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INVESTIGATION OF THE LEAST SQUARES APPROACH TO THE CALCULATION OF ANGULAR ACCELERATION FROM LINEAR ACCELEROMETRY

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ABSTRACT

The measurement of angular acceleration depends on differences in linear acceleration measured at two locations on a rigid body. The use of the least squares method to determine this difference was investigated by using more than two accelerometers along a single axis. The statistical approach consistently out-performed the technique of simply taking the difference between two linear accelerometers in the determination of angular accelerations, velocities, and displacements. However, errors in signal amplitude, frequency content, and phase are still noticeable with this technique. These errors do not appear to result from expected sources. That is, low accelerometer sensitivity, fixture vibration, cross-axis sensitivity and cyclic noise seem to be at a minimum. The prevailing factors are the time delay between the film and transducer data, and random noise. The accelerometers are not able to accurately represent an instantaneous start. Also, the signal conditioning process may add a few milliseconds of delay. In general, the least squares approach to obtaining angular acceleration from linear accelerometry is a straightforward, convenient method that provides greater accuracy than previously available techniques, without a significant increase in complexity.

FOREWORD

In a recent study of motor vehicle related crash injuries it is estimated that nearly 62 percent of these injuries involve head, neck, and facial trauma. The determination of angular acceleration is important in understanding head kinematics, brain injury mechanisms and in evaluating the Head Injury Criterion (HIC). An effective device for measuring three-dimensional angular acceleration directly does not exist. At present, 3-D angular acceleration is measured by means of a uniquely configured array of linear accelerometers. Angular acceleration is computed directly from the data from this array, without the need to integrate a system of non-linear ordinary differential equations. Although the method yields stable angular acceleration, it is based on the computation of a small difference between two accelerometer readings which can be relatively large, depending upon the impact environment. To reduce measurement errors inherent in this method, multiple accelerations were measured and the method of least squares was used to determine the desired difference. An improvement in the computed acceleration was achieved by this new method.

BACKGROUND

The extension of experimental impact biomechanics to general 3-D motion requires the measurement of angular acceleration of body segments in 3-D. In theory, it is simple to describe such rigid-body kinematics. Straightforward differential equations have been derived to describe such motion. Although the experimental determination of angular acceleration is thought to be a problem that has been solved in principle, it remains a technical challenge to overcome errors encountered in practical application.

To the present, many researchers have relied upon videometric methods of collecting biodynamic data. Kinematic quantities are calculated from displacement data captured on film by high-speed cameras. This process is quite tedious in that it involves time-consuming manual digitizing of the filmed event. Moreover, there are several sources of error in this method including those introduced by single and double differentiation of position data which may yield inaccurate and unreliable results.

In the analysis of planar motion, Mertz (1967), showed that it was feasible to quantify angular acceleration of a rigid body using linear accelerometer data. Many investigators have demonstrated that translational accelerometers, appropriately positioned within a moving reference frame could, theoretically, yield both angular and translational accelerations with respect to the moving frame (body fixed basis).

It has been shown that a minimum of five linear accelerometers are necessary to compute all three components of angular acceleration of the body on which the transducers are mounted, but a sixth accelerometer is required to derive all three linear acceleration components for a complete definition of rigid body motion.

Padgaonkar et al (1975) demonstrated the accuracy and stability of a nine-accelerometer configuration for 3-D angular acceleration analysis. In doing so, they proved the inadequacy of the formerly developed six-transducer scheme. However, the method contains inherent problems of inaccuracy, particularly when angular acceleration is to be measured in the presence of high linear accelerations. Angular acceleration about any axis is obtained by taking the difference between a pair of linear accelerometers. One of the causes of inaccuracy is the fact that this methods attempts to find a small difference between two numbers which can be very large at times.

Viano et al (1985) have developed a recursive formula for a fifteen accelerometer approach which yields angular displacement data after numerical solution of integro-differential equations. The angular velocity and acceleration and translational acceleration may be determined once the displacement has been obtained.

In developing accelerometric techniques, many important aspects contribute to the overall integrity of the measurement system. From an analytical standpoint, it is necessary to avoid numerical analysis and integration as part of the solution technique. Non-commutativity of finite rotations and integration of previous time step data to obtain velocity values from acceleration data contribute to an accumulation of error.

Furthermore, from an experimental approach, there are many possible sources of error. The transducers themselves suffer from cross-axis sensitivity, that is, undesired output is generated from stimulation orthogonal to the accelerometers´ sensitive axes. Because most methods rely on the difference in measured linear acceleration between two points, mismatching of transducer pairs, signal noise, and mounting fixture vibrations all contribute to errors in measurement and calculation. Large relative motions between accelerometers due to vibration are common during high-energy direct impact when extracorporal transducer fixtures are employed. Finally, a low mass, highly rigid mounting fixture must be fabricated such that all accelerometer seismic masses are accurately aligned and spaced.

OBJECTIVES

The objective of this study is to demonstrate that an improvement in the accuracy of the computed angular acceleration can be achieved by using multiple accelerometers to determine the difference in linear acceleration by the method of least squares. It is a two-dimensional study to demonstrate the underlying principle and is not a complete three-dimensional analysis.

METHODOLOGY

To justify the use of the least squares technique as a method of solving for angular acceleration three objectives must be accomplished. First, a well-founded theoretical background must be developed. Second, the performance of the technique must be proved under realistic test conditions. Third, the superiority of the technique over existing methods must be shown.

The theoretical background for computing angular acceleration without the need to integrate a set of non-linear differential equations was provided by Padgaonkar et al (1975). In this two-dimensional study, four accelerometers were used and the difference in linear acceleration was estimated from the linear regression coefficient of the least squares line of linear acceleration versus radius for each instant in time. The angular acceleration time history for this planar case was then computed. To validate the hypothesis that the angular acceleration computed by the least squares method is more accurate than taking the differences in linear acceleration between two accelerometers, it is necessary to obtain kinematic data from an independent source. One such source is displacement data obtained from high spead film. The time history of angular displacement of the accelerometer mount was obtained from the arctangent of the linear regression coefficient of position coordinates of four photo targets attached to the mount. An angular acceleration time history was then obtained by differentiating the displacement data twice. In this way, film data could be compared to transducer output.

EXPERIMENTAL METHOD

The two-dimensional test was carried by spinning a rigid bar about a transverse axis. High angular acceleration was achieved by releasing the rotating arm which was spring loaded initially. The motion took place in a horizontal plane to permit an overhead recording of the motion using a high speed video system made by Kodak - the SP 2000, which is capable of taking 2,000 pictures per second.

The test fixture consisted of a fixed aluminum plate which was 1/2 inch thick and 12 inches square. A vertical shaft in the center of this plate acted as a pivot for the rotating arm consisting of a second plate and an accelerometer mount. The moving plate could rotate relative to the fixed one on a ball bearing axle. It was also 1/2-inch thick and was 14 inches long and 4 inches wide. The accelerometer mount was a 9-inch long bar, 1/2-inch square in cross-section. It was bolted to the top plate which could be spring loaded to produce angular acceleration. Figure 1 is a sketch of the mount designed to carry up to 6 linear uniaxial accelerometers. The seismic mass of each accelerometer was located on the axis passing through the center of rotation of the fixture. The mount was supported on a moving plate to ensure planar motion and to minimize vibration. The moving surfaces between the two plates were lubricated with molybdenum.

Although the accelerometer mount could accommodate a maximum of 6 accelerometers, only four were used at a time. The spacing was designed to allow two sets of radii of rotation. The transducers could be arranged to have two with one-inch radii and two with two-inch radii. Alternately, the radii could be changed to 2 and 4 inches for each pair of accelerometers. The mount

