9

REFERENCE FRAMES AND DIRECT HEAD IMPACTS

Guy S. Nusholtz
Nabih M. Alem
John W. Melvin
Richard L. Stalnaker

INTRODUCTION

The diverse studies of impact and motion of human and human analogues result in mechanical quantities described in reference frames which vary from one instrumentation method to another.

In order to ensure precise comparison of mechanical responses between subjects, it is necessary to refer all results to a "standard" anatomical frame which may be easily identified. On the other hand, it is impractical to require that transducers be aligned with this anatomical frame, since this would create problems which may not be satisfactorily solved.

An alternative is to mount transducers in an arbitrary and convenient reference frame, then describe the transformation necessary to convert the data from this frame to the desired anatomical one (1)*

We now have three reference frame to worry about: the instrumentational, the anatomical, and the inertial (lab). This paper presents a fourth frame used at HSRI, one that is related to the kinematics of impact.

Instrumentation Reference Frame

The method used at HSRI to measure the 3-D motion of the head employs nine accelerometers mounted on the head in three clusters. Each cluster is a triaxial unit which measures the components of the acceleration vector at its center in three orthogonal directions, which are the same for the other two clusters. These orthogonal instrumentation directions are arbitrarily chosen for convenience of mounting and are such that the centers Q1, Q2 and Q3 of the three triaxial clusters will lie on the axes $\hat{E}1$, $\hat{E}2$ and $\hat{E}3$ of the instrumentation frame, at known distances R1, R2 and R3 from the origin P, as shown in Figure 1.

Given the coordinates of the origin P, and those of two points Q1 and Q2, the unit vectors $\hat{E}1$ and $\hat{E}2$ are determined by normalizing

* Number in parentheses indicate reference at end of paper.

