Evaluation of Two Stage Crash Warning System on Inner City Busses

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ABSTRACT
Pedestrian injuries and fatalities are on the rise. Vulnerable road users are particularly difficult for transit bus drivers to see because of both the many obstructions to their view of the forward roadway and the limited visibility of events near the side of the bus (here we discuss only pedestrians but our discussion generalizes to other vulnerable road users such as bicyclists). In response to the rise in pedestrian collisions, systems have been developed to warn transit bus drivers of the potential presence and location of pedestrians whom they might strike if they (the drivers) do not take preemptive action. If the collision warning systems only alerted the driver of a crash threat when he or she was not aware of it and never warned the driver unnecessarily, they would seem to provide an unconditional benefit. However, warnings are often issued after a driver has observed the threat, possibly distracting the driver (here referred to as unnecessary warnings). In a highly trusted system, unnecessary warnings could lead drivers to respond abruptly in ways that increase the risk of injury to standing and seated passengers. In this paper, we review factors that influence when and how a driver responds to unnecessary warnings and identify the need for a framework for better understanding of the effects of imminent and cautionary warnings found in two-stage pedestrian warning systems.

INTRODUCTION

The National Highway Traffic Safety Administration (NHTSA) recently reported increases in the percentage of pedestrian fatalities beginning in 2008 following 5 years of stable percentages (11% of all traffic fatalities). In 2014 the percentage of fatalities reached 15% (NHTSA, 2016). Individual transit agencies have paid millions annually for claims related to crashes with tragic and costly pedestrian strikes (Rouse, 2013). Pedestrian safety and economic factors motivate efforts to reduce the number of these events.

NHTSA (2014, 2015, 2016) reported that in 2012, 2013 and 2014, respectively, fatal crashes more commonly occurred when the pedestrian was struck by the sides of buses and large trucks compared to other types of vehicle. Correspondingly fewer crashes occurred with the front suggesting that bus and large truck drivers' situational awareness may not always extend to crash threats in proximity to the sides of the vehicle. Most of these crashes occurred at night. These findings suggest that a system that reliably warns bus drivers of pedestrians who are in the path of the bus, and that provide cues to where they are located, could benefit pedestrian safety. Drivers who receive an alarm may be more quick to identify the pedestrian who triggered the alarm than they would without the alarm, especially under circumstances when the pedestrian is partially hidden by darkness or location. This would be consistent with prior research has shown faster response times to signals when participants were cued to the location of the signal in the visual field (Posner, Snyder, & Davidson, 1980).

Additional benefits are potentially introduced by two-stage warning systems. They may further improve driver response to the warning system by increasing driver alertness when a potential pedestrian crash threat exists. In a two-stage warning, a cautionary threat warning is followed by an imminent warning when the operator does not take an appropriate action after the cautionary warning (Campbell, Richard, Brown & McCallum, 2007).

However, there are potential problems with pedestrian collision warning systems. First, such warning systems are often somewhat unreliable. Researchers have shown that that as the reliability of warnings decreases, operators actually perform better without warnings than they do with warnings (Wickens and Dixon, 2007). Second, the algorithms used to trigger the warnings must typically trade off missed hazards against false alarms (FAs). As the number of misses decrease, not only will the number of false alarms increase but so too will the number of unnecessary warnings (threats which the driver has identified before the warning is activated). The driver then must consider how to adjust his or her attention so that the mix of unnecessary warnings and necessary warnings is satisfactory for the driver. Third, the use of cautionary warnings, while potentially of benefit, could cause the driver to pay too much attention to potential threats which never materialize, thereby missing early detection of imminent threats.

The first of the above three issues has been widely addressed in the warning literature. Here we address the issues raised by unnecessary warnings and two-stage systems. The way the driver deals with unnecessary warnings and two-stage systems will influence the probability that a driver will strike a pedestrian when an alarm is activated and when it is not. These in turn influence the expected costs of a warning system with a given mix of hits, misses, false alarms and correct rejections. The warning system needs to facilitate a driver's optimal response by adjusting the alarm settings (the mix of alarm hits, misses, false alarms and correct rejections). By optimal response here we mean

one which minimizes the costs associated with fatalities and severe injuries.

NECESSARY AND UNNECESSARY WARNINGS

To begin the discussion, we want to speak generally to the effects of necessary and unnecessary warnings.