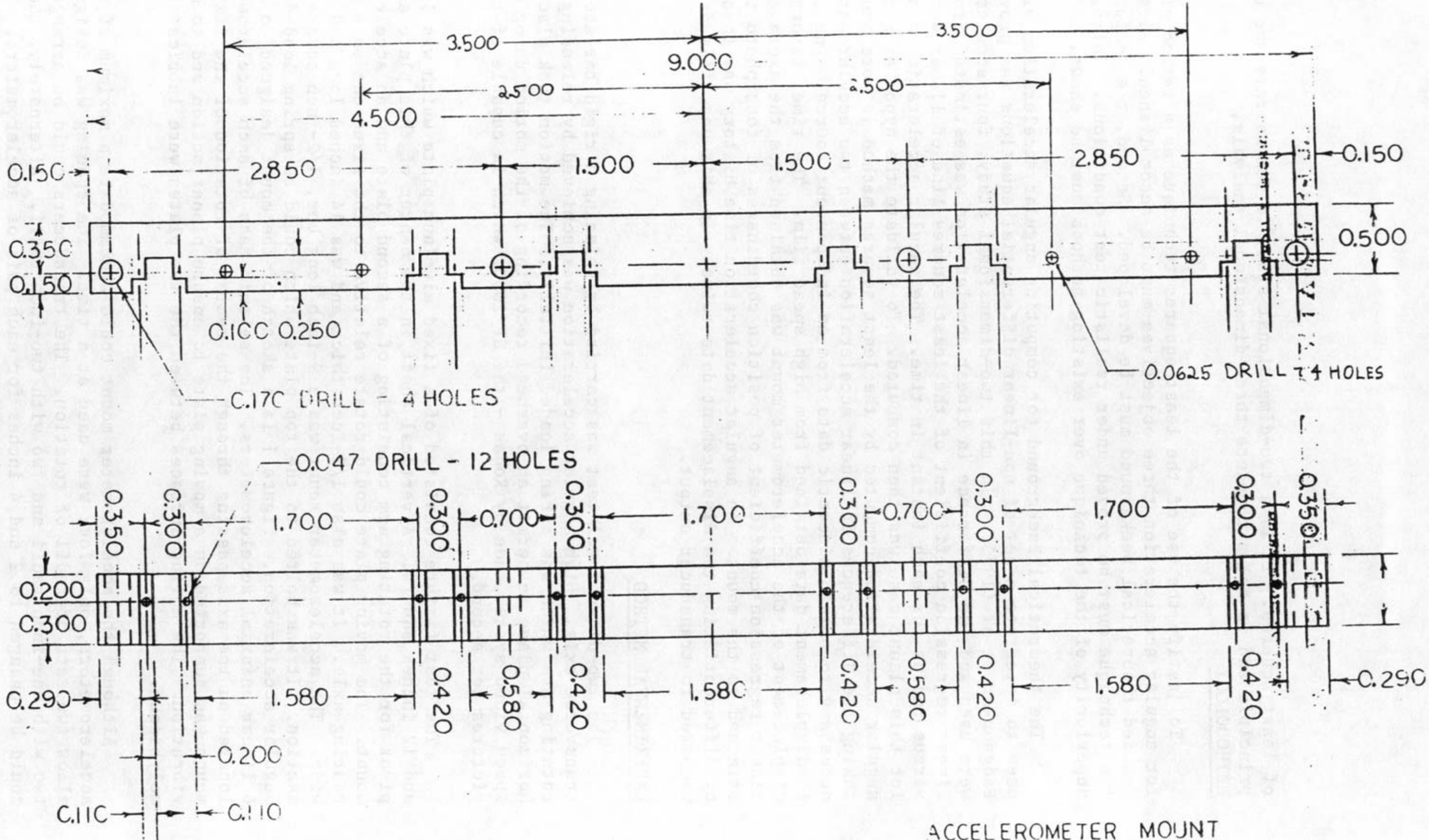


Figure 1 - Drawing of accelerometer mount for angular acceleration measurement

ACCELEROMETER MOUNT
MAT'L ALUMINUM
SCALE FULL
BY G. S. LOCKE

was also equipped with light emitting diodes (LED) placed along the same axis as the seismic mass of the accelerometers. They were located at known distances from the common center of rotation to facilitate the comparison of angular kinematic data.

Endveco Model 7264-2000 piezoresistive linear accelerometers were used. Each accelerometer required two resistors to complete the Wheatstone bridge. These resistors were chosen to match each the resistance of the active arms of the accelerometer so that a well balanced bridge was obtained.

The drop test method of calibration was used for each accelerometer. A Kistler model 808A piezoelectric accelerometer, used only for calibration, was the standard. The standard and the accelerometers to be calibrated were mounted to a calibration fixture and dropped to obtain a physical conversion factor for each transducer.

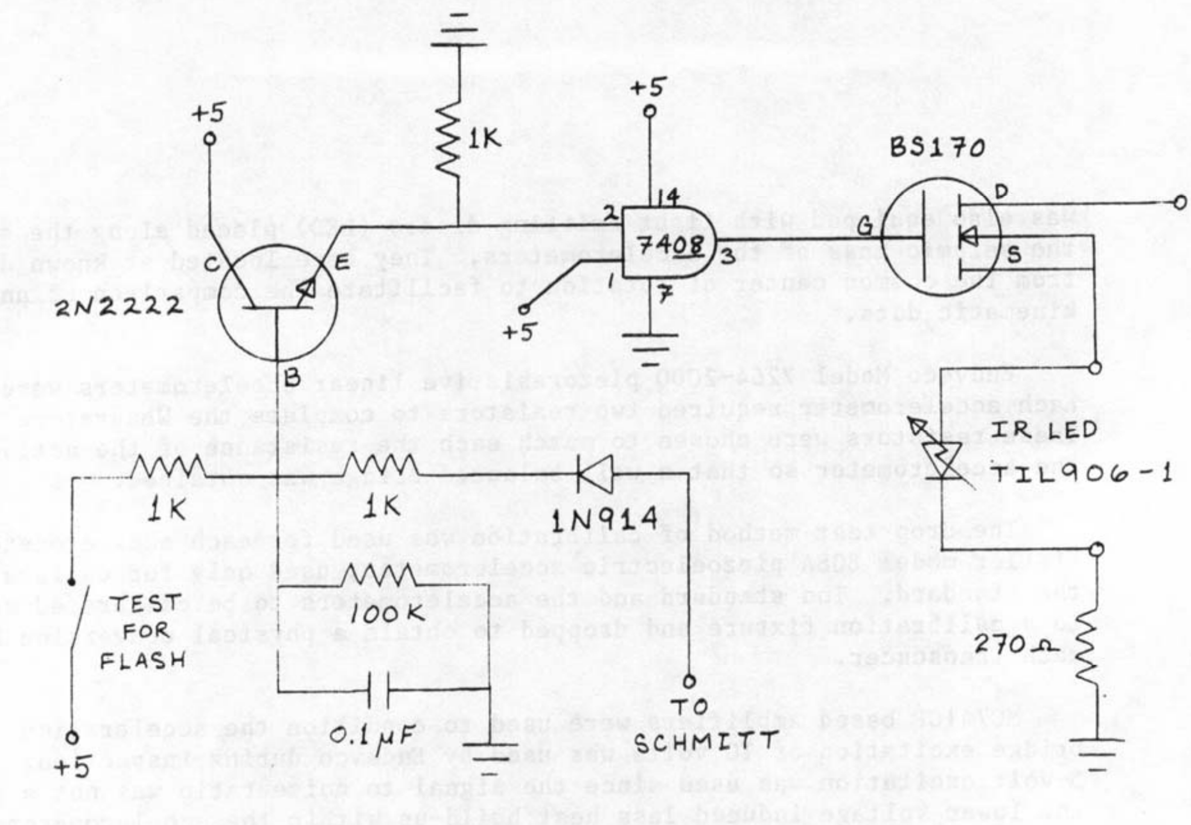
MC741CP based amplifiers were used to condition the acceleration signal. A bridge excitation of 10 volts was used by Endevco during inspection. However, a 5-volt excitation was used since the signal to noise ratio was not a problem and the lower voltage induced less heat build-up within the accelerometers. The lower voltage also reduces both the thermal sensitivity shift and the thermal zero shift of the accelerometer circuit.

The infrared LED's were controlled by the circuits shown in Figure 2. Their motion was recorded on a high speed video system (SP2000) at 1000 fps. LED motion was analyzed by an automatic target tracking program loaded on to an IBM PC/AT. The angular position data were fed through a linear regression program and double differentiated to yield angular acceleration for comparison with the results computed from accelerometer data.

A total of 16 tests were conducted. Four of the tests were discarded due to mechanical or electronic failures, leaving a total of 12 valid test runs. The four accelerometers used assumed two configurations. In one case, they were located at radii of +4, +2, -2, and -4 inches, while in the second, the radii were +2, +1, -1, and +2 inches. These are referred to as the "outer" and "inner" positions respectively. The motion of the bar was also varied. Each test involved oscillatory motion in one of three distinct categories: low g (<10g), high g (>15g), and high g complex (dual spring). To generate these acceleration amplitudes, the spring tensions on the fixture were varied by changing the magnitude of the initial rotational displacement of the fixture. The experiment was performed in the following order:

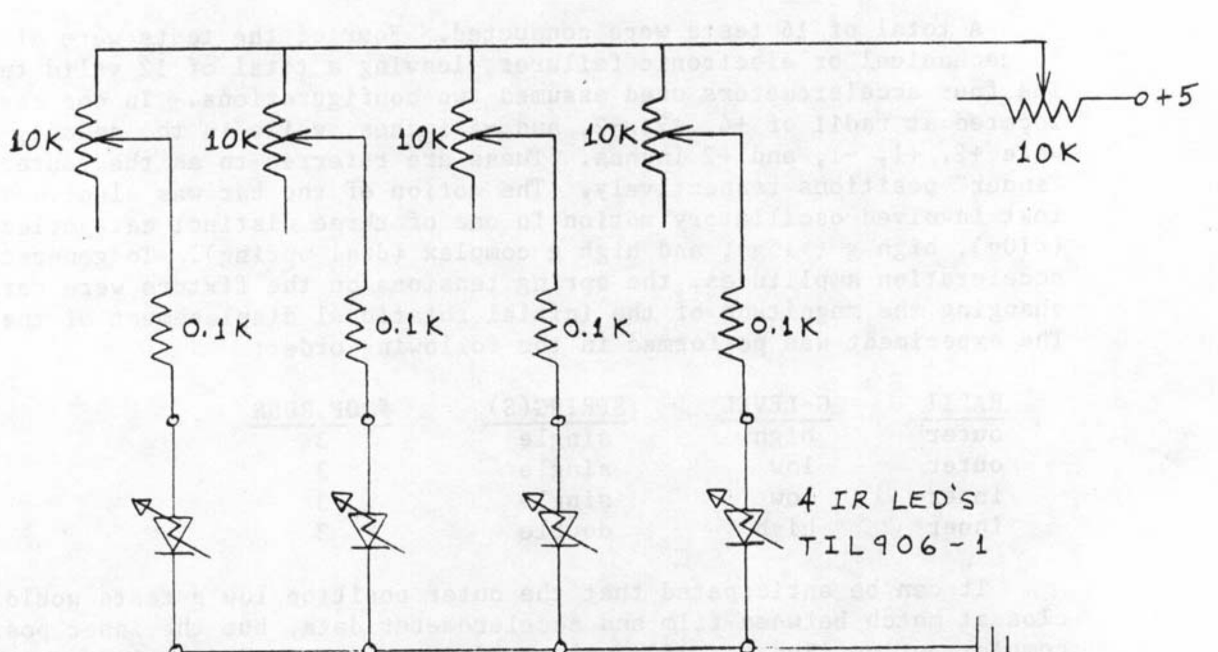
RADII	G-LEVEL	SPRING(S)	# OF RUNS
outer	high	single	3
outer	low	single	3
inner	low	single	3
inner	high	double	3

It can be anticipated that the outer position low g tests would produce the closest match between film and accelerometer data, but the inner position high g complex motion (Run No. 16) would produce the most true-to-life test conditions.