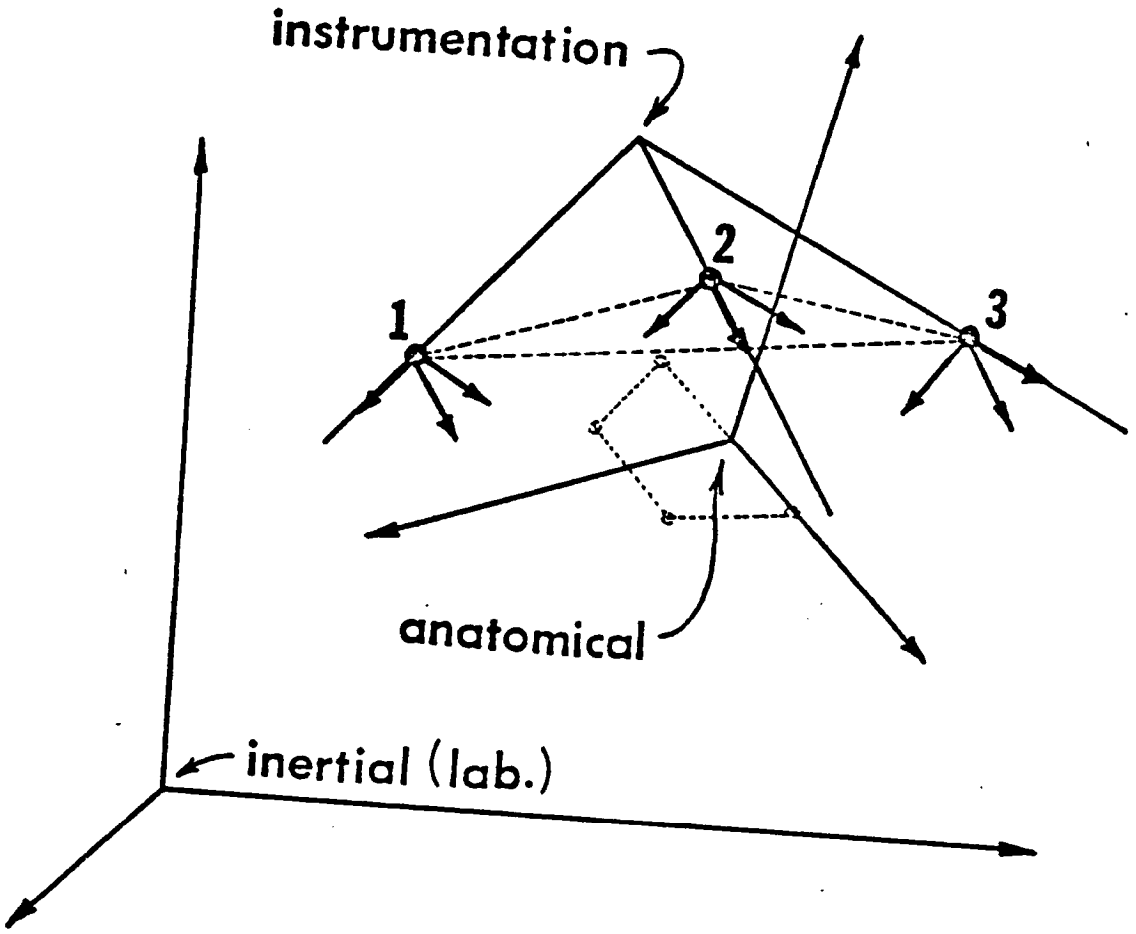


FIGURE 1. INSTRUMENTATION REFERENCE FRAME, AND THE LOCATION OF THE THREE TRIAXIAL ACCELEROMETERS FOR THE HSRI 3-D MOTION MEASUREMENT.

the distances PQ1 and PQ2. The third unit vector $\hat{E}3$ is then obtained by cross-multiplying $\hat{E}1$ by $\hat{E}2$. However, it is experimentally more practical to supply the coordinates of all three points Q1, Q2 and Q3, as well as the distances from them to the origin P, then compute the coordinates of other points necessary to define the instrumentation frame completely in the 3-D space.

The anatomical $\hat{i}$-axis is defined along the intersection of the Frankfort and midsagittal planes in the posterior-to-anterior (P-A) direction. The $\hat{j}$-axis is defined along the line joining the two superior edges of the auditory meati, in the right-to-left (R-L) direction. This $\hat{j}$-axis, which lies in the frankfort plane, is perpendicular to the midsagittal plane at the "anatomical center," which is taken as the origin of the anatomical frame (Figure 2). Finally, the $\hat{k}$-axis is defined as the cross-product of the unit vectors of $\hat{i}$- and $\hat{j}$-axes, and therefore, will lie in the midsagittal plane perpendicular to the Frankfort plane, and will be in the inferior-to-superior (I-S) direction.

Thus, the anatomical reference frame ($\hat{i}$, $\hat{j}$, $\hat{k}$) can be completely defined once the four anatomical landmarks are specified.

Laboratory Reference Frame

It is desired to describe the instrumentation ($\hat{E}1$, $\hat{E}2$, $\hat{E}3$) in terms of the anatomical ($\hat{i}$, $\hat{j}$, $\hat{k}$) unit vectors:

$$\begin{pmatrix} \hat{E}1 \\ \hat{E}2 \\ \hat{E}3 \end{pmatrix} = [E] \begin{pmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{pmatrix} \tag{1}$$

where [E] is an orthogonal transformation matrix made up with the nine unknown direction cosines. This matrix may be determined by first expressing each of the ($\hat{E}1$, $\hat{E}2$, $\hat{E}3$) and ($\hat{i}$, $\hat{j}$, $\hat{k}$) in terms of an arbitrary frame ($\hat{I}$, $\hat{J}$, $\hat{K}$):

$$\begin{pmatrix} \hat{E}1 \\ \hat{E}2 \\ \hat{E}3 \end{pmatrix} = [U] \begin{pmatrix} \hat{I} \\ \hat{J} \\ \hat{K} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{pmatrix} = (V) \begin{pmatrix} \hat{I} \\ \hat{J} \\ \hat{K} \end{pmatrix} \tag{2}$$

