10

CROSS-AXIS EFFECTS ON THE MEASUREMENT OF ANGULAR ACCELERATION USING LINEAR ACCELEROMETERS

Anthony S. Hu
Physical Science Laboratory
New Mexico State University

Arnold K. Johnson
National Highway Traffic Safety Administration
Department of Transportation

Abstract--An instrument currently used in biomechanical experiments for the measurement of rigid body motion is the nine-accelerometer module which provides linear and angular acceleration data in three dimensions. However, validation results were restricted to planar motion measurements. This paper discusses the errors induced in the derivation of angular acceleration by cross-axis sensitivity and principal-axis misalignment of the linear accelerometers. The study reveals some insight into the limitations of the nine-accelerometer module and provides the criteria for the selection of linear accelerometers.

INTRODUCTION

In biomechanical experiments, a commonly used instrument for kinematic measurement is the nine-accelerometer instrumentation module [1]. This module utilizes nine linear accelerometers arranged in an ingenious 3-2-2-2 configuration, thus, providing three elegant linear equations which relate the accelerometer measurements to the angular accelerations of the module.

Validation results [1], [2] using both hypothetical and experimental data indicated that good major axis angular acceleration and velocity data may be obtained for planar motion if the state-of-the-art linear piezoresistive accelerometers are used. In oblique runs, especially in near planar motion, minor axis angular accelerations generally contain large errors induced by the cross-axis sensitivity and the principal-axis misalignment of the linear accelerometers. These effects may be severe enough to hamper an experiment where angular velocity data are required to transform the instrumented data in a body-fixed coordinate system through rigid-body assumption. In near planar motion where the angular displacement is relatively small, the resultant linear acceleration after rigid-body transformation may be acceptable [2]. This is because the resultant linear acceleration is dominated by a single component in a plane in this special case. If other coordinate transformations are used, the angular displacement data may be severely limited in range and accuracy because of the large errors in the directional cosines.

Titlow* has made an analysis of the nine-accelerometer module error due to accelerometer major axis data resolution and gain errors. He was able to express the angular acceleration errors in terms of these linear accelerometer errors as a function of the module size.

Not so obvious is that the commonly used piezoresistive linear accelerometers have significant cross-axis sensitivity (1%) and sensitive-axis misalignment (1 degree). The purpose of this paper is to explicitely express the angular acceleration errors in terms of these two factors. This will provide some insights into the limitations of the nine-accelerometer module and will provide guidance for the selection of the linear accelerometers.

---

*"Resolution and Accuracy Estimates for Nine-Axis Accelerometer Configuration" prepared by J. Titlow under DOT-TSC-1095 Contract, "Design and Fabrication of Microminature Accelerometer/Transmitter" with Konigsberg Instruments, Inc., 200 East Foothill Boulevard, Pasadena, California 91107.

MATHEMATICAL MODEL

The 3-2-2-2 nine-accelerometer module [1] is configured as shown in Fig. 1 with the bold arrows indicating the nine accelerometers. The lower case a_1, a_2, ... a_9 indicate the true acceleration when measured with nine ideal accelerometers. Note that a_{10}, a_{11}, and a_{12} are not related to any accelerometers but they represent the true accelerations in axes x, y, and z at cluster locations 1, 2, and 3 respectively. The lower case ω_x, ω_y, and ω_z indicate the true angular velocities about axes x, y, and z respectively. Each cluster of sensors is located in one of the four positions as shown in the figure.

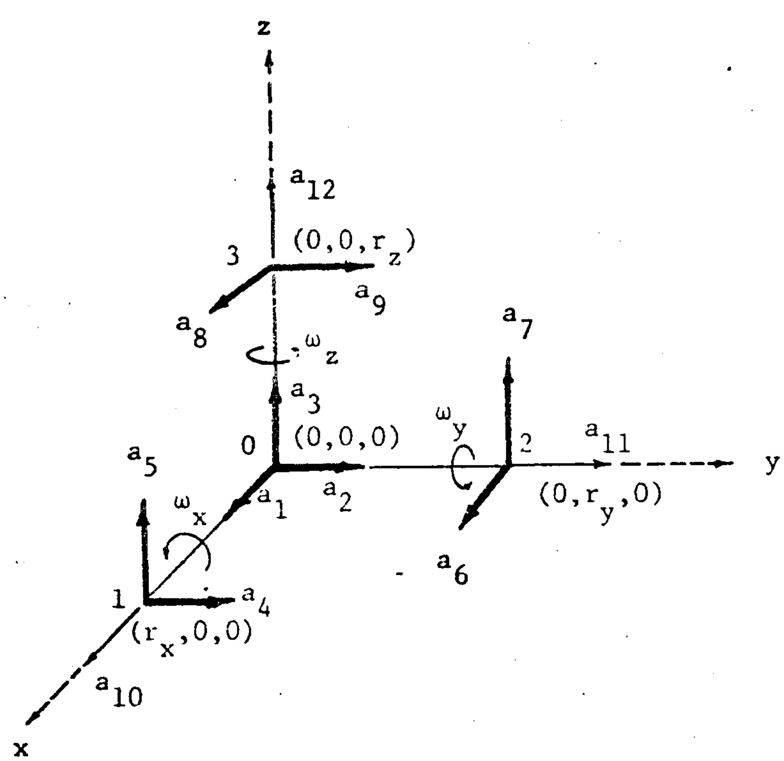


Fig. 1 Nine-accelerometer configuration.