CONTROL CIRCUIT AND DEVICE TRIGGER

Figure 2 - Control circuits for LED's on the accelerometer mount



LED INTENSITY BALANCE CIRCUIT

RESULTS

In general, the least squares method of computing angular acceleration was superior to simply taking the difference in acceleration between any pair of accelerometers. The effectiveness of the least squares method in determining angular accelerations was best demonstrated by varying data quality, using the results from a single test. To accomplish this four different sets of linear acceleration data were used. Two were from an actual test (Run No. 16), and two were hypothetically created. The real data set, referred to as "clean" and shown in Figure 3 was noise free or "de-spiked". Another set, referred to as "spiked" is shown in Figure 4. It contained one channel of noisy data (Accelerometer #3). The hypothetical data sets either were rendered noisy by superposition of a high frequency sinusoidal signal (Figure 5) or by the introduction of a 90-deg phase shift to one of the accelerometer channels (Figure 6).

Run 16 was selected for analysis because the motion was more complex than a a single sinusoidal oscillation of the mount and it was a more realistic simulation of an impact situation. The use of hypothetical data can reveal differences between methods. Thus, such data were also examined in detail. In an effort to quantify the similarities and differences between the least squares and the two accelerometer approaches for determining angular acceleration from linear acceleration data a number of comparisons were made:

A. Displacement (film) data were double-differentiated and compared to angular acceleration determined from linear accelerometry (Data in Run 16A, Figure 3) for analysis of acceleration time histories. The angular acceleration computed by the least squares method (LSSQ) is compared with that from displacement data (FILM) in Figure 7. The computed angular accelerations from the spiked data set (noise in Channel 3) are shown in Figure 8 and those from data set with superimposed noise on Channel 3 are shown in Figure 9. The curve designated by the symbol AVE represents data computed from the difference in acceleration based on a pair of accelerometers.

B. Displacement data were differentiated and compared to integrated angular acceleration data for analysis of velocity time histories, using the spiked data in Channel 3. The three curves shown in Figure 10 and designated as LSSQ, AVE and FILM represent velocities computed by the least squares and the two-accelerometer method and from displacement data respectively.

C. Displacement data were compared to double-integrated angular acceleration data for analysis of displacement time histories, using the spiked data set. These data are shown in Figure 11.

D. Cross-correlation integrals for each method were performed against film data and were compared to film auto-correlation (for angular acceleration). These correlations are shown in Figure 12 for the spiked data set. Cross-correlation is useful in disclosing cyclic patterns, relative power levels, and general time shifts. The maximum, minimum, and terminal values for the acceleration, velocity, and displacement traces are useful for determining discrete percentage differences between "calculated" (least squares and two-accelerometer) and "actual" (film) data. Therfore, frequency, phase, and amplitude information from the least squares and averaging methods are