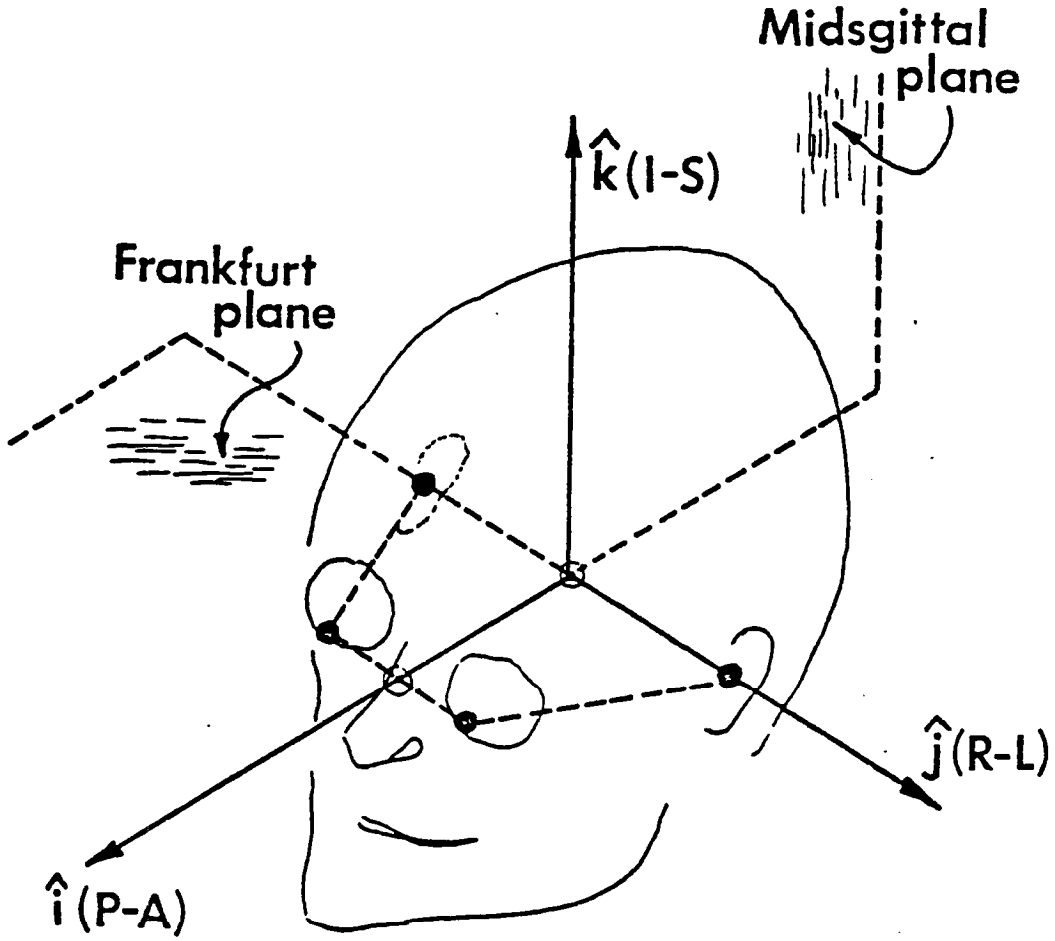


FIGURE 2. HEAD ANATOMICAL REFERENCE SYSTEM, BASED ON THE SKULL'S FRANKFORT AND MIDSAGITTAL PLANES.

then eliminating the $(\hat{I}, \hat{J}, \hat{K})$ between the two expressions to obtain the matrix [E]:

$$[E] = [U]^{-1} [V] \tag{3}$$

Since [U] and [V] are determined from coordinates of several points, the arbitrary frame $(\hat{I}, \hat{J}, \hat{K})$ will simply be the laboratory frame in which these coordinates are measured. The x-ray method used at HSRI to measure these points automatically defines the laboratory frame.

Frenet Triad

For any moving point, such as the anatomical center of the head, the absolute position, velocity and acceleration vectors are given in the laboratory reference frame as:

$$\begin{aligned} \vec{R} &= x(t)\,\hat{I} + y(t)\,\hat{J} + z(t)\,\hat{K} \\ \dot{\vec{R}} &= \dot{x}(t)\,\hat{I} + \dot{y}(t)\,\hat{J} + \dot{z}(t)\,\hat{K} \\ \ddot{\vec{R}} &= \ddot{x}(t)\,\hat{I} + \ddot{y}(t)\,\hat{J} + \ddot{z}(t)\,\hat{K} \end{aligned} \tag{4}$$

As the point moves in space, it travels along a trajectory defined by x(t), y(t) and z(t). This trajectory may be thought of as a skewed "one-way track" which is fixed in the laboratory $(\hat{I}, \hat{J}, \hat{K})$ reference frame.