Forward Equations

From Fig. 1, the linear acceleration along a_i, for i = 1, 2, ... 12, can be expressed as follows

$$a_1 = \ddot{x} \tag{1-1}$$
$$a_2 = \ddot{y} \tag{1-2}$$
$$a_3 = \ddot{z} \tag{1-3}$$
$$a_4 = \ddot{y} + \omega_x\omega_y r_x + \dot{\omega}_z r_x \tag{1-4}$$
$$a_5 = \ddot{z} + \omega_x\omega_z r_x - \dot{\omega}_y r_x \tag{1-5}$$
$$a_6 = \ddot{x} + \omega_x\omega_y r_y - \dot{\omega}_z r_y \tag{1-6}$$
$$a_7 = \ddot{z} + \omega_y\omega_z r_y + \dot{\omega}_x r_y \tag{1-7}$$
$$a_8 = \ddot{x} + \omega_x\omega_z r_z + \dot{\omega}_y r_z \tag{1-8}$$
$$a_9 = \ddot{y} + \omega_y\omega_z r_z - \dot{\omega}_x r_z \tag{1-9}$$
$$a_{10} = \ddot{x} - \omega_y^2 r_x - \omega_z^2 r_x \tag{1-10}$$
$$a_{11} = \ddot{y} - \omega_x^2 r_y - \omega_z^2 r_y \tag{1-11}$$
$$a_{12} = \ddot{z} - \omega_x^2 r_z - \omega_y^2 r_z \tag{1-12}$$

Inverse Equations

Using Equations (1-1) through (1-9), $\dot{\omega}_x$, $\dot{\omega}_y$, and $\dot{\omega}_z$ can be calculated [1] to obtain:

$$\dot{\omega}_x = (a_7 - a_3)/2r_y - (a_9 - a_2)/2r_z \tag{2-1}$$
$$\dot{\omega}_y = (a_8 - a_1)/2r_z - (a_5 - a_3)/2r_x \tag{2-2}$$
$$\dot{\omega}_z = (a_4 - a_2)/2r_x - (a_6 - a_1)/2r_y \tag{2-3}$$

Let $r_x = r_y = r_z = r$. Equation (2) is simplified in matrix form

$$\begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} = \frac{1}{2r} \begin{bmatrix} 0 & 1 & -1 & 0 & 0 & 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 & -1 & 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \end{bmatrix} \quad (3)$$

If the accelerometers were ideal, then the electrical signals are a_i ($i = 1, 2, \ldots 9$), and the calculated angular accelerations will have no error.

EFFECTS OF CROSS-AXIS SENSITIVITY

Definition of Cross-Axis Sensitivity

For an ideal accelerometer, the output signal (bold line) is a cosine function of the angle θ between the accelerometer principal axis and the acceleration input axis as illustrated in Fig. 2. The output signal should be zero when the acceleration is applied perpendicular to the sensitive axis ($\theta = \pm 90^{o}$). But for an actual accelerometer, this output signal is not zero, as shown in the dashed curve. The ratio of this value to the value when the applied acceleration is in line with the sensitive axis is defined as the cross-axis sensitivity coefficient s_{ij} where the first subscript is the accelerometer number and the second subscript is the acceleration input axis. Its magnitude may be between 0.01 and 0.05. It is assigned positive if the cross product of the input acceleration and sensor principal axis is positive (right hand rule) and vice versa. For example, if the acceleration along vector a_1 (x-axis) creates a positive voltage reading in the accelerometer along vector a_2, then the coefficient s_{2x} is positive.

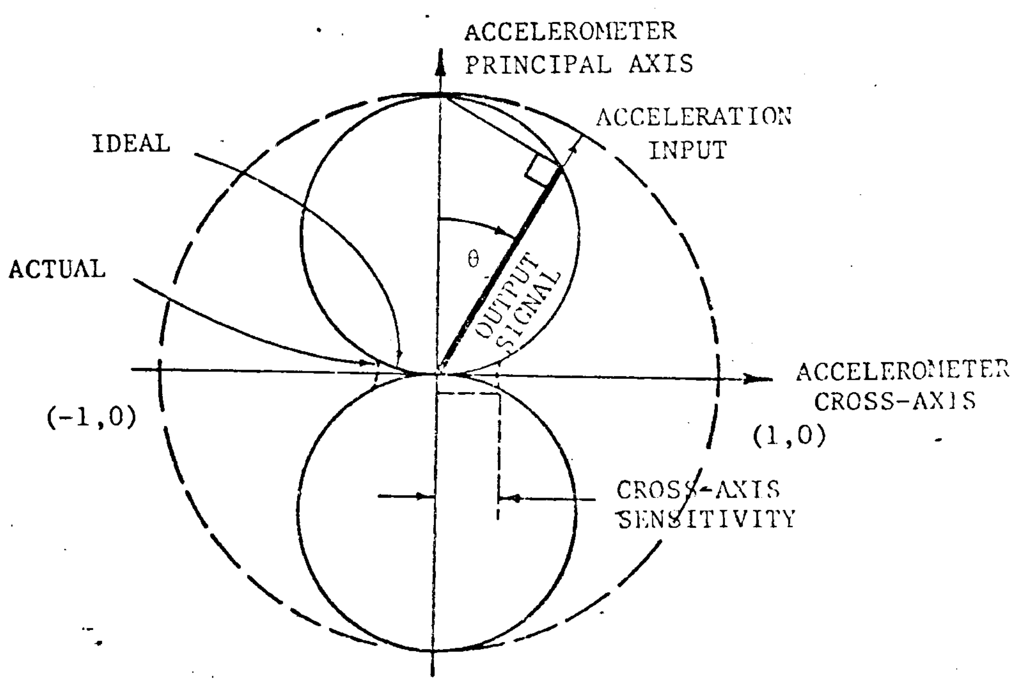


Fig. 2 Illustration of cross-axis sensitivity.