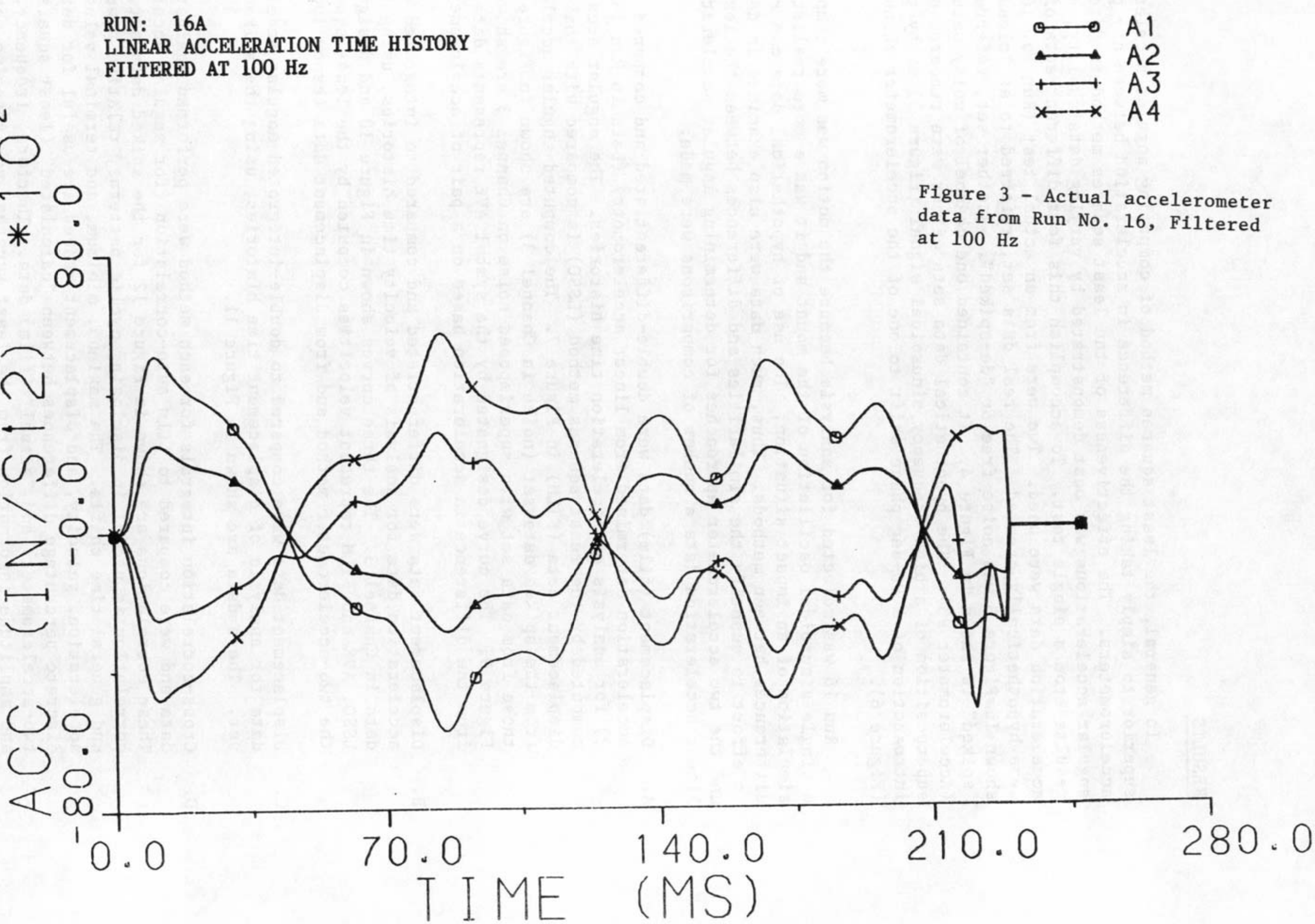


Figure 3 - Actual accelerometer data from Run No. 16, Filtered at 100 Hz

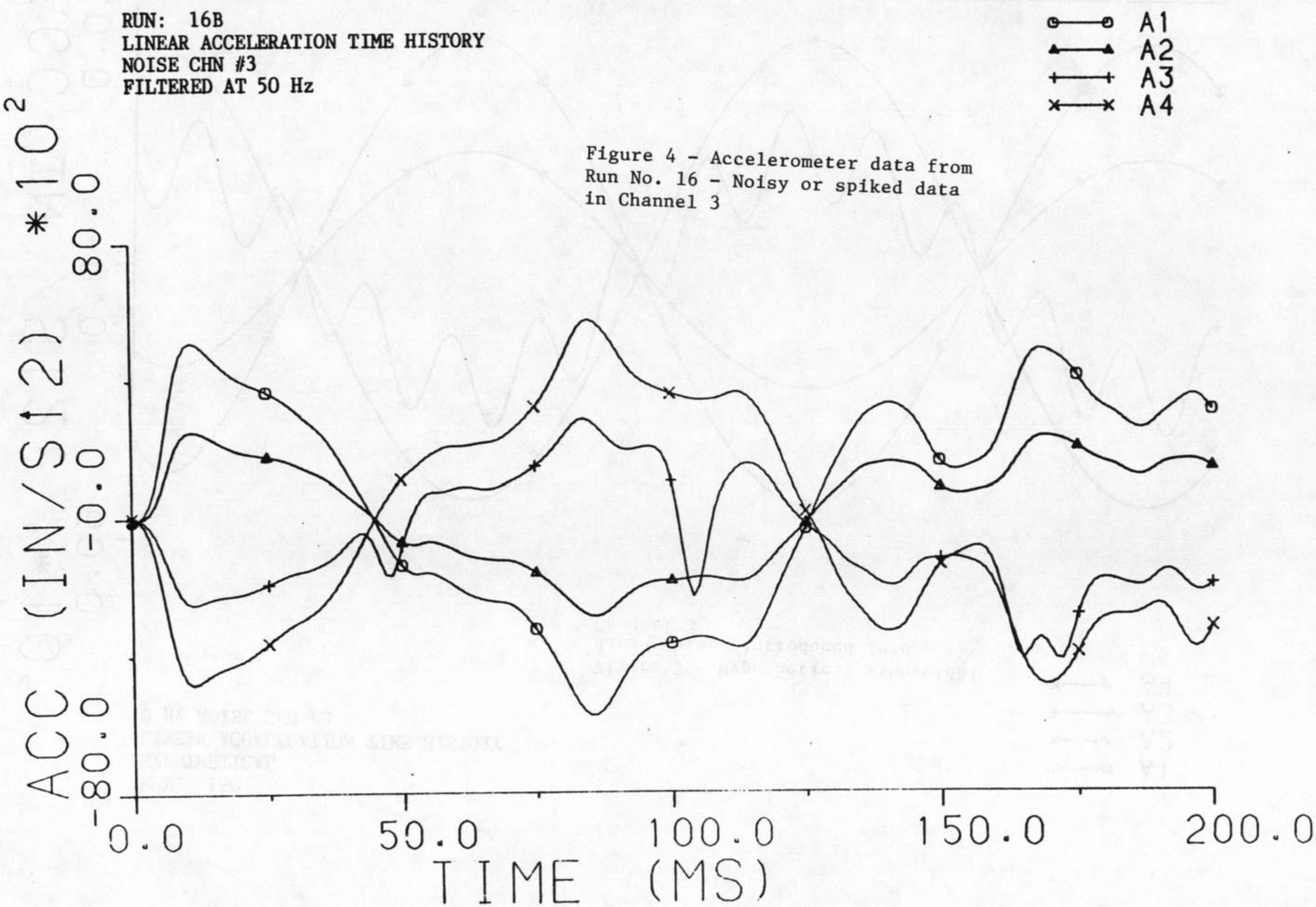


Figure 4 - Accelerometer data from Run No. 16 - Noisy or spiked data in Channel 3