The distance traveled along the trajectory is a function of time, s(t). This function determines the position of any point on the trajectory, and the tangent to trajectory curve at this point is defined as a unit vector:

$$\hat{T} = \frac{d\vec{R}}{ds} \tag{5}$$

but since $\dot{\vec{R}} = \dfrac{d\vec{R}}{dt}$

then

$$\dot{\vec{R}} = \frac{d\vec{R}}{ds} \cdot \frac{ds}{dt} = \frac{ds}{dt}\ \hat{T} = \nu(t)\ \hat{T} \tag{6}$$

The quantity $\nu(t)$ is called the speed of the point, and is equal to the derivative of $s(t)$ with respect to time. It is clear that $\nu(t)$ is precisely the resultant of the velocity vector, i.e.,

$$\nu(t) = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} \tag{7}$$

Any plane containing the tangent $\hat{T}$ is called the tangent plane to the trajectory. Any line perpendicular to the tangent $\hat{T}$ is called a normal line, and the plain containing all the normal lines is called the normal plane. Since the derivative of any vector is normal to the vector, the <u>principle normal</u> of the curve is defined as:

$$\hat{N} = \kappa \frac{d\hat{T}}{ds}, \quad \kappa \geq 0 \tag{8}$$

where $\hat{N}$ is perpendicular to $\hat{T}$ and κ chosen so that $\hat{N}$ is a unit vector. Finally,, the cross product $\hat{T} \times \hat{N}$ defines a third vector, perpendicular to both the tangent and principal normal, and is called the <u>binormal</u>:

$$\hat{B} = \hat{T} \times \hat{N}. \tag{9}$$

The three orthogonal unit vectors ($\hat{T}$, $\hat{N}$, $\hat{B}$) shown in figure 3 form a right-hand triad, called the Frenet triad. As the rigid body moves in 3-D space, each body point moves along a space trajectory; therefore, each point of the rigid body leaves behind a unique track characterized by individual Frenet triads attached to that trajectory. For a given trajectory,, the collection of its triads is often called a "field," which is stationary in 3-D space.

Curvature and Torsion of a Trajectory

It is often useful to study, in 3-D space, the turning and twisting of the trajectory of a point on the rigid body such as the anatomical center. The Frenet formulas provide mathematical measurements of the turning and twisting, given the inertial components of the position, velocity and acceleration vectors of the point of interest.

Let P be a point on the given trajectory and P' a neighboring point shown in Figure 3, such that:

$$\text{arc}\ (P\ ,P') = \Delta S \tag{10}$$

and let $\hat{T}$ and $\hat{T}'$ be the tangent vectors to the trajectory at P and P' respectively, such that the angle between $\hat{T}$ and $\hat{T}'$ is:

$$(\hat{T}\ ,\ \hat{T}') = \Delta\alpha \tag{11}$$

The definition of a derivative of $\hat{T}$ is given by:

$$\frac{d\hat{T}}{ds} = \lim_{\Delta s \to 0} \frac{\Delta\alpha}{\Delta s} \tag{12}$$

and that of the normal unit vector $\hat{N}$ is given by

$$\hat{N} = \kappa \frac{d\hat{T}}{ds} \ . \tag{13}$$

But $\hat{N}$ is a unit vector, κ must be the resultant (absolute value) of the tangent, so that