Designation of Errors

If the true kinematic acceleration is a_i and the measured acceleration is A_i, (for $i = 1, 2, \cdots 9$) then they are related by Equation (4).

$$A_1 = (a_1) + (-s_{1y}a_2 + s_{1z}a_3) \equiv a_1 + \Delta a_1 \tag{4-1}$$

$$A_2 = (a_2) + (s_{2x}a_1 - s_{2z}a_3) \equiv a_2 + \Delta a_2 \tag{4-2}$$

$$A_3 = (a_3) + (-s_{3x}a_1 + s_{3y}a_2) \equiv a_3 + \Delta a_3 \tag{4-3}$$

$$A_4 = (a_4) + (s_{4x}a_{10} - s_{4z}a_5) \equiv a_4 + \Delta a_4 \tag{4-4}$$

$$A_5 = (a_5) + (-s_{5x}a_{10} + s_{5y}a_4) \equiv a_5 + \Delta a_5 \tag{4-5}$$

$$A_6 = (a_6) + (-s_{6y}a_{11} + s_{6z}a_7) \equiv a_6 + \Delta a_6 \tag{4-6}$$

$$A_7 = (a_7) + (-s_{7x}a_6 + s_{7y}a_{11}) \equiv a_7 + \Delta a_7 \tag{4-7}$$

$$A_8 = (a_8) + (-s_{8y}a_9 + s_{8z}a_{12}) \equiv a_8 + \Delta a_8 \tag{4-8}$$

$$A_9 = (a_9) + (s_{9x}a_8 - s_{9z}a_{12}) \equiv a_9 + \Delta a_9 \tag{4-9}$$

Note that A_i can be divided into two parts: The first part is the true kinematic acceleration and the second part is the error acceleration caused by the non-zero cross-axis sensitivity coefficients.

Calculated Angular Accelerations

Since an investigator does not know the true kinematic acceleration a_i, he, therefore, uses the measured value of A_i. Equation (3) becomes Equation (5).

$$\dot{\Omega}_x = \frac{1}{2r}(A_7 - A_3 - A_9 + A_2) \tag{5-1}$$

$$\dot{\Omega}_y = \frac{1}{2r}(A_8 - A_1 - A_5 + A_3) \tag{5-2}$$

$$\dot{\Omega}_z = \frac{1}{2r}(A_4 - A_2 - A_6 + A_1) \tag{5-3}$$

where $\dot{\Omega}_j$ (j = x, y, z) is the calculated angular acceleration.

Using Eqs. (1), (4), and (5), the calculated angular acceleration in matrix form becomes

$$\dot{\boldsymbol{\Omega}} = \dot{\boldsymbol{\omega}} + \Delta\dot{\boldsymbol{\omega}} \tag{6'}$$

where

$$\dot{\boldsymbol{\omega}} = \begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} = \frac{1}{2r} \begin{bmatrix} 0 & 1 & -1 & 0 & 0 & 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 & -1 & 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \end{bmatrix} \tag{7}$$

and

$$\Delta\dot{\boldsymbol{\omega}} = \begin{bmatrix} \Delta\dot{\omega}_x \\ \Delta\dot{\omega}_y \\ \Delta\dot{\omega}_z \end{bmatrix} = \frac{1}{2\gamma} \begin{bmatrix} (s_{2x} + s_{3x} - s_{7x} - s_{9x}) & -(s_{3y} - s_{7y}) & -(s_{2z} - s_{9z}) \\ -(s_{3x} - s_{5x}) & (s_{1y} + s_{3y} - s_{5y} - s_{8y}) & -(s_{1z} - s_{8z}) \\ -(s_{2x} - s_{4x}) & -(s_{1y} - s_{6y}) & (s_{1z} + s_{2z} - s_{4z} - s_{6z}) \end{bmatrix}$$

$$\cdot \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} + \frac{1}{2} \begin{bmatrix} -(s_{7y} + s_{9z}) & -s_{9z} & -s_{7y} \\ -s_{8z} & -(s_{5x} + s_{8z}) & -s_{5x} \\ -s_{6y} & -s_{4x} & -(s_{4x} + s_{6y}) \end{bmatrix} \cdot \begin{bmatrix} \omega_x^2 \\ \omega_y^2 \\ \omega_z^2 \end{bmatrix}$$

$$+ \frac{1}{2} \begin{bmatrix} -s_{7x} & 0 & -s_{9x} \\ -s_{5y} & -s_{8y} & 0 \\ 0 & -s_{6z} & -s_{4z} \end{bmatrix} \cdot \begin{bmatrix} \omega_x\omega_y \\ \omega_y\omega_z \\ \omega_z\omega_x \end{bmatrix}$$

$$+\frac{1}{2}\begin{bmatrix} 0 & -s_{9x} & s_{7x} \\ s_{8y} & 0 & -s_{5y} \\ -s_{6z} & s_{4z} & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} \tag{8}$$

Equation (7) is identical to Equation (3), which is the true angular acceleration. Equation (8) is the angular acceleration errors due to the non-zero cross-axis sensitivity of the actual accelerometers. These errors are functions of linear accelerations, squares of angular velocities, cross products of angular velocities, and angular accelerations. The cross coupling effect is obvious.

