINUED

```
600 GØ TØ 620
610 F1=-K(3)*D
620 F(1,2)=F1*C(1,1)-F2*C(2,2)
630 F(2,2)=F1*C(2,1)+F2*C(1,2)
640 F(1,1)=-F(1,2)
650 F(2,1)=-F(2,2)
660 F(3,1)=F1*C-F2*(X1*C(1,2)+Y1*C(2,2))
670 F(3,2)=-F1*V3
680 REM                    TIRE FØRCES
690 IF K(4)=0 THEN 870
700 X1=ABS(V(1,1)*C(1,1)+V(2,1)*C(2,1))
710 Y1=V(2,1)*C(1,1)-V(1,1)*C(2,1)
720 F3=0
730 FØR J=1 TØ 2
740 B=Y1+B(J)*V(3,1)
750 IF B=0 THEN 840
760 IF X1<>0 THEN 790
770 B=G(4)*SGN(B)
780 GØ TØ 820
790 B=B/X1
800 IF ABS(B)>=D(4) THEN 770
810 B=K(4)*B
820 F3=F3-B
830 F(3,1)=F(3,1)-B*B(J)
840 NEXT J
850 F(1,1)=F(1,1)-F3*C(2,1)
860 F(2,1)=F(2,1)+F3*C(1,1)
870 RETURN
880 END
```

CTØCC

```
100 IF Z9<>0 THEN 160
110 PRINT "CTØCC  ";
120 PRINT "1330  7";
130 PRINT "  8 69"
140 D9=(G9/K9)↑2
150 RETURN
160 X1=S(1,3)-S(1,1)
170 Y1=S(2,3)-S(2,1)
180 X2=X1*C(1,1)+Y1*C(2,1)
190 Y2=Y1*C(1,1)-X1*C(2,1)
200 S1=X2-X3
210 S2=Y2-Y3
220 X1=S1↑2+S2↑2
230 IF X1<=D9 THEN 270
240 IF X1<-D9 THEN 310
250 D9=-X1
260 K9=G9/SQR(X1)
270 F1=-K9*S1
280 F2=-K9*S2
290 F(1,3)=F1*C(1,1)-F2*C(2,1)
300 F(2,3)=F2*C(1,1)+F1*C(2,1)
310 IF Ø4>1 THEN 370
320 IF S1<P(1) THEN 360
330 IF S1>P(2) THEN 360
340 IF S2<P(3) THEN 360
350 IF S2<=P(4) THEN 370
360 Ø4=2
370 RETURN
380 END
```

CTDVØU

```
100 IF Z9<>0 THEN 230
110 PRINT "CTDVØU ";
120 PRINT "1330  7";
130 PRINT "  8 69"
140 SET W=5.1
150 SET T=1.3,S
160 SET S=3.1
170 SET H=4.0,S
180 SET V=3.2
190 FØR J=1 TØ 7
200 PRINT
210 NEXT J
220 Z9=1
230 IF Ø4<>2 THEN 280
240 Ø6=1
250 Ø4=3
260 Ø7=1
270 GØ TØ 430
280 GØ TØ (390,320,390,350,390,290),Ø5
290 Ø6=5
300 Ø5=1
310 GØ TØ 370
320 Ø6=2
330 Ø5=3
340 GØ TØ 370
350 Ø6=3
360 Ø5=5
370 Ø7=2
380 GØ TØ 430
390 IF Ø1>0 THEN 970
400 Ø1=T3
410 Ø7=3
420 Ø6=4
430 IF Ø8=0 THEN 450
440 PRINT
450 Ø8=1
460 IF Ø2>0 THEN 670
470 IF Ø2=0 THEN 500
480 Ø2=0
490 GØ TØ 540
500 FØR J=1 TØ 10
510 PRINT AA;
520 NEXT J
530 PRINT
540 PRINT
550 PRINT H(1);H(2);H(3);H(4);
560 IF H(5)=0 THEN 580
570 PRINT " REV. ";H(5);
580 PRINT
590 PRINT
```