ALERT ACTIVATED BEFORE DRIVER DETECTS THREAT (NECESSARY WARNINGS)

Imagine that the driver has not seen the threat. A warning sounds. The driver's attention is immediately redirected toward the threat. We refer to this as a necessary warning. The driver is both less likely to hit the pedestrian than a driver who is given no warning and the driver is more likely to trust the system. This increase in trust can have both benefits and costs.

It will have benefits in scenarios where the warning system can detect the threat. However, it is likely to have costs in scenarios where a threat exists and the warning system could not detect the threat, but a vigilant driver could detect the threat. An example of such a scenario is given in the figure below:
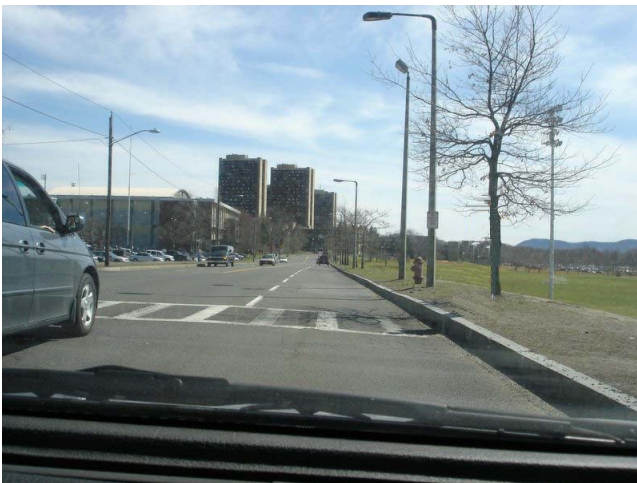

Figure 1. Pedestrian hidden on the left

An attentive driver would recognize that a pedestrian could emerge from behind the van stopped in the adjacent travel lane immediately in front of the marked midblock crosswalk. A

sensor system cannot see behind the van and so would not be able to warn the driver of a potential threat.

ALERT ACTIVATED AFTER DRIVER DETECTS THREAT: UNNECESSARY WARNINGS

Now imagine that there is a crash threat, the driver detects the threat, and then the alert occurs. This might be considered an unnecessary warning (UW). For Lees & Lee (2007), an unnecessary warning is "an alarm associated with a situation judged hazardous by the designer, but not by the driver. The driver can understand what triggered the alert" (p. 1267). Also, "UWs are predictable, understood, not considered useful." In contrast, false alarms (FAs) are said to occur at random from a driver's perspective. They are "non-useful, unintended by designer, unpredictable." Braitman, et al. (2010) describe UWs similarly: "where the technology functioned as intended but drivers did not think they were at risk of a crash or were already aware of the situation" (p. 276). As an example of the former situation, imagine a warning is issued when a bus is approaching a pedestrian and the pedestrian is moving towards the bus, but the pedestrian is waiving on the bus. As an example of the latter situation, cases where drivers are already aware of the threat, they may begin to brake before the warning is issued. In fact, in an aviation context, participants responded to a majority of situations before they received an alert about them (Friedman-Berg, et al., 2008).

In short, there appear to be at least two forms of unnecessary warning: those where the driver understands why the warning was issued, but he or she has not responded because the situation is one which the driver would define as a safe (UW1), and those where a warning is issued after the driver detects and may have already responded to the threat(UW2) . In contrast, in a situation where a false alarm is issued, the warning system responds unexpectedly in a safe situation. In support of the meaningfulness of the UW1-FA distinction, Cotté, Meyer, & Coughlin (2001) found that

providing older participants visible reasons for a warning issued in a safe situation led to lower subjective false alarm ratings, implying higher tolerance, than when no visible reasons were provided to the participants. Comparing driver response to UW1 and FAs, Lees & Lee (2007) found that drivers braked and reduced their speed in response to a critical collision situation more with a warning system that had produced the UWs than with one that had produced FAs, and they also trusted the UW system more. Distracted drivers also responded to the UW-prone system prior to the time when the warnings were issued, similar to undistracted drivers (i.e., UW1s resulted in UW2s). The UW1s were presented in response to situations that momentarily appeared to require a response, but then resolved themselves without an evasive maneuver, or in response to situations that only required a gradual response. FAs were issued in advance of these situations.