<u>Planar Motion</u>

Assume that the nine-accelerometer module undergoes a planar motion in the x-z plane. This implies the following:

$$\ddot{y} = 0 \tag{9-1}$$
$$\omega_x = \omega_z = 0 \tag{9-2}$$
$$\dot{\omega}_x = \dot{\omega}_z = 0 \tag{9-3}$$

The true and the error angular accelerations are expressed in Equations (10) and (11) respectively.

$$\dot{\omega}_x = 0 \tag{10-1}$$
$$\dot{\omega}_y = \frac{1}{2r}(-a_1 + a_3 - a_5 + a_8) \tag{10-2}$$
$$\dot{\omega}_z = 0 \tag{10-3}$$

$$\Delta\dot{\omega}_x = \frac{1}{2r}\left[(S_{2x} + S_{3x} - S_{7x} - S_{9x})\,\ddot{x} - (S_{2z} - S_{9z})\,\ddot{z}\right]$$

due to linear acceleration

$$+\frac{1}{2}\left[-S_{9z}\,\omega_y^2\right]$$

due to angular velocity squared

$$+\frac{1}{2}\left[-S_{9x}\,\dot{\omega}_y\right] \qquad (11\text{-}1)$$

due to angular acceleration

$$\Delta\dot{\omega}_y = \frac{1}{2r}\left[-(S_{3x} - S_{5x})\,\ddot{x} - (S_{1z} - S_{8z})\,\ddot{z}\right]$$

$$+\frac{1}{2}\left[-(S_{5x} + S_{8z})\,\omega_y^2\right] \qquad (11\text{-}2)$$

$$\Delta\dot{\omega}_z = \frac{1}{2r}\left[-(S_{2x} - S_{4x})\,\ddot{x} + (S_{1z} + S_{2z} - S_{4z} - S_{6z})\,\ddot{z}\right]$$

$$+\frac{1}{2}\left[-S_{4x}\,\omega_y^2\right]$$

$$+\frac{1}{2}\left[S_{4z}\,\dot{\omega}_y\right] \qquad (11\text{-}3)$$

Although $\Delta\dot{\omega}_x$ and $\Delta\dot{\omega}_z$ are not considered under the planar motion assumption, they are never zero. This has been demonstrated by Padgoadnkar [1] in his planar motion examples.

Pure Rotational Planar Motion

An investigator may ignor $\dot{\Omega}_x$ ($\dot{\Omega}_x = \dot{\omega}_x + \Delta\dot{\omega}_x$) and $\dot{\Omega}_z$ ($\dot{\Omega}_z = \dot{\omega}_z + \Delta\dot{\omega}_z$), and consider $\dot{\Omega}_y$ ($\dot{\Omega}_y = \dot{\omega}_y + \Delta\dot{\omega}_y$) as his only measurement. Thus, his experiment has an error of $\Delta\dot{\omega}_y = -\frac{1}{2}(s_{5x} + s_{8z})\omega_y^2$ under pure rotational planar motion conditions.

Let the angular velocity be $\omega_y(t) = \omega_{y\,max}\sin(\omega t)$, where ω is the oscillating frequency, then $\dot{\omega}_y(t) = \omega\omega_{y\,max}\cos(\omega t)$. Equation (11-2) becomes

$$\Delta\dot{\omega}_y(t) = -\frac{1}{2}(s_{5x} + s_{8z})\,\omega_{y\,max}^2\,\sin^2(\omega t)$$

$$= \underbrace{\frac{-(s_{5x} + s_{8z})\,\omega_{y\,max}^2}{4}}_{\text{constant error}} + \underbrace{\frac{(s_{5x} + s_{8z})\,\omega_{y\,max}^2}{4}\cos(2\omega t)}_{\text{second-harmonic error}} \qquad (12)$$

The constant error term in $\dot{\omega}_y$ will create errors proportional to time-squared in angular displacement. For example, if $s_{5x} = s_{8z} = 0.01$, and $\omega_{y\,max} = 100$ rad/sec, then this constant error is 50 rad/sec^2. This error will, in turn, accumulate an angular velocity error of 5 rad/sec and an angular displacement error of 0.25 radians after 0.1 second of an experiment.

The second-harmonic distortion is illustrated in Fig. 3. Maximum errors occur when $\dot{\omega}_y$ passes through its zeros and peaks but does not affect the peak-to-peak value of $\dot{\omega}_y$. This often misleads the investigator to believe that his experiment has very little error. This harmonic distortion can also be observed in Padgoankar's validation data [1] where the positive half cycles of the angular acceleration had longer zero-crossing time than those of the corresponding negative half cycles.