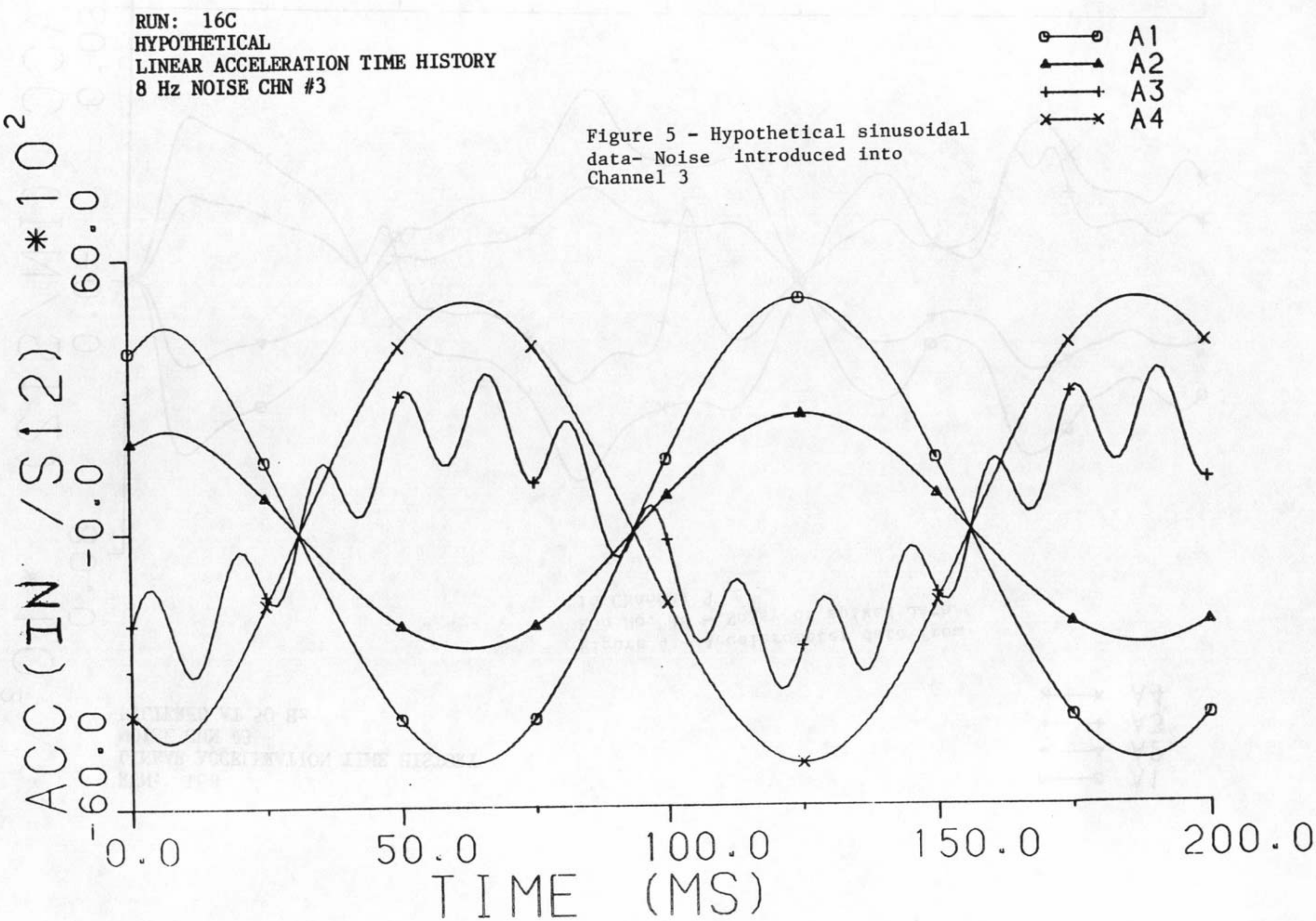


Figure 5 - Hypothetical sinusoidal data- Noise introduced into Channel 3

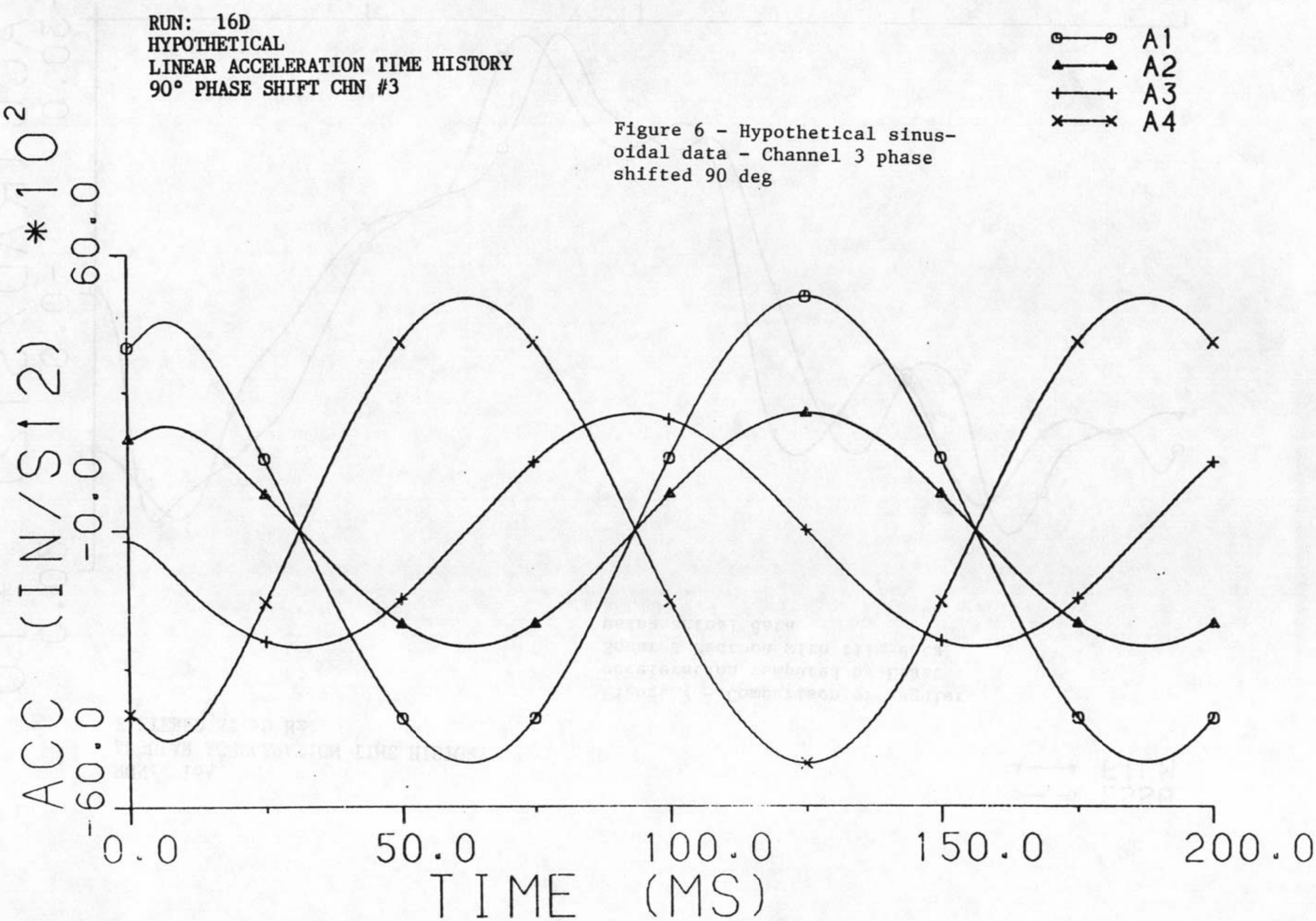


Figure 6 - Hypothetical sinusoidal data - Channel 3 phase shifted 90 deg

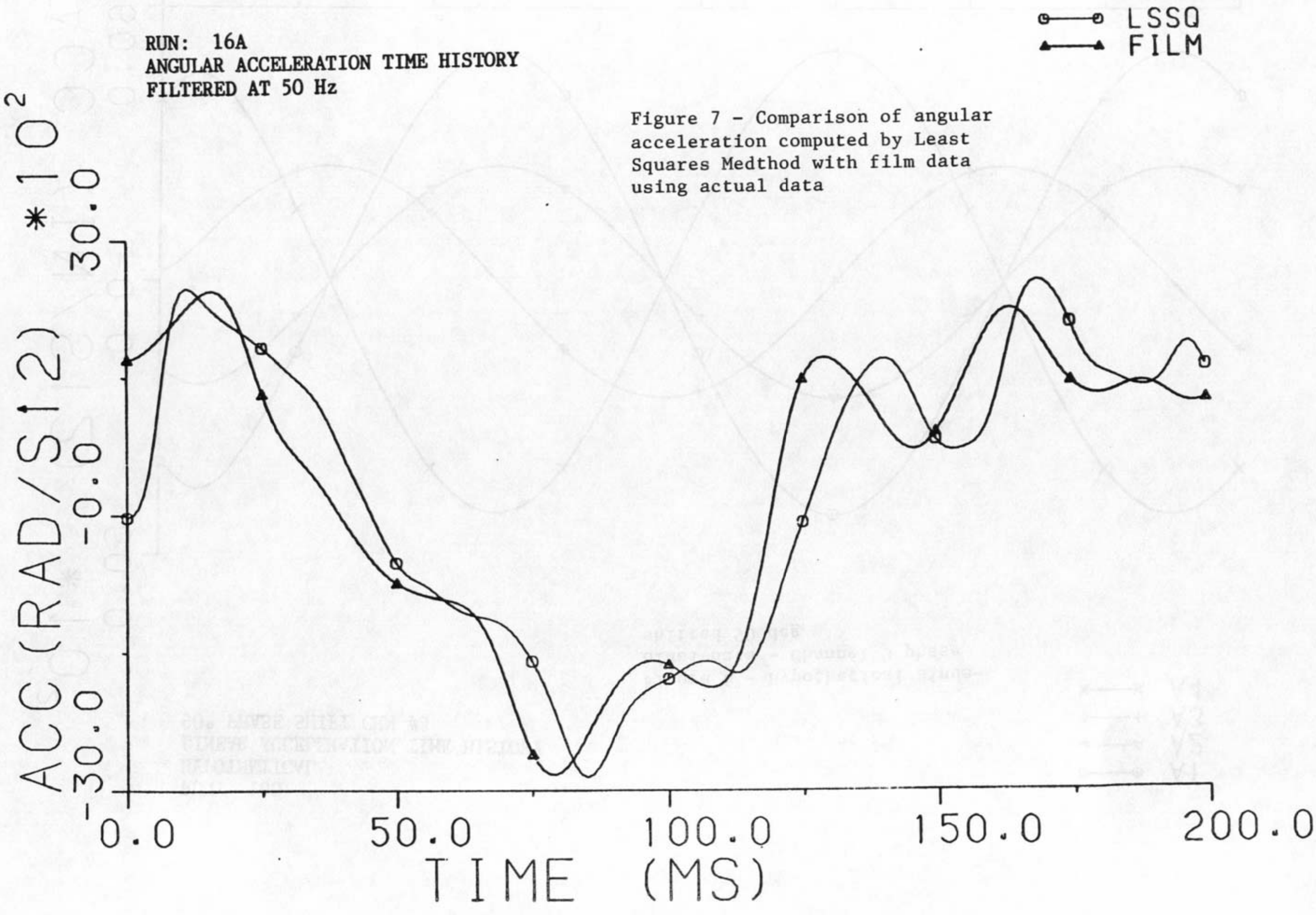


Figure 7 - Comparison of angular acceleration computed by Least Squares Medthod with film data using actual data

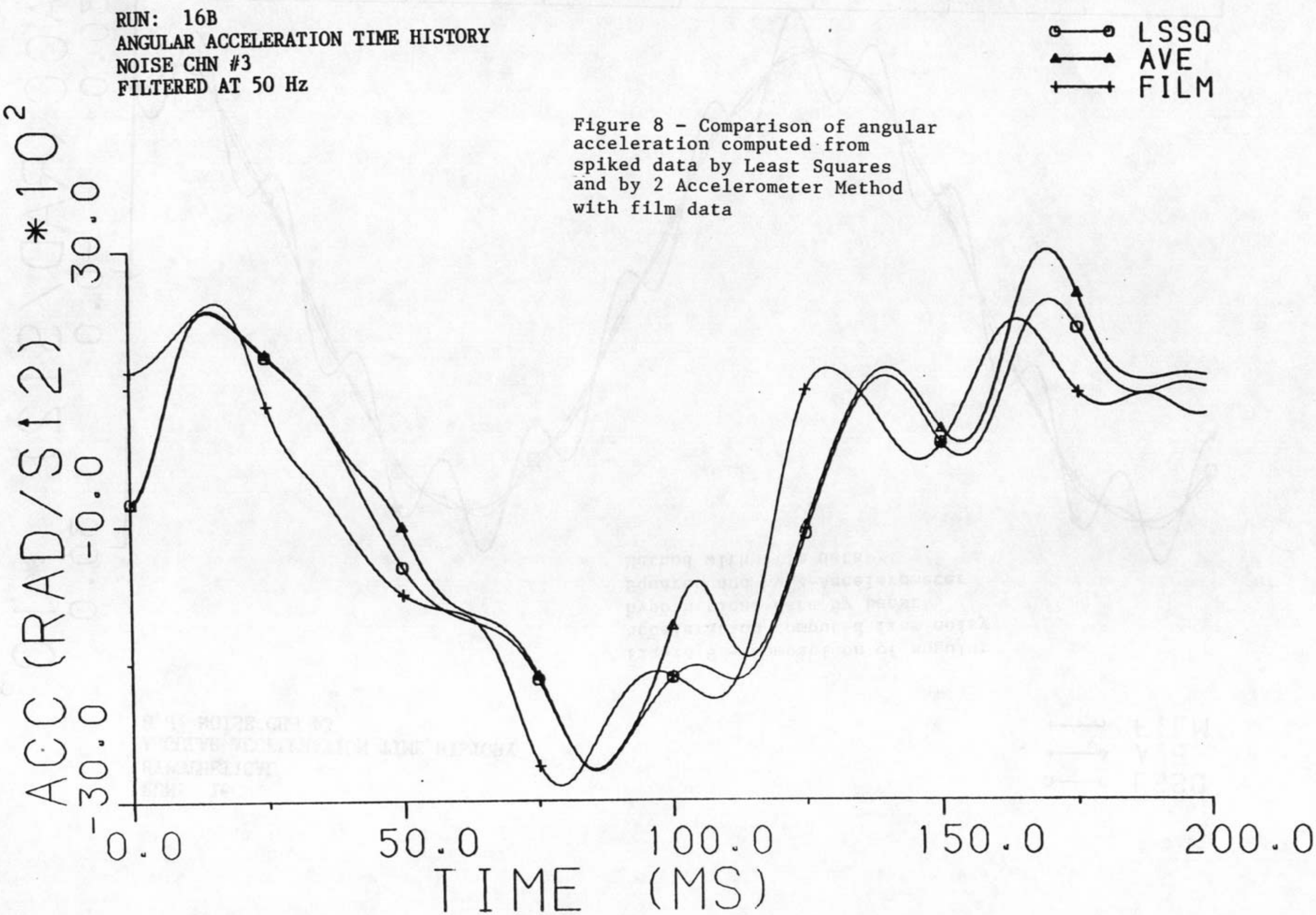


Figure 8 - Comparison of angular acceleration computed from spiked data by Least Squares and by 2 Accelerometer Method with film data