Unnecessary warnings are here classified with hits for several reasons. First, they are predictable and unlike FAs the driver understands why they occur. Second, Lees & Lee (2007) and Cotté et al. (2001) found that they differ from FAs in effects on simulated driving, on trust for the warnings, and in how drivers regard them. Rather, like hits they build trust. Third, UW2s are equivalent to hits, particularly relatively late warnings that are only issued after a driver has had an opportunity to respond and that are designed primarily for inattentive drivers. Whereas false alarms are undesirable, Källhammer, et al. (2016), following Farber & Paley (1993), suggested that the warning system should issue warnings where there is not a clear need because the situation would not lead to a collision, but where the driver still perceives them as "relevant and useful" (p. 2). They do not consider them unnecessary at all. Alarms in situations like these are not hits, but share more attributes with them than with FAs.

MISSES AND FALSE ALARMS

We are interested in how changes in the mixture of necessary and unnecessary warnings produced by changes to the warning threshold influences how a driver decides to behave. We cannot come to a complete picture of what is happening unless we also consider how a driver treats changes in false alarms and misses since the expected number of both will also change as the warning threshold changes.

FALSE ALARMS

False alarms create problems for several reasons. In a simulation study of imperfect forward collision warnings, Maltz & Shinar (2004) found that a relatively high FA rate led to more braking when it was not necessary. Hard braking was generally not found in Maltz & Shinar's study because the alarm system only presented FAs when time headway exceeded 6 seconds. FAs issued for pedestrians could occur at shorter range and present a particular problem for transit bus drivers because hard braking can result in discomfort or injuries to riders. Thus, drivers must carefully but quickly weigh the possibility of hard braking in response to false warnings with the necessity of an appropriate response to true warnings when they do not observe a pedestrian crash threat. Another consequence of false alarms is that they reduce trust in the warning system (Lees & Lee, 2007).

MISSES

Maltz & Shinar (2004) found that a warning system's high FA rate affected driver responses to a potential collision whereas their manipulation of miss rate only showed a marginal effect. Dixon, Wickens, & Chang (2004) found that a FA-prone system and a miss-prone system both reduced performance, with the miss-prone system producing slower detection times than no warning when the warning system missed a target, while FA-prone system affected both tasks and even when it

generated a hit. These effects occurred under the higher workload concurrent task conditions. We hypothesize that when drivers place a high level of trust in the warning system because it infrequently issues FAs, complacency could increase the probability of a strike, given a miss (this follows from Bliss & Acton's (2003) finding that drivers who experienced 50% FAs crashed less than drivers with 25% or no FAs). Presumably the driver with the more unreliable system predicted the threats because he or she trusted the system less, and was therefore paying more attention:

$$P(strike \mid Miss) = g(Hit, alert\ before\ threat\ acquisition)$$

TWO-STAGE WARNINGS
Next we want to speak generally about two-stage warnings. As noted earlier, Källhammer et al. (2016) suggested that UWs are useful. They suggest that UWs are useful because actual collision situations are rare. When actual collisions are rare drivers may have little or no experience with a warning and not know how to respond. It is here that UWs can serve a purpose because in the absence of an actual collision they can provide applicable experience of what to expect when a warning does appear. Källhammer et al. reported that participants found UWs acceptable if they were issued when a pedestrian was in the street even when there was no threat of collision. Two-stage warnings would appear to provide similar benefits. The initial "cautionary" warning provides an early warning to the driver that is often unnecessary because the threat never becomes (or perhaps the driver has already identified the potential threat). Again, the cautionary warning, like the unnecessary imminent warning, can serve as a learning experience.

Habibovic & Davidsson (2011) recommend cautionary warnings in two-stage pedestrian warning systems and, in particular, a cautionary warning followed by an imminent crash warning if the hazardous situation continues. The current paper proposes an analysis framework that concerns the distinctions between alarm hits (UW1s or UW2s depending on the driver's behavior) and alarm FAs for both cautionary and imminent warnings that cue the driver to the location of a pedestrian hazard. In Insurance Institute of Highway Safety surveys of user experience with various automotive safety warnings, the percentage of drivers who reported FAs or UWs has been considerably higher than the percentage who reported that the warnings were annoying. In fact, nearly all drivers surveyed found the warnings useful (Braitman, McCartt, Zuby & Singer, 2010; Eichelberger & McCartt, 2014, 2016; Cicchino & McCartt, 2015). For example, Braitman, et al. report that 43% of respondents said that they received forward collision warnings when they did not perceive a crash risk, while only 21% said that they dislike the false or unnecessary warnings. Most (61%) of the respondents in Cicchino & McCartt (2015) said that they received warnings when they did not perceive a crash risk, while only 12% agreed that it was annoying. These surveys did not distinguish between FAs and UWs, but from these results suggest that many of the reported FAs/UWs were actually UWs. UWs may represent many of the warnings drivers receive, but their effect on key Signal Detection Theory metrics is uncertain.