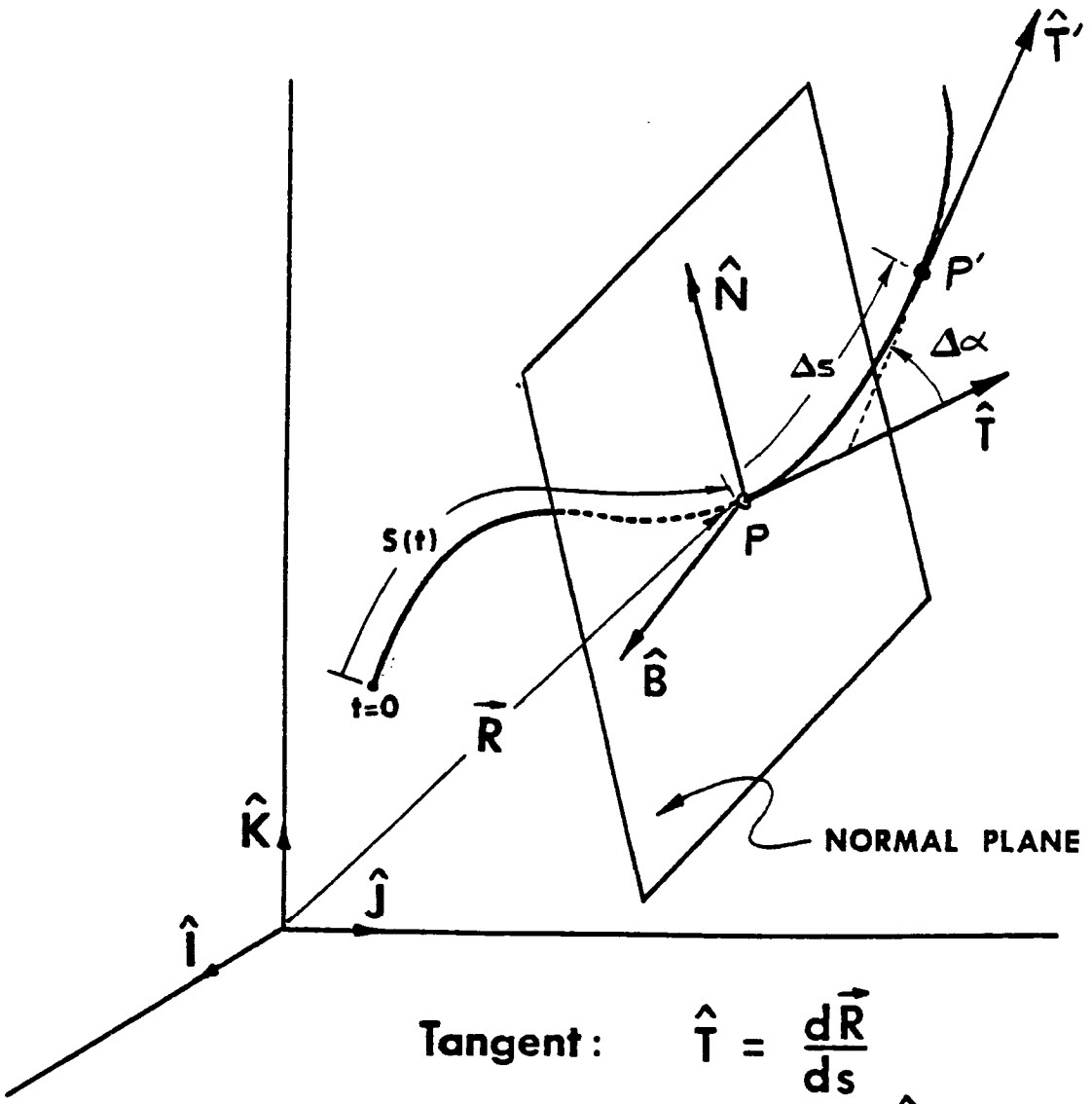
$$\kappa = \left| \frac{d\hat{T}}{ds} \right| \lim_{\Delta s \to 0} \frac{\Delta\alpha}{\Delta s} \ . \tag{14}$$

Therefore, as P_1 approaches P_0, the above limit is defined as the curvature of the trajectory at point P_0. The radius of curvature ρ is defined as the inverse of κ:

$$\rho = \frac{1}{\kappa} = \left| \frac{ds}{d\theta} \right| \tag{15}$$

$$\frac{d^2\vec{R}}{ds^2} = \frac{d\hat{T}}{ds} = \frac{1}{\rho}\ \hat{N} \tag{16}$$

A slow turn of the trajectory is characterized by a large radius ρ



Tangent: $\hat{T} = \dfrac{d\vec{R}}{ds}$

Normal: $\hat{N} = \dfrac{1}{\rho}\dfrac{d\hat{T}}{ds}$

Binormal: $\hat{B} = \hat{T} \times \hat{N}$

Figure 3. Frenet Frame

(or small curvature κ), while small radius (large κ) indicates a sharp turn of the trajectory. A numerical value of the radius ρ is therefore extremely useful in quantitatively describing the sharpness of the turns in the trajectory of interest.

In order to calculate the radius of curvature ρ, consider the velocity vector of a point along the trajectory,

$$\dot{\vec{R}} = \frac{d\vec{R}}{dt} = \frac{ds}{dt}\,\hat{T} \tag{17}$$

The acceleration vector may be obtained by differentiating $\dot{\vec{R}}$:

$$\ddot{\vec{R}} = \frac{d\dot{\vec{R}}}{dt} = \frac{ds}{dt} \cdot \frac{d\hat{T}}{dt} + \frac{d^2s}{dt}\,\hat{T} \tag{18}$$

but

$$\frac{d\hat{T}}{dt} = \frac{d\hat{T}}{ds} \cdot \frac{ds}{dt} = \frac{1}{\rho}\,\hat{N} \cdot \frac{ds}{dT} \tag{19}$$

so that

$$\ddot{\vec{R}} = \frac{1}{\rho}\left(\frac{ds}{dt}\right)^2 \hat{N} + \frac{d^2s}{dt}\,\hat{T} \tag{20}$$