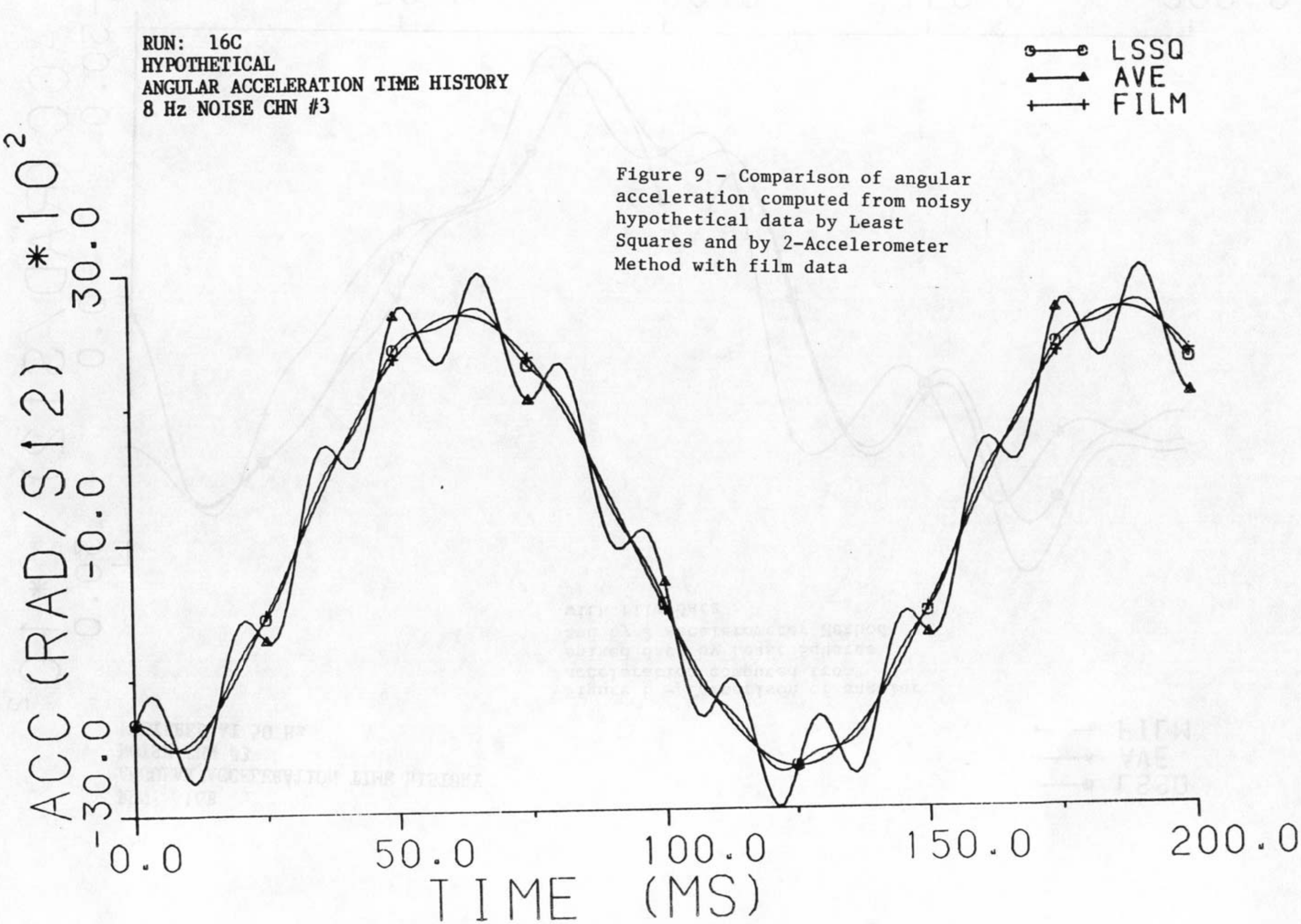


Figure 9 - Comparison of angular acceleration computed from noisy hypothetical data by Least Squares and by 2-Accelerometer Method with film data

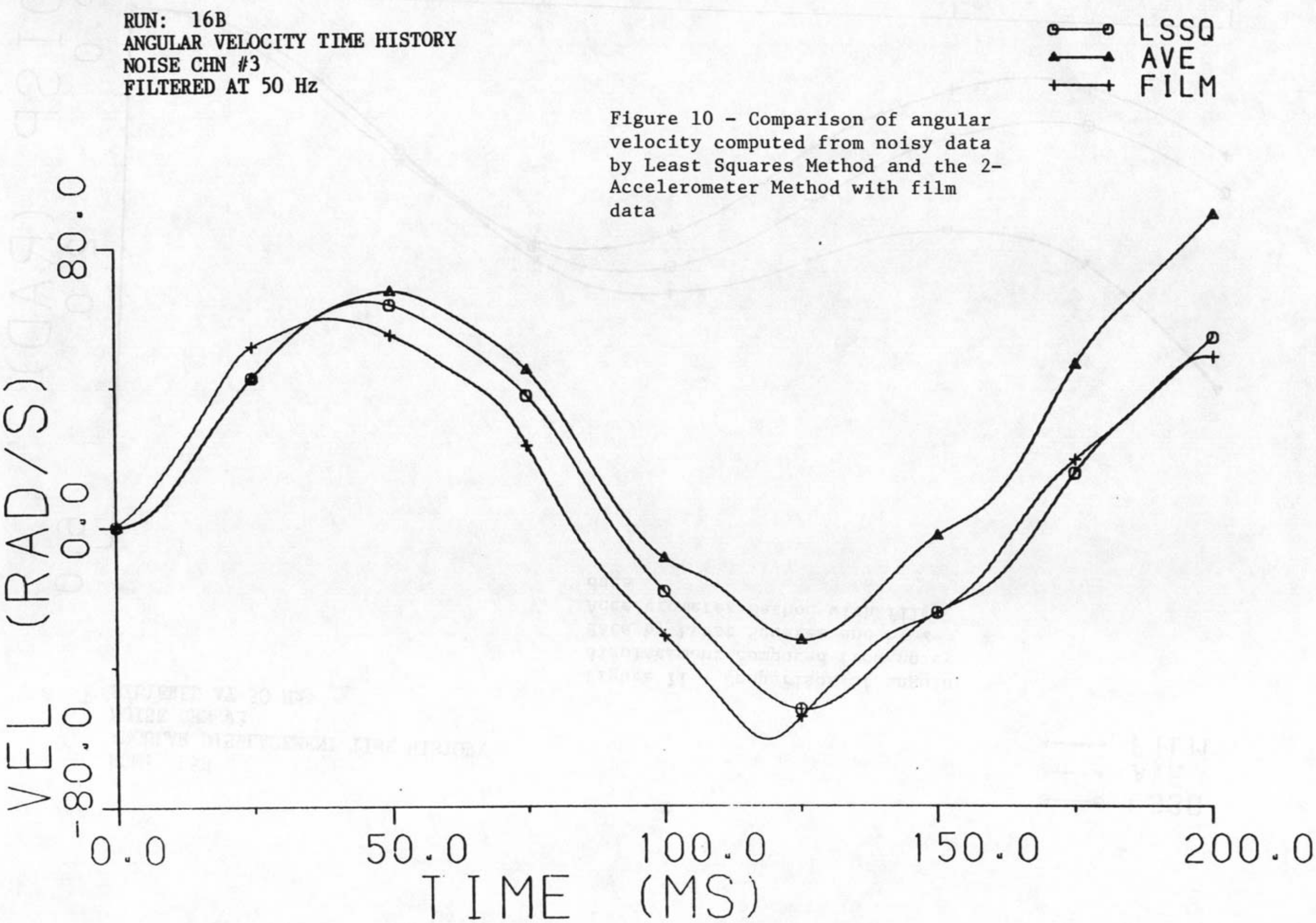


Figure 10 - Comparison of angular velocity computed from noisy data by Least Squares Method and the 2-Accelerometer Method with film data

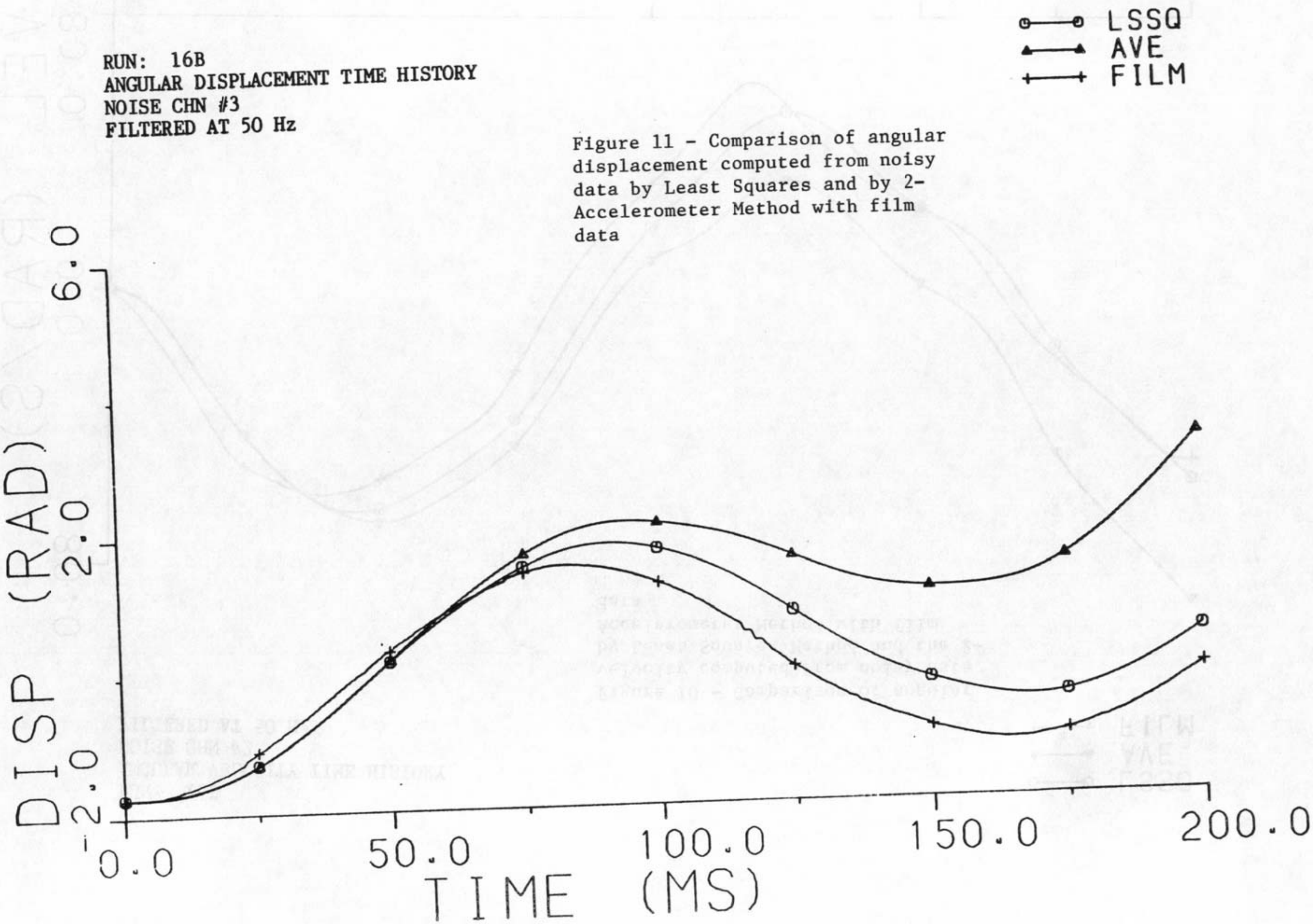


Figure 11 - Comparison of angular displacement computed from noisy data by Least Squares and by 2-Accelerometer Method with film data