To achieve an optimal response, one that minimizes cost, it may be necessary to design a warning system that occasionally fails to warn a driver of a pedestrian in order to provide a low false alarm rate. However, when the false alarm rate is adjusted suitably downward so that the expected benefits exceed the costs, the cost of misses may rise to the point where the overall costs of a warning system may exceed its overall benefits. Thus, drivers may perform better without the warning system than with the warning system and a rational driver would disengage it. Cautionary and "unnecessary"

warnings understood as more similar to hits than to false alarms, as well as the cautionary warnings in two-stage warning systems, need to be considered as well for an accurate calculation of the cost impact of a pedestrian warning system.   Hypothetically, the cautionary false alarm is not as consequential as the false alarm of an imminent warning.

EXPECTED COSTS

ONE-STAGE WARNINGS

To have a framework for thinking about the effect of unnecessary warnings and two-stage warning systems on the expected costs, we need to provide a rudimentary formula that contains the terms that go into the computation of the expected costs. To begin, we need to define several terms, similar to those used in signal detection theory. Let N(Hit), N(Miss), and N(FA) be, respectively, the expected number of hits, misses and false alarms in some number of bus miles traveled, say 100 miles. A hit will be defined as an alarm which is activated when the bus and pedestrian are on a collision course and they will collide in, say, x seconds (the threshold is chosen arbitrarily here) unless some action is taken, where x is relatively short for an imminent alarm and longer for a cautionary alarm. A miss is defined as an alarm which is not activated when the bus and pedestrian are on a collision course. And a false alarm is defined as an alarm which is activated when the bus and pedestrian are not on a collision course within the warning time of the cautionary or imminent warning. Let P(strike|Hit), P(strike|Miss) and P(strike|FA) be the probabilities, respectively, of an operator striking a pedestrian given the warning is a hit, miss, or false alarm. Finally, let C(strike) be the cost of a strike. We keep things simple by assuming that the cost of a strike does not vary across the different speeds and situations in which a strike can occur.

Given the above, the expected cost of a trip over 100 miles of bus travel can be written as a function of the above terms:

$$E[C(trip)] = N(Hit)P(strike|Hit)C(strike) + N(Miss)P(strike|Miss)C(strike) + N(FA)P(strike|FA)C(strike)$$

Equation 1

We now want to describe an example where we show how the costs vary as a function of the expected number of hits, misses and false alarms.   As a start, we assume that the conditional probability of a pedestrian strike does not vary as a function of the mix of hits, misses, and false alarms. Suppose our first system is a fairly conservative warning system, where we have only 2 instances in every 100 miles where the driver receives a false alarm (top panel, Table 1). Our second system is a less conservative one where we have 3 instances in every 100 miles where the driver receives a false alarm (middle panel). And our third system is the least conservative system where we have 20 instances in every 100 miles where the driver receives a false alarm. The disadvantage of the less conservative systems is obvious. The advantage is that the misses are reduced almost to zero.

Table 1. Warning Hits, Misses and False Alarms. (Top panel most conservative, very few false alarms. Bottom panel least conservative, the most false alarms.)

True threats	Alarm activates	Alarm does not activate
10	8	2
No threats		
100	2	98

True threats	Alarm activates	Alarm does not activate
10	9	1
No threats	Alarm activates	Alarm does not activate
100	3	97

True threats	Alarm activates	Alarm does not activate
10	9.9	0.1[1]
No threats	Alarm activates	Alarm does not activate
100	20	80

It is now easy enough using Equation 1 to compute the expected cost of a trip if we assign values to the various quantities in Equation 1. One possible assignment is included below:

Category	Quantity
Miles	100
P(strike|Hit)	0.001
P(strike|Miss)	0.100
P(strike|FA)	0.010
C(strike)	$2,000,000