Thus, the acceleration vector may be broken along two orthogonal components, one along the tangent (called tangential acceleration) and one along the principal normal (called the normal acceleration). By cross multiplying the above equation by the unit $\hat{T}$ vector, we have:

$$\hat{T} \times \ddot{\vec{R}} = \frac{1}{\rho}\left(\frac{ds}{dt}\right)^2 \hat{T} \times \hat{N} \tag{21}$$

but since $\hat{T}$ and $\ddot{\vec{R}}$ are defined by the components of the absolute velocity and acceleration vector, it is possible to express all terms in the above equation as functions of $(\dot{x}, \dot{y}, \dot{z})$ and $(\ddot{x}, \ddot{y}, \ddot{z})$ to obtain the radius of curvature ρ:

$$\rho^2 = \frac{(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)}{(\dot{x}\ddot{y} - \dot{y}\ddot{x})^2 + (\dot{y}\ddot{z} - \dot{z}\ddot{y})^2 + (\dot{z}\ddot{x} - \dot{x}\ddot{z})^2} \tag{22}$$

Next, we turn our attention to the binormal vector $\hat{B}$ defined earlier. The derivative of $\hat{B}$ with respect to s may be shown to be:

$$\frac{d\hat{B}}{ds} = \frac{d}{ds}(\hat{T} \times \hat{N}) = \hat{T} \times \frac{d\hat{N}}{ds} \tag{23}$$

Thus, in addition to being perpendicular to $\hat{B}$, the derivative is also perpendicular to $\hat{T}$ and, subsequently, must be along the normal direction $\hat{N}$, i.e.,

$$\frac{d\hat{B}}{ds} = \tau\hat{N} \tag{24}$$

where τ is called the <u>torsion</u> of the trajectory, and may be either positive or negative. The radius of torsion λ is defined as the inverse of the torsion τ, and, in a manner similar to the curvature, may be shown to be

$$\lambda = \frac{1}{\tau} = \frac{ds}{d\beta} = \lim_{\Delta s \to 0} \frac{\Delta s}{\Delta\beta} \tag{25}$$

where $\Delta\beta$ is the angle between two adjacent binormals to the curve at two points separated by Δs.

It is possible to show that the radius of torsion λ may be calculated from the velocity $\dot{\vec{R}}(\dot{x}, \dot{y}, \dot{z})$ acceleration $\ddot{\vec{R}}$ $(\ddot{x}, \ddot{y}, \ddot{z})$ and jerk $\dddot{\vec{R}}$ $(\dddot{x}, \dddot{y}, \dddot{z})$ vectors first by forming the mixed (triple) product

$$m = (\dot{\vec{R}}, \ddot{\vec{R}}) \cdot \dddot{\vec{R}} = -\frac{1}{e^2}\left(\frac{ds}{dt}\right)^6 \frac{1}{\lambda} \tag{26}$$

so that λ is calculated by

$$\lambda = \frac{|\dot{\vec{R}} \times \ddot{\vec{R}}|^2}{(\dot{\vec{R}} \times \ddot{\vec{R}}) \cdot \dddot{\vec{R}}} \tag{27}$$

The radius of torsion λ is a quantitative measure of the amount of turning that the trajectory makes at a given point. Because of its similarity with the curvature, the torsion is sometimes called the second curvature. The only difference between the two is that the radius of curvature is along the normal vector, while the radius of torsion is in the binormal direction. Since torsion involves the jerk vector (derivative of acceleration), and since jerk is difficult to measure experimentally and must be obtained by numerical differentiation of the measured acceleration, the computed radius of torsion carries errors which are amplified by the differentiation scheme.

The key to the successful study of the geometry of a given trajectory is to use the Frenet triad ($\hat{T}$, $\hat{N}$, $\hat{B}$) whenever possible, instead of the inertial laboratory frame ($\hat{I}$, $\hat{J}$, $\hat{K}$), for the field of Frenet Triads is full of information about the trajectory, while the laboratory reference frame contains none at all.

Some examples of the way this frame can be used are the following. These examples come from a series of direct head impacts in which the subject was positioned in front of the HSRI pneumatic cannon with the head surface to be impacted approximately normal to the cannon impacting surface, and the centerline of the impact approximately through the head center of gravity.

Padded rear impact

Examination of the tangential and normal component in Figure 4 shows right away that during impact the anatomical center moves almost exactly in a straight line until almost the end of impact. At this point not only does it move off the line of impact (normal acceleration) but there is also a negative acceleration along the tangent, indicating a force acting on the skull other than that caused by the impactor.