CTDVØU CØNTINUED

```
600 PRINT "        ØCCUPANT"
610 PRINT
620 PRINT "  TIME X PØS Y PØS  X VEL  Y VEL  SPEED  X ACC";
630 PRINT "  Y ACC   ACC"
640 PRINT "  SEC.   IN    IN    FPS    FPS    FPS    G-S";
650 PRINT "    G-S    G-S"
660 Ø3=0
670 IF Ø3>0 THEN 730
680 Ø3=10
690 Ø2=Ø2+1
700 PRINT
710 WRITE BINARY2,-5,1
720 WRITE BINARY3,-5,1
730 PRINT T;
740 X1=V(1,3)-V(1,1)
750 X2=V(2,3)-V(2,1)
760 V1=(X1*C(1,1)+X2*C(2,1))/12
770 V2=(X2*C(1,1)-X1*C(2,1))/12
780 V3 =SQR(V1↑2+V2↑2)
790 V4=(A(1,3)*C(1,1)+A(2,3)*C(2,1))/386.4
800 V5=(A(2,3)*C(1,1)-A(1,3)*C(2,1))/386.4
810 V6=SQR(V4↑2+V5↑2)
820 PRINT S1;S2;V1;V2;V3;V4;V5;V6;
830 FØR J=1 TØ 2
840 WRITE BINARY J+1,T
850 FØR K=1 TØ 2
860 WRITE BINARY J+1,S(K,J),V(K,J)/17.6,A(K,J)/386.4
870 NEXT K
880 WRITE BINARY J+1,S(3,J)*57.3
890 NEXT J
900 WRITE BINARY 2,D1,Ø6
910 WRITE BINARY 3,D2,Ø6
920 Ø2=Ø2+1
930 Ø3=Ø3-1
940 IF Ø6=4 THEN 960
950 PRINT BB(Ø6);
960 GØ TØ (280,390,970),Ø7
970 Ø1=Ø1-T2
980 RETURN
990 END
```

CTBUF

```
100 IF Z9<>0 THEN 150
110 PRINT "CTBUF  ";
120 PRINT "0830 18";
130 PRINT "  7 69"
140 RETURN
150 J=2
160 FØR K=1 TØ 10
170 PRINT AA;
180 NEXT K
190 IF J=4 THEN 580
200 PRINT
210 PRINT
220 PRINT H(1);H(2);H(3);H(4);
230 IF H(5)=0 THEN 250
240 PRINT " REV. ";H(5);
250 PRINT
260 PRINT
270 IF J=2 THEN 300
280 PRINT "   STRIKING";
290 GØ TØ 310
300 PRINT "    STRUCK";
310 PRINT " VEHICLE"
320 PRINT
330 PRINT "  TIME   X PØS   Y PØS   YAW  X VEL  YVEL  X ACC";
340 PRINT "  Y ACC";
350 PRINT "    PEN."
360 PRINT "  SEC.    IN      IN     DEG   MPH   MPH    G-S    G-S";
370 PRINT "       IN."
380 READ BINARY J,T9
390 IF T9<0 THEN 490
400 READ BINARY J,W1,V1,V3,W2,V2,V4,V5
410 READ BINARY J,V6,V7
420 PRINT T9;W1;W2;V5;V1;V2;V3;V4;
430 PRINT V6;
440 IF V7=4 THEN 470
450 PRINT BB(V7)
460 GØ TØ 380
470 PRINT
480 GØ TØ 380
490 IF T9=-4 THEN 550
500 READ BINARY J,Ø2
510 FØR K=1 TØ Ø2
520 PRINT
530 NEXT K
540 GØ TØ 380
550 RESTØRE FILE J
560 J=J+1
570 GØ TØ 160
580 IF T>T1 THEN 610
590 Ø2=-1
```

CTBUF CØNTINUED

```
600 RETURN
610 INPUT T1
620 08=0
630 IF T<=T1 THEN 590
640 STØP
650 END
```

CTDATS

```
100 IF Z9<>0 THEN 140
110 PRINT "CTDATS ";
120 PRINT "1345  7";
130 PRINT "  8 69"
140 WRITE BINARY 4,:
150 T,T1,T2,C,E,K9,P,X3,Y3,B(1),B(2),:
160 T3,G9,:
170 K1,K8,Ø1,Ø4,Ø5,CC,D3,D5,Z9
180 FØR K=1 TØ 3
190 FØR J=1 TØ 3
200 WRITE BINARY 4,:
210 S(J,K),V(J,K),A(J,K),M(J,K),F(J,K)
220 NEXT J
230 NEXT K
240 FØR J=1 TØ 5
250 WRITE BINARY 4,:
260 H(J),:
270 K(J),G(J),D(J),P(J),BB(J)
280 NEXT J
290 RETURN
300 END
```

CTDATB

```
100 READ BINARY 4,:
110 T,T1,T2,C,E,K9,P,X3,Y3,B(1),B(2),:
120 T3,G9,:
130 K1,K8,Ø1,Ø4,Ø5,AA,D3,D5,Z9
140 CC=AA
150 FØR K=1 TØ 3
160 FØR J=1 TØ 3
170 READ BINARY 4,:
180 S(J,K),V(J,K),A(J,K),M(J,K),F(J,K)
190 NEXT J
200 NEXT K
210 FØR J=1 TØ 5
220 READ BINARY 4,:
230 H(J),:
240 K(J),G(J),D(J),P(J),BB(J)
250 NEXT J
260 RESTØRE FILE 4
270 IF Z9<>0 THEN 310
280 PRINT "CTDATB ";
290 PRINT "1345  7";
300 PRINT "  8 69"
310 RETURN
320 END
```