Using Equation 1, for the top, middle and bottom panels of Table 1 one finds the expected trip costs are, respectively:

Table 2. Expected Trip Costs: Variation with Strike Criterion

Trip	Expected Costs
Top Panel	$456,000
Middle Panel	$278,000
Bottom Panel	$439,800

Identifying the optimal threshold in order to minimize the costs in **Error! Reference source not found.** would not be a hard problem to solve if the probability of a strike conditional on a hit, miss or false alarm did not vary as the mixture of hits, misses and false alarms varied. However, there is every reason to believe that it will vary, both because of the driver's desire to arrive at an optimal mix of necessary and unnecessary alarms and because of the existence of a two-stage warning system. The goal of the next several sections is to explore how the variation in the frequency of necessary and unnecessary alarms is expected to influence these conditional probabilities.

TWO-STAGE WARNINGS
Two-stage warnings from a computational perspective are a straightforward extension of Equation 1. However, instead of the three conditioning events being hits, misses, and false alarms, they are now a function of nine conditioning events. Those nine conditioning events correspond to all sequences of hits, misses and false alarms of first the cautionary alarm and then the imminent alarm.

PROBABILITY OF A STRIKE
Now that we understand the general effects on drivers' behavior of unnecessary warnings and two-stage warnings we can go on to describe how they affect both the change in the probability of a strike given the mix of hits, misses and false alarms and, in turn, the expected costs as the alarm threshold is

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[1] The readers should note that this is the expected number of alarms, not the number of alarms on a given trip of 100 miles. So, if we took 1000 trips each of 100 miles and the alarm activated on 9,900 where a threat was present and failed to activate on 100 where a threat was present then the expected (average) number of misses per 100 miles would be 0.1

changed. To begin, we will show how a change in the alarm threshold affects the expected number of threat mitigation responses as the threshold is decreased.

THREAT MITIGATION RESPONSES

A simple example can help make clear what we are hoping to do. In particular, imagine two warning thresholds, one set at 3 seconds (left panel, Table 3) and one set at 2 seconds (right panel). The expected number of alarm hits, misses and false alarms are depicted, respectively, in the upper left, upper right and bottom left of Table 3. As the threshold decreases, the expected number of a false alarm decreases and, correspondingly, the expected number of a misses increases. This is exactly what is depicted in Table 3 below as one moves from a warning system with a the 3 second threshold to a warning system with a 2 second threshold.

Table 3. Two warning systems: Alarm hits, misses and false alarms. (Three second and two second thresholds)

	3 Seconds			2 Seconds	
	Yes	No		Yes	No
Threat	18	2	Threat	12	8
No Threat	10	NA	No Threat	6	NA

Note that the above table needs to be embellished by the occurrence of unnecessary alarms. Recall that unnecessary alarms occur after a driver has identified a threat. Some or all of the above hits could be unnecessary alarms. For purposes of this example, let's assume that the driver is paying attention to the forward roadway and sides of the bus 2/3 of the time, beginning at 4 seconds from a potential threat, and is attending to other necessary tasks some 1/3 of the time as he or she approaches the threat. Then, for the moment that the driver always himself or herself identifies the threat, the expected number of unnecessary alarms with the 3 second threshold is 12 and the expected number of unnecessary alarms with the 2 second threshold is 8.   This is displayed below in Table 4 on, respectively, the left and right panels.

Table 4. Two warning systems: Alarm hits, misses and false alarms and driver necessary and unnecessary alarms. (Three second and two second thresholds)

	3 Seconds				2 Seconds		
	Yes	NA/UA	No		Yes	NA/UA	No
Threat	18	6	2	Threat	12	4	8
		12				8	
No Threat	10		10	No Threat	6		14

Finally, we want to know the system mitigation response. In particular, we want to know the expected number of threats which actually lead to a mitigation response by the driver. All unnecessary alarms are defined as ones where the driver has perceived the threat before the alarm sounds. We can assume that the driver will make a mitigation response here. All necessary alarms are generated by the driver not paying attention. To keep things simple, let's assume that only 50% of the UAs lead to a mitigation response. Finally, along the same lines with respect to misses we can assume that the driver is not paying attention 1/3 of the time and therefore will make no mitigation response for 1/3 of the misses (and always make a mitigation response for the other misses). Thus, the expected number of mitigation responses to the necessary and unnecessary alarms are as presented in the left

hand panel of Table 5 for a warning with a three second threshold and as presented in the right hand table for a warning with a two second threshold

Table 5. Two warning systems: Mitigation responses. (No change in attention level.)