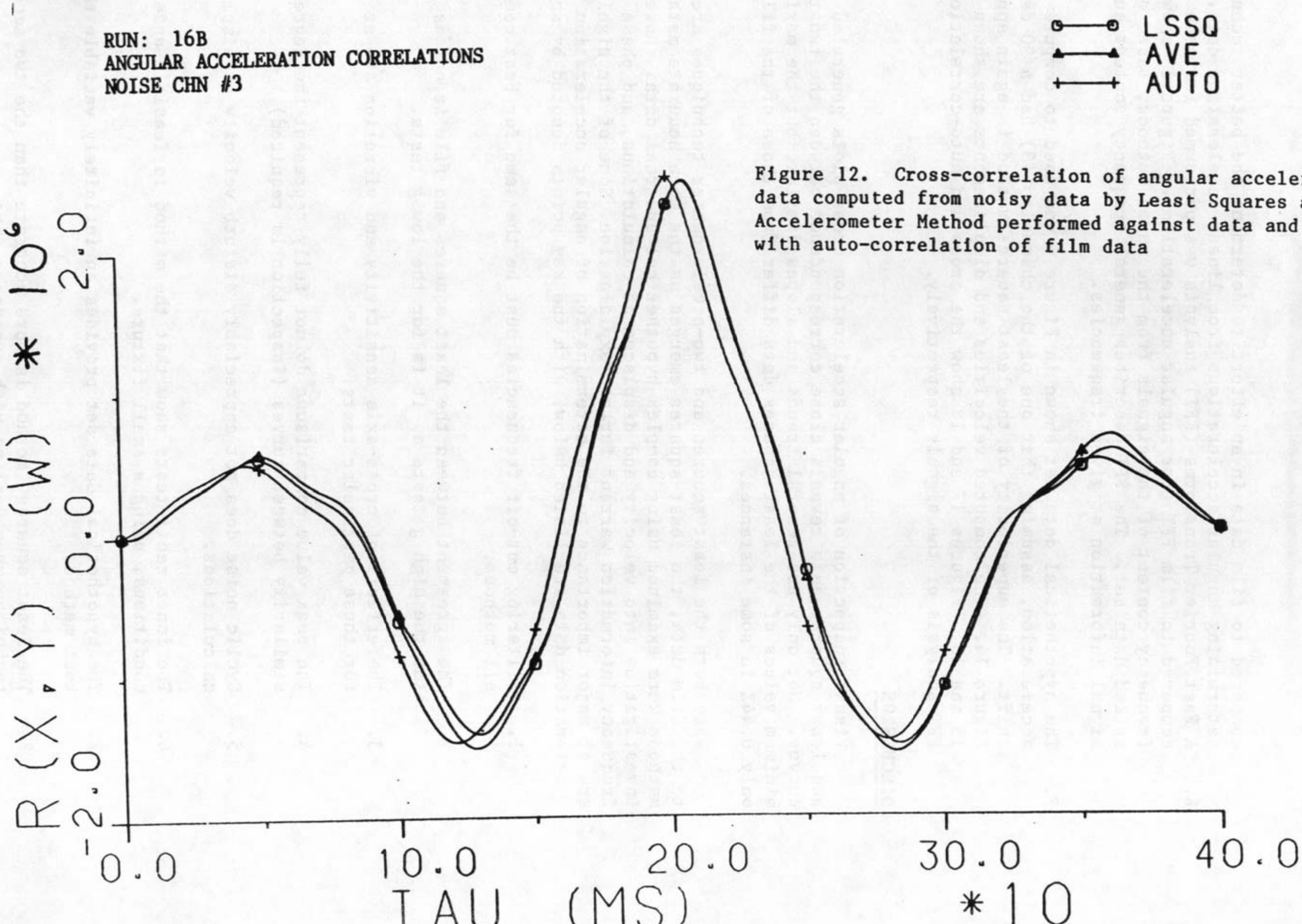


Figure 12. Cross-correlation of angular acceleration data computed from noisy data by Least Squares and 2-Accelerometer Method performed against data and compared with auto-correlation of film data

compared to film data in an effort to determine the better technique for determining angular accelerations from linear acceleration data.

E. A Fast Fourier Transforms (FFT) analysis was performed for each method and compared to film FFT (for angular acceleration). Figure 13 shows the frequency content of the signals from the three methods, based on the spiked data set. The FFT describes general frequency content and level of signal information at given frequencies.

F. The hypothetical data set shown in Figure 6 was used to compute angular acceleration, assuming that one of the channels (#3) had a 90 deg phase shift. The superiority of the least squares method is again apparent in Figure 14. The computed velocities and displacements are shown in Figures 15 and 16. Figures 17 and 18 show the cross and auto-correlations and the FFT analysis of the signals respectively.

DISCUSSION

Visual inspection of angular acceleration cross-plots generated from film and least squares data reveals close correspondence between the two types of curves. Not only are general trends and slopes similar, but the maximum and minimum values of the least squares data differ from those of the film data by only 0.44% in some instances.

When both the least squares and two-accelerometer techniques are compared to the film data, the least squares emerges as the more accurate method. Both methods were examined using complex hypothetical and real data. However, the investigation into velocity and displacement calculations, and phase and frequency information warrant further explaination. Some of the highlights which are of major importance to the determination of angular acceleration from linear acceleration data are listed below, with the key points denoted by an asterisk:

1. Filtering cut-off frequencies must be the same for best comparison of all methods.

2. The agreement between the least squares and film is not as close for the the high g tests as it is for the low g tests.

3. The effects of cross-axis sensitivity and vibration are at a minimum for these particular tests.

4. The peak value comparisons do not fully represent the degree of similarity between curves (inspection is required).

5. Cyclic noise does not appreciably disturb velocity or displacement calculations.

6. The inner radii tests show that the method is feasible under high g conditions, using a small fixture.

7. The hypothetical data set provides an infinitely variable theoretical test method.

8.* The least squares method is more accurate than the two accelerometer technique when dealing with erroneous data.