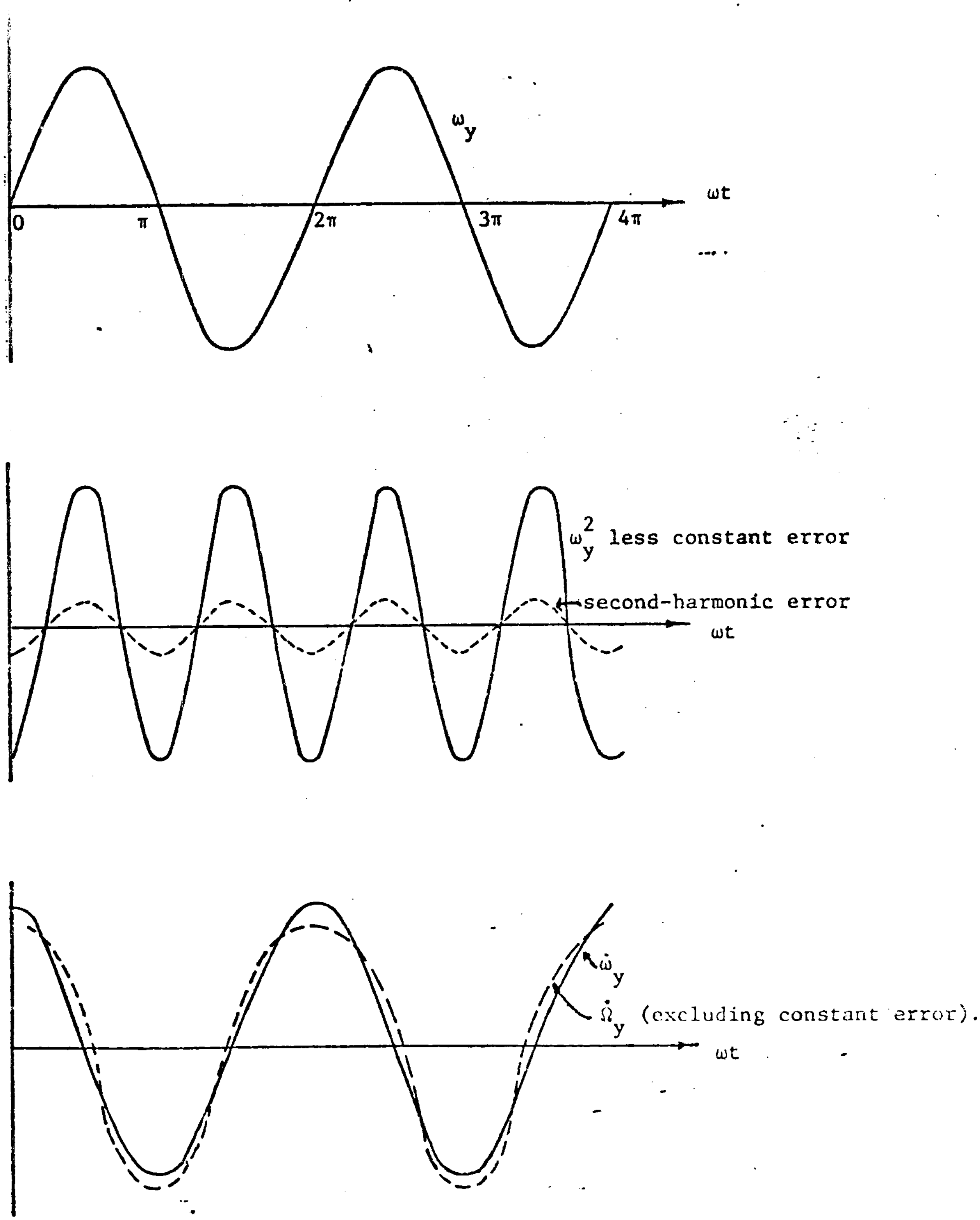


Fig. 3 Second-harmonic distortion.

Pure Translational Planar Motion

Again, Equation (11) can be simplified by restricting the planar motion to a pure translational motion in the x-z plane. In this case, the measured angular acceleration has an error of

$$\frac{-1}{2r}\left[(s_{3x}-s_{5x})\ddot{x}-(s_{1z}-s_{8z})\ddot{z}\right]$$

For example, if r = 7.62 cm (3 inches), $\ddot{x}=\ddot{z}=50g$, and the magnitude of the cross-axis sensitivity is 0.01, then the worst case is $\Delta\dot{\omega}_y = 128\ \text{rad/sec}^2$.

Near Planar Motion

If this module undergoes an oblique motion but is dominated by the x-z plane motion, then Equation (11) is also valid for the near planar motion where the cross product terms are much less than the squared angular velocity terms. The error analysis of $\Delta\dot{\omega}_y$ is identical to that in the planar motion but the values of $\Delta\dot{\omega}_x$ and $\Delta\dot{\omega}_z$ can no longer be ignored. The following discusses the nature of $\Delta\dot{\omega}_x$. Similar discussion applies for $\Delta\dot{\omega}_z$.