	3 Seconds				2 Seconds		
	Yes	NA/UA	No		Yes	NA/UA	No
Threat	18	3	1.7	Threat	12	2	5.5
		12				8	
No Threat	10		10	No Threat	6		14

It is important to note here the two critical factors that explain why Table 4 and Table 5 are different with respect to the entries in the necessary alarms (yellow highlighting). The first is the level of attention. This influences the number of necessary and unnecessary warnings. And the necessary warnings, being more risky, are not all ones which are followed by a mitigation response. Rather, this is influenced by the second factor, proportion of necessary alarms to which the driver cannot make a mitigation response because the alarm occurs too late.

It is equally important to understand the factors that explain why Table 4 and Table 5 are different with respect to the entries in misses (green highlighting). Again, the level of attention is a key factor. It is assumed that all missed alarms which occur in scenarios where the driver was paying attention lead to mitigation responses. But what about missed alarms which occur in scenarios where the driver was not paying attention? To keep things simple, we will assume that this is equal to the proportion of necessary alarms to which a driver could not make a mitigation response.

SHIFT OF ATTENTION

There is a problem with the above analysis that the reader may have discovered on his or her own at this point. In particular, it was assumed that the driver kept constant the level of attention paid to the forward roadway and sides of the bus as the threshold was decreased. Now we need to ask ourselves whether this is reasonable. To be clear, by changing the level of attention the driver cannot change the expected number of alarm hits, misses and false alarms. But the driver can change the expected number of necessary alarms and misses which lead to mitigation actions by varying the attention given to the forward roadway and sides of the bus.

In this case, it would appear reasonable that if the threshold is lowered and therefore the expected number of misses increases, the driver would pay more attention to the forward roadway and sides of the bus.   The driver has limited cognitive resources. These resources can be divided among multiple tasks, that division depending on many things. As the likelihood of missing something on the forward roadway increases, it would only be natural for the driver to shift his or her attentional capacity to scanning the forward and sides of the bus. This would decrease the number of system misses since the driver is now paying more attention in the 2 second threshold. In terms of the above example, the driver might increase the level of attention in the 3 second threshold of 67% to a level of attention in the 2 second threshold of 90%.

Note that this would lead to an increase in the expected number of mitigation actions. When there was no change in the level of attention given to the forward roadway and sides of the bus, the expected number of mitigation actions decreases from 16.7 with a 3 second threshold (the sum of the number of threats in the NA/UA/No cells, i.e., 3, 12, 1.7, in the left hand

panel of Table 5) to 15.5 with a 2 second threshold (right hand panel). But, if the level of attention actually increases as the threshold decreases then the expected number of mitigation actions increases from 16.7 (left panel, Table 6) to 18.65 (Table 6) even as the number of misses increases.

Table 6. Two warning systems: Mitigation responses. (Increase in attention with decrease in threshold. Left panel -- attention 67%; right panel -- attention 90%.)

3 Seconds				2 Seconds		
	Yes	NA/UA	No		Yes	NA/UA	No
Threat	18	3 / 12	1.7	Threat	12	0.6 / 10.8	7.25
No Threat	10		10	No Threat	6		14

The story is not quite complete because we have not considered how the decrease in unnecessary alarms might also affect the likelihood that the driver attends to the forward roadway and side of the bus. This decrease is from 12 with a three second threshold (left panel, Table 6) to 10.8 with a two second threshold (right panel). Note that this is also accompanied by a decrease in the number of necessary alarms, from 3 to 0.6. It seems unlikely that this would prompt the driver to modify his or her level of attention given that a decrease in the number of necessary and unnecessary alarms are both good things (or presumably so).

PROBABILITY OF A STRIKE

Finally, we want to consider how the probability of a strike changes as the alarm threshold decreases for each of the four conditions: necessary alarm, unnecessary alarm, miss and false alarm. Note that we now need to add one other factor which the reader may already have

guessed was missing. In particular, as the warning threshold is decreased, the likelihood of a strike when the driver is not paying attention would arguably increase. We kept this likelihood the same, both for necessary alarms and for misses. The likelihood of a strike given that the driver is paying attention should not change as the threshold is decreased. By definition, if the driver is paying attention when the alarm is activated is not relevant. Of course, we realize that when fully articulated, the relation may be more complex.