Rigid impact

Figure 5 shows a rigid impact in which the skull was fractured (depression under the impactor). The skull is loaded very rapidly. The force then drops, during which the tangent drops to zero; the normal increases rapidly, indicating that the large unfractured portion of the skull is not in complete contact with the impactor and that the only acceleration is from the change in $\hat{T}$. This short-lived change of $\hat{T}$ (rotation) ends when the impactor comes in complete contact with the skull, evidenced by a rapid increase of the angular velocity. Once the unfractured portion of the skull is back in complete contact, the angular velocity drops and the tangential acceleration increases.

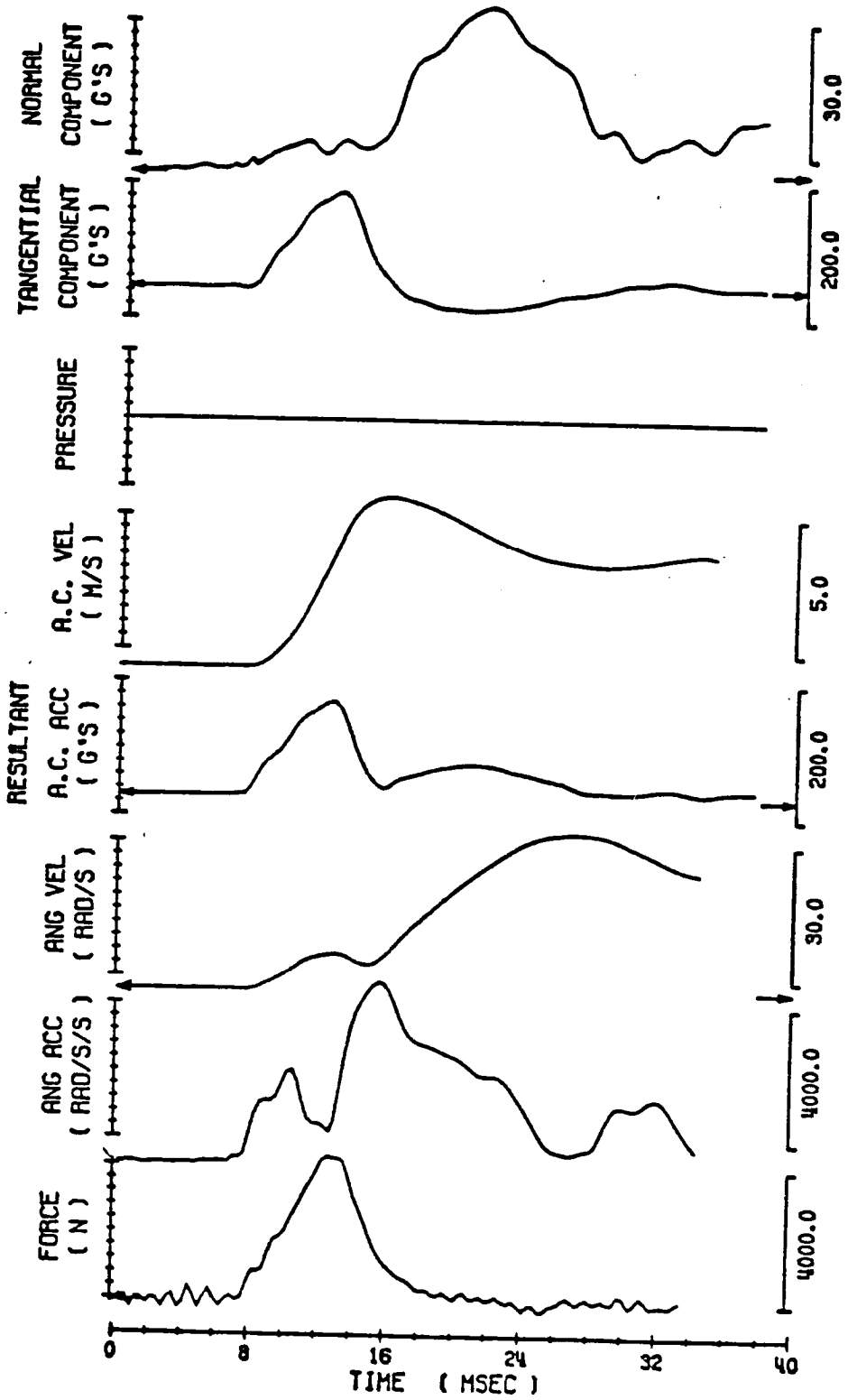


Figure 4 DATA SUMMARY SHEET OF 76A135

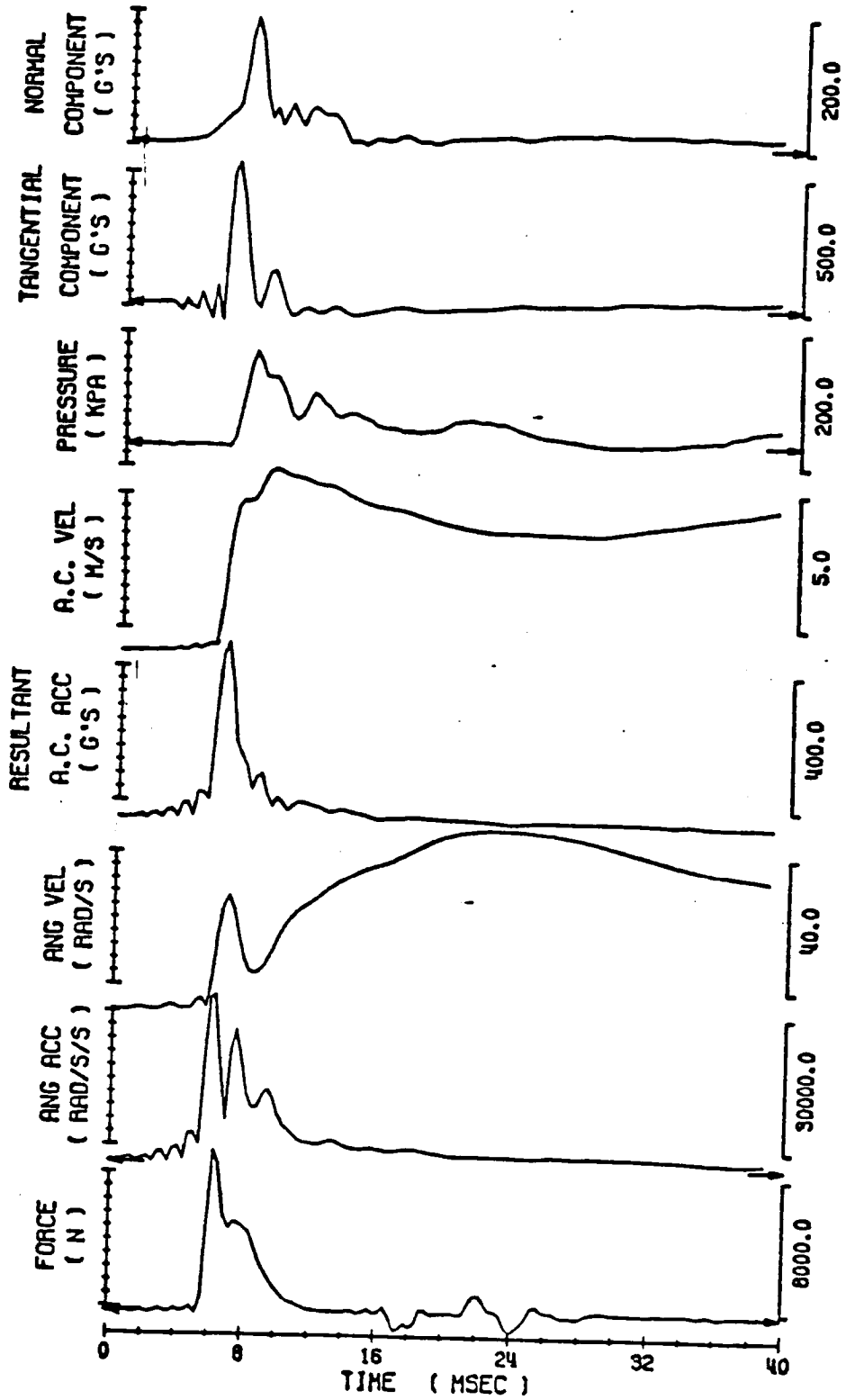


Figure 5 DATA SUMMARY SHEET OF 76A145

REFERENCES

1. Thomas, D. J., Robbins, D. H., Eppinger, R. H., King, A. I., and Hubbard, R. P., "Guidelines for the comparison of human and human analogue biomechanical data," A report of an ad hoc committee, Ann Arbor, Michigan, December 6, 1974.