The error $\Delta\dot{\omega}_x$ for a pure rotational motion is derived from Eq. (11-1).

$$\Delta\dot{\omega}_x \simeq \frac{-1}{2}\left(s_{9x}\dot{\omega}_y - s_{9z}\omega_y^2\right) \tag{13}$$

where the term $(s_{9z}\omega_y^2)$ consists of a constant error and a second-harmonic error similar to those of Equation (12). The additional term $(s_{9x}\dot{\omega}_y)$ may appear to be insignificant when compared with $\dot{\omega}_y$ but it is significant when compared with $\dot{\omega}_x$ since this is a near planar motion. For example, Padgoankar's [1] near planar motion ($\pm$15 degrees in pitch, $\pm$2 degrees yaw and roll, all oscillated at the same frequency) hypothetical data indicated that $\Delta\dot{\omega}_x$ was 20% of $\dot{\omega}_y$ and 200% of $\dot{\omega}_x$ for a cross-axis sensitivity of 0.05. The phasing relationship between $\dot{\omega}_y$ and $\Delta\dot{\omega}_x$ was also in agreement with $s_{9x}\dot{\omega}_y$ for a negative s_{9x}.

The error $\Delta\dot{\omega}_x$ for a <u>pure translational</u> motion is shown in Equation (14).

$$\Delta\dot{\omega}_x \simeq \frac{1}{2r}\left[(s_{2x} + s_{3x} - s_{7x} - s_{9x})\,\ddot{x} - (s_{2z} - s_{9z})\,\ddot{z}\right] \qquad (14)$$

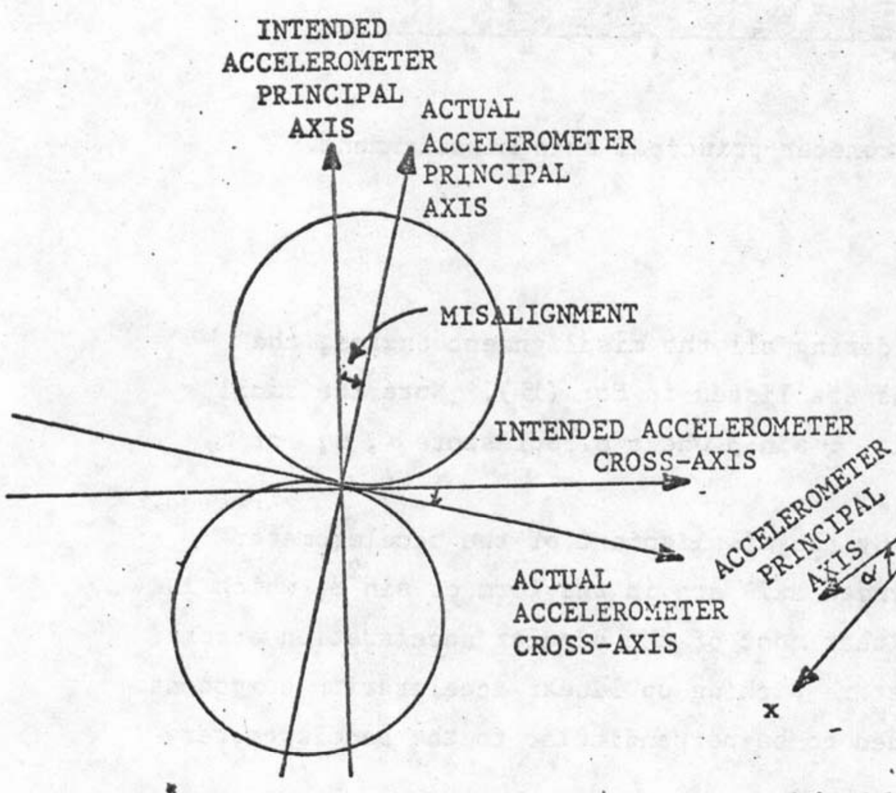
Again, this error may be insignificant when compared with $\dot{\omega}_y$, but it is significant when compared with $\dot{\omega}_x$ since this is a near planar motion. For example, if $r = $ ~~3 inches~~ 7.62 cm, $\ddot{x} = \ddot{z} = 50g$, and the magnitude of the cross-axis sensitivity is 0.01, the worst case is $\Delta\dot{\omega}_x = 192\ \mathrm{rad/sec}^2$.

A recent paper [2] appears to indicate that the nine-accelerometer module is suitable for 3-D near planar motion application. However, this was a special case in which: (1) The angular acceleration and velocity were compared to film data in planar motion only. This was a 2-D application. The large errors in minor axes were clearly demonstrated but were not used in the comparison. These errors did not appear to be large as compared to the major axis data. The maximum angular acceleration error was 40% and the maximum angular velocity error was 30% between the two techniques at the head center of gravity. (2) The angular displacement was small where the resultant linear acceleration could be closely approximated by the PA-axis component alone. This was a 1-D application. In fact, in his key example, the resultant acceleration could be approximated by the PA-axis component with only 8% error.