At any rate, if we stick with the assumption that the likelihood of a mitigation action when the threshold decreases changes only when the driver is not paying attention, then we need to modify the probability in the above example, 0.5, to something smaller, say 0.2. The final predictions expected mitigation actions for this example with a three second threshold are listed in the left panel below (Table 7). Those for a two second threshold are listed in the right panel below.

Table 7. Two warning systems: Mitigation responses. (Increase in attention with decrease in threshold. Left panel -- attention 67%; right panel -- attention 90%.)

3 Seconds				2 Seconds		
	Yes	NA/UA	No		Yes	NA/UA	No
Threat	18	3 / 12	1.7	Threat	12	0.24 / 10.8	7.22
No Threat	10		10	No Threat	6		14

One would now need to translate both the occurrence of a mitigation action and the nonoccurrence of a mitigation action into the probability of a strike in the three conditions (hit, miss false alarm) in order to substitute values back into Equation 1. However, it is clear

that we do not have a single probability of a strike conditional on a hit nor do we have a single probability of a strike conditional on a miss. So we need to expand several of the terms in Equation 1 in order to be able to substitute back into the equation.

Let's start with the probability of a strike given a miss. We have said that this conditional probability depends on whether the driver is or is not paying attention. The expansion is given below in Equation 2. If the driver is paying attention, then the product of interest is on the left hand side of Equation 2. So, in the two second threshold warning example discussed above, $P(attend \mid Miss) = 0.9$. The quantity, $P(strike \mid Miss, attend)$ needs to be determined empirically but is presumably very small, if only because the probability of a mitigation response given that the driver is attending we have set equal to 1.

$$P(strike \mid Miss) = P(strike \& attend \bigcup strike \& not\ attend \mid Miss)$$
$$= P(strike \mid Miss, attend)P(attend \mid Miss) + P(strike \mid Miss, not\ attend)P(not\ attend \mid Miss)$$

Equation 2

If the driver is not paying attention, then the product of interest is on the right hand side of Equation 2. By complementation it follows that $P(not\ attend \mid Miss) = 0.1$. Finally, we need to compute the conditional probability $P(strike \mid Miss, not\ attend)$. This final conditional probability is presumably close to 1.

The computation of the conditional probability of a strike, given a hit, would proceed along similar lines with the proviso that one will need to differentiate between necessary and unnecessary alarms.

In summary, what we know about necessary and unnecessary warnings as well as what we believe about how drivers would adjust their level of attention to the forward roadway and sides of the bus means that we can provide a computational model which could in theory be used to identify the quantities referenced in Equation 1 and, by extension, the mixture of alarm hits, misses and false alarms which minimized the expected cost. However, such could not occur until estimates of the actual values that one needs in order to derive a value for the expected cost associated with a given mixture of hits, misses and false alarms are available. This awaits further experimentation.

TWO-STAGE WARNINGS
Computing the conditional probability of a strike, as complex as it might be from the standpoint of estimating the precise quantities based on actual human behavior, becomes still more difficult when two stage warnings are involved. However, conceptually the factors influencing how the driver might change the level of attention, and therefore, ultimately, the likelihood of a strike, remain the same.

CONCLUSIONS
In conclusion, a framework for estimating warning system effectiveness needs to incorporate the driver's perception of the warnings and the context in which they are issued.   These perceptions affect their trust in the warning system, and the level of trust is fundamental, potentially increasing the probability of a crash following a miss when trust is very high, while increasing the likelihood of disuse if trust is very low. For each type of perceived warning: hit, FA, miss, UW1, UW2, and cautionary warning, the framework could include:
- Alarm status
- Pedestrian crash threat status
- Does the driver perceive the alarm?
- Does the driver respond?
- Is the response before or after the warning?
- Does the driver perceive the pedestrian crash threat?
- Does the driver perceive another pedestrian, not the crash threat?

- How to classify the driver's detection (hit, miss, FA)
- How does the driver classify the alarm (hit, miss, FA, UW)?
- What is the utility of the alarm (useful, reassuring, nuisance, harmful)?
- Effect on trust, use, misuse, disuse
- How to classify the alarm (hit, miss, FA, UW)

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