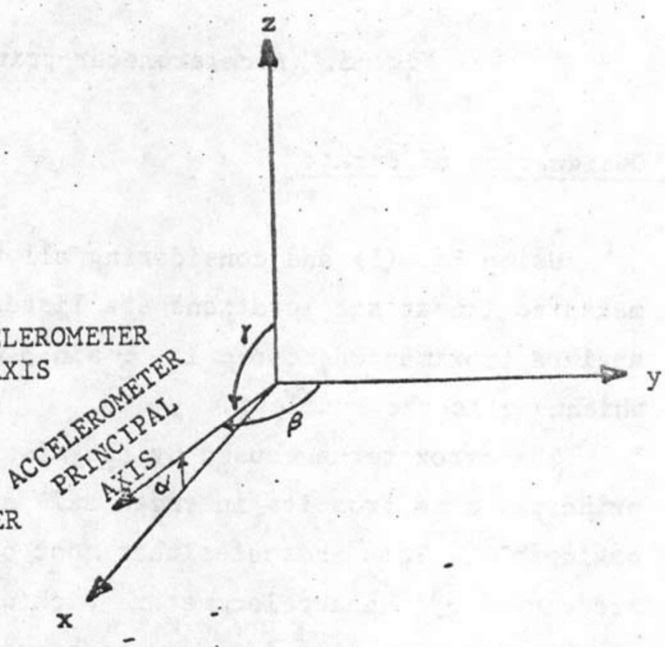
EFFECTS OF MISALIGNMENT

Definition of Misalignment

The sensitive (principal) axis of an actual accelerometer can not be defined precisely by the manufacturer. In addition, the module block can not be machined and maintained precisely in the intended coördinate system. Figure 4 shows the output electrical signal when an accelerometer principal axis is misaligned. Typical one-sigma value of the misalignment [3] is generally larger than one degree, see Figure 5. In subsequent derivations, α, β, and γ are used to define the directional cosines from the accelerometer principal axis to the x, y, and z axis respectively.



(a) Planar misalignment



(b) Oblique misalignment

Fig. 4 Illustration of accelerometer misalignment.

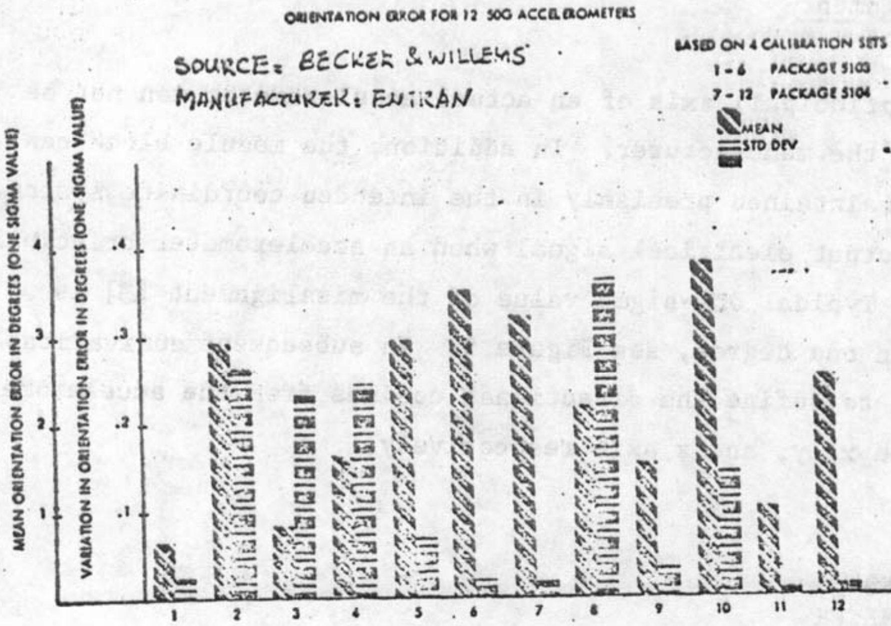


Fig. 5. Accelerometer principal axis misalignment.

Designation of Errors

Using Eq. (1) and considering all the misalignment angles, the measured linear accelerations are listed in Eq. (15). Note the small angle approximation $\cos\phi \simeq 1 - \frac{1}{2}\sin^2\phi$ where ϕ represents α, β, or γ, whichever is the smallest.

The error terms caused by the misalignment of the accelerometer principal axis from its intended axis are in the form of $\sin^2\phi$, which is neglegible. This indicates that most of the angular acceleration errors are caused by the accelerometers picking up linear acceleration components on the plane which is intended to be perpendicular to the accelerometer principal axis.

After dropping the second order terms in Eq. (15), this equation has similar form as that of Eq. (4). It is clear that these two problems are similar in nature. The analysis of the misalignment problem then becomes the problem of cross-axis sensitivity when one equates a 0.6-degree accelerometer principal axis misalignment from its intended axis to a 1%

$$A_1 = (a_1) + [(\tfrac{-1}{2}\sin^2\alpha_1)a_1 + (\cos\beta_1)a_2 + (\cos\gamma_1)a_3] \equiv a_1 + \Delta a_1 \qquad (15\text{-}1)$$

$$A_2 = (a_2) + [(\cos\alpha_2)a_1 + (\tfrac{-1}{2}\sin^2\beta_2)a_2 + (\cos\gamma_2)a_3] \equiv a_2 + \Delta a_2 \qquad (15\text{-}2)$$

$$A_3 = (a_3) + [(\cos\alpha_3)a_1 + (\cos\beta_3)a_2 + (\tfrac{-1}{2}\sin^2\gamma_3)a_3] \equiv a_3 + \Delta a_3 \qquad (15\text{-}3)$$

$$A_4 = (a_4) + [(\cos\alpha_4)a_{10} + (\tfrac{-1}{2}\sin^2\beta_4)a_4 + (\cos\gamma_4)a_5] \equiv a_4 + \Delta a_4 \qquad (15\text{-}4)$$

$$A_5 = (a_5) + [(\cos\alpha_5)a_{10} + (\cos\beta_5)a_4 + (\tfrac{-1}{2}\sin^2\gamma_5)a_5] \equiv a_5 + \Delta a_5 \qquad (15\text{-}5)$$

$$A_6 = (a_6) + [(\tfrac{-1}{2}\sin^2\alpha_6)a_6 + (\cos\beta_6)a_{11} + (\cos\gamma_6)a_7] \equiv a_6 + \Delta a_6 \qquad (15\text{-}6)$$

$$A_7 = (a_7) + [(\cos\alpha_7)a_6 + (\cos\beta_7)a_{11} + (\tfrac{-1}{2}\sin^2\gamma_7)a_7] \equiv a_7 + \Delta a_7 \qquad (15\text{-}7)$$

$$A_8 = (a_8) + [(\tfrac{-1}{2}\sin^2\alpha_8)a_8 + (\cos\beta_8)a_9 + (\cos\gamma_8)a_{12}] \equiv a_8 + \Delta a_8 \qquad (15\text{-}8)$$

$$A_9 = (a_9) + [(\cos\alpha_9)a_8 + (\tfrac{-1}{2}\sin^2\beta_9)a_9 + (\cos\gamma_9)a_{12}] \equiv a_9 + \Delta a_9 \qquad (15\text{-}9)$$

accelerometer cross-axis sensitivity. It should be noted that this 0.6-degree misalignment is perhaps the state-of-the-art specification for piezoresistive accelerometers [3] because the principal axis is difficult to define.

CONCLUSIONS

It appears that the accelerometer cross-axis sensitivity or axis misalignment imposes limitations on the 9-accelerometer module for the application in biomechanical experiments. Some of these limitations are:

Planar Motion -- The rotational kinematics creates two error terms in the calculated angular acceleration, Eq. (12). The constant error accumulates angular velocity and angular displacement errors while the second-harmonic error creates distortion in the calculated angular acceleration. The translational kinematics creates an error in the calculated angular acceleration, Eq. (11-2), proportionally. However, calculated data of angular acceleration and angular velocity in the local coordinate system are generally valid for impact experiment [1],[2].

Oblique Motion -- In oblique experiments, the calculated angular accelerations have errors due to the non-zero cross-axis sensitivity or the principal axis misalignment of the accelerometers. See Eq. (8). This is especially true for oblique but near planar motion where the kinematics in one plane dominates the others. The rotational kinematics creates three error terms in the minor axis angular acceleration, Eqs. (11) and (12). The first term is coupled from the major axis angular acceleration; the second term is a constant error; and the third term is the second-harmonic distortion. The translational kinemetics creates an error in the calculated angular acceleration, Eq. (11), proportionally. These errors become significant in the minor axes but not necessarily in the major axis because of the near planar nature. They can easily overcome the true angular accelerations in the minor axes. Even with moderate errors, the translation of data through a rigid body, say, to the head center of gravity, may contain large error because this requires the knowledge of angular velocities. Chou [2] seems to have validated the transformed head resultant linear acceleration data under near oblique condition. But this was a special case where the minor axis components and their associated errors did not contribute significantly to the resultant. In other words, the resultant acceleration can be closely approximated by a simple planar motion assumption. Furthermore, in his key example (15 Km/hr) the head angular excursion was limited to ± 15 degrees. This further restricted the motion to a near 1-D linear motion where the dominating acceleration component was in the PA-axis. For complete kinematic application, a coordinate transformation in three dimensions is almost impossible because the elements of the directional cosine matrix consist of the sums of the products of the trigonometric functions of the three angles. A small angle with large error (in percentage) will offset the values of the matrix elements, especially when one or more of the angles changes sign or reaches ± 90 degrees.

Other error analyses involve the accelerometer principal axis error (linearity, hysteresis, resolution, threshold, noise, thermal drift, frequency response, etc.), seismic mass center error, data channel error, and computational error. They are not included in this paper. The basic problem of the nine-accelerometer module lies in the fact that it measures the differences of paired accelerations while the accelerometers undergo severe impact environment. It requires extremely accurate accelerometer tracking data. In addition, the measured angular acceleration errors caused by the linear accelerations are inversely proportional to the size of the module, thus limiting the reduction of the module size.

If it should be determined that current nine accelerometer system error is unacceptable, there are at least two possible alternatives for biomechanics applications. The first method is to use better quality accelerometers such as the servo grade force-balanced accelerometers. This may improve the measurement accuracy sufficiently to provide data within acceptable limits. A second method is to use angular sensing devices such as the rate gyros. This method is not new. Previous applications were limited because of the weight of the sensors. Recent development of angular rate measuring devices has reduced the size (1"Dx1.5"L) and weight (2oz.) significantly. For example, one device senses inertial angular rates based on the principal of coriolis acceleration. Another example is the magneto-hydrodynamic two-axis rate sensor. It should be pointed out, however, that other type errors may be introduced in the application of rate sensors and that an analysis such as the foregoing for such application would be desirable.

REFERENCES

1. Padgoankar, A. J., Krieger, K. W., and King, A. I., "Measurement of Angular Acceleration of a Rigid Body Using Linear Accelerometers," J. of Applied Mechanics, pp. 552-556, September 1975.

2. Chou, C. C. and Sinha, S. C., "On the Kinematics of the Head Using Linear Acceleration Measurements," J. of Biomechanics, vol. 9, pp. 607-613, September 1976.

3. Becker, E. and Willems, G., "An Experimentally Validated 3-D Interial Tracking Package for Application in Biodynamic Research," Proceedings of Nineteenth Stapp Car Crash Conference, pp. 899-930, 1975.