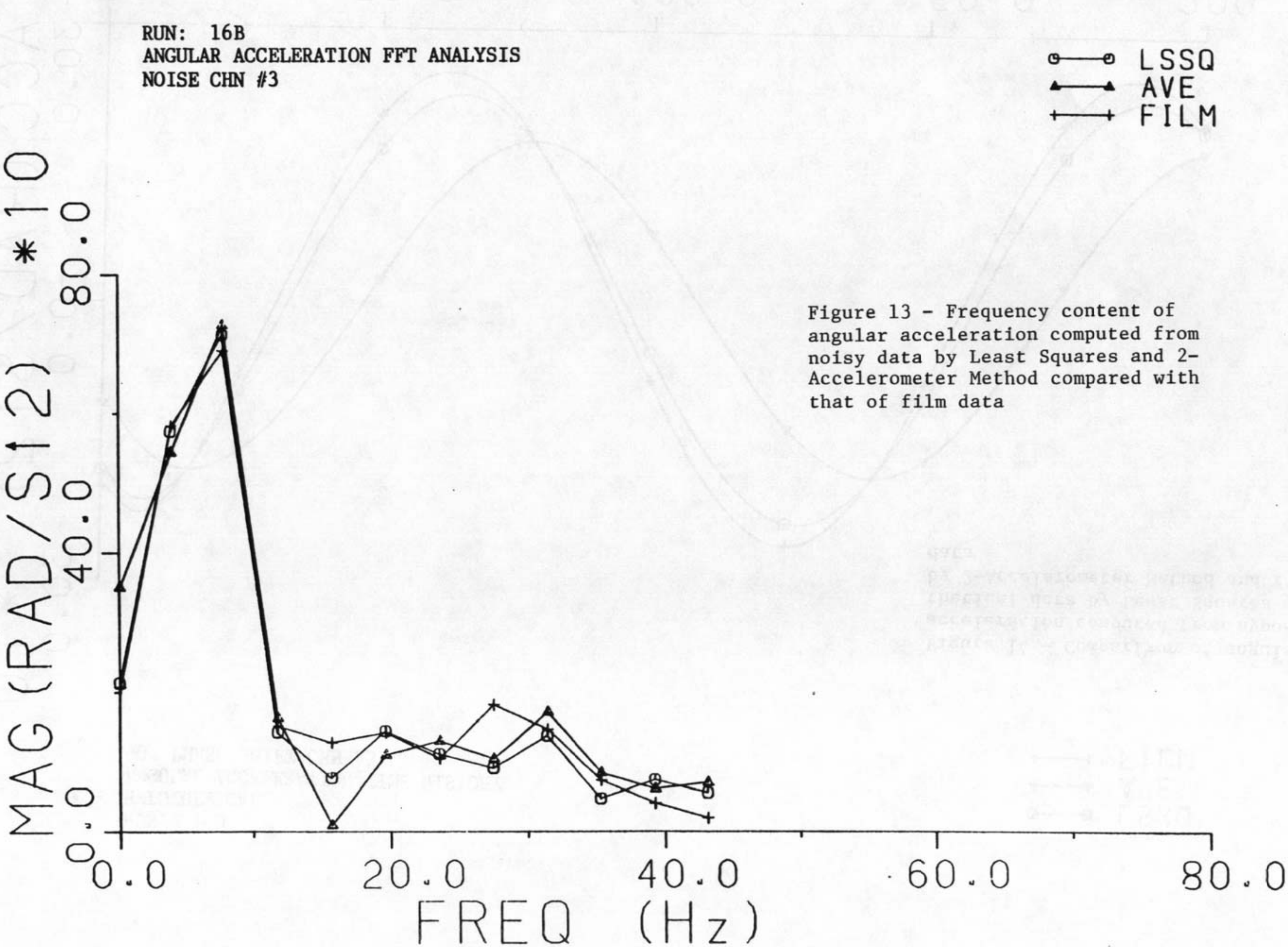


Figure 13 - Frequency content of angular acceleration computed from noisy data by Least Squares and 2-Accelerometer Method compared with that of film data

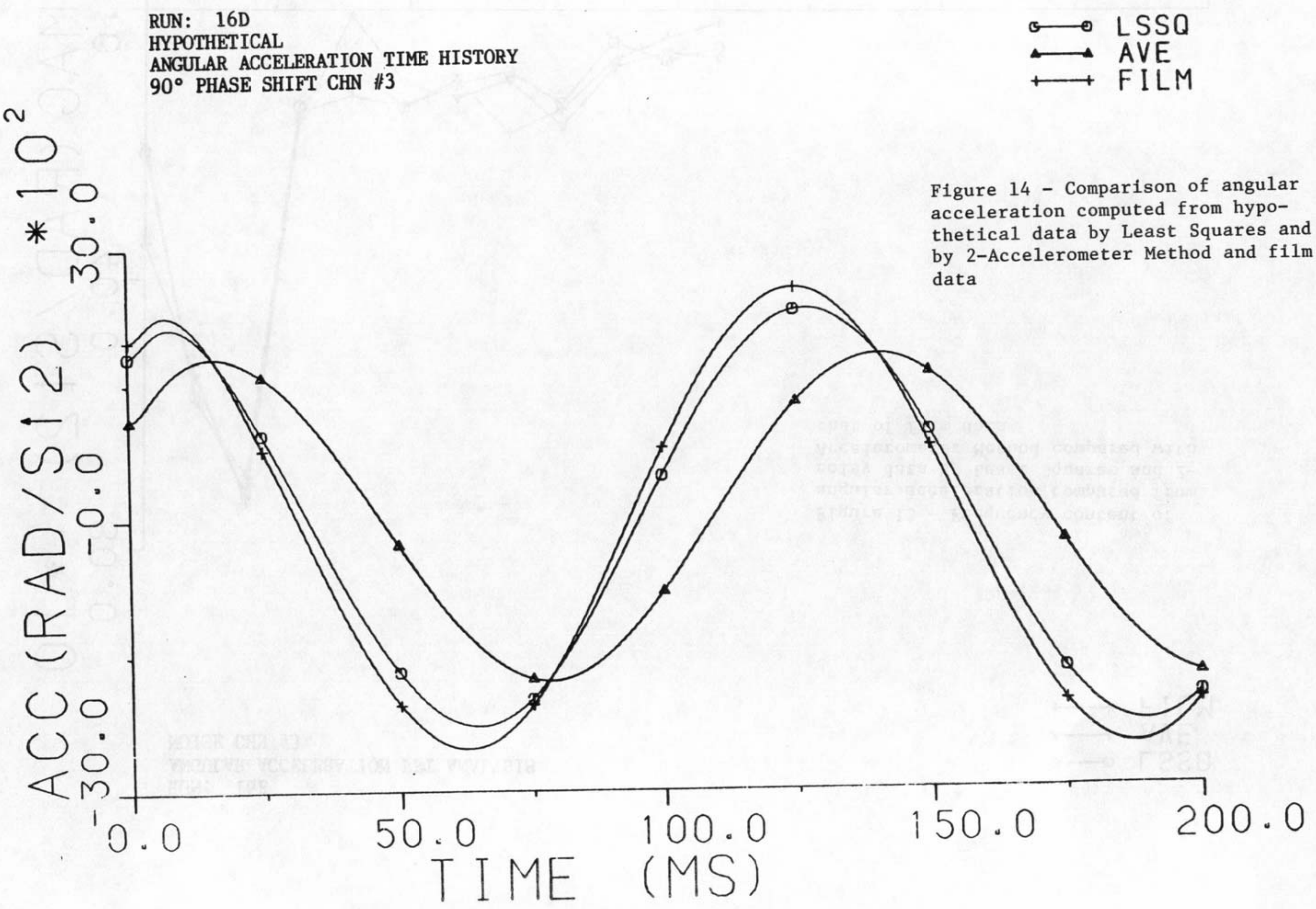


Figure 14 - Comparison of angular acceleration computed from hypothetical data by Least Squares and by 2-Accelerometer Method and film data

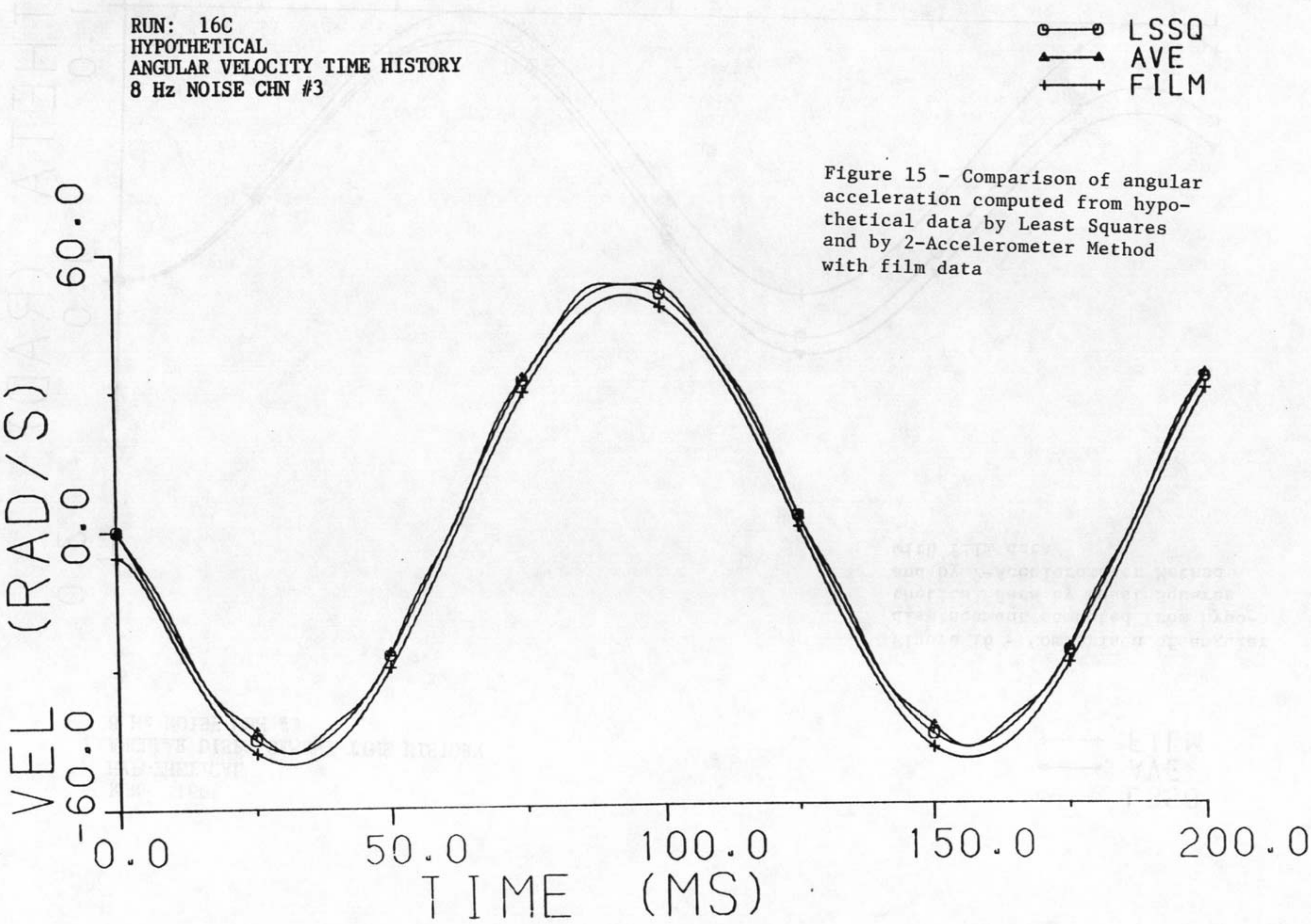


Figure 15 - Comparison of angular acceleration computed from hypothetical data by Least Squares and by 2-Accelerometer Method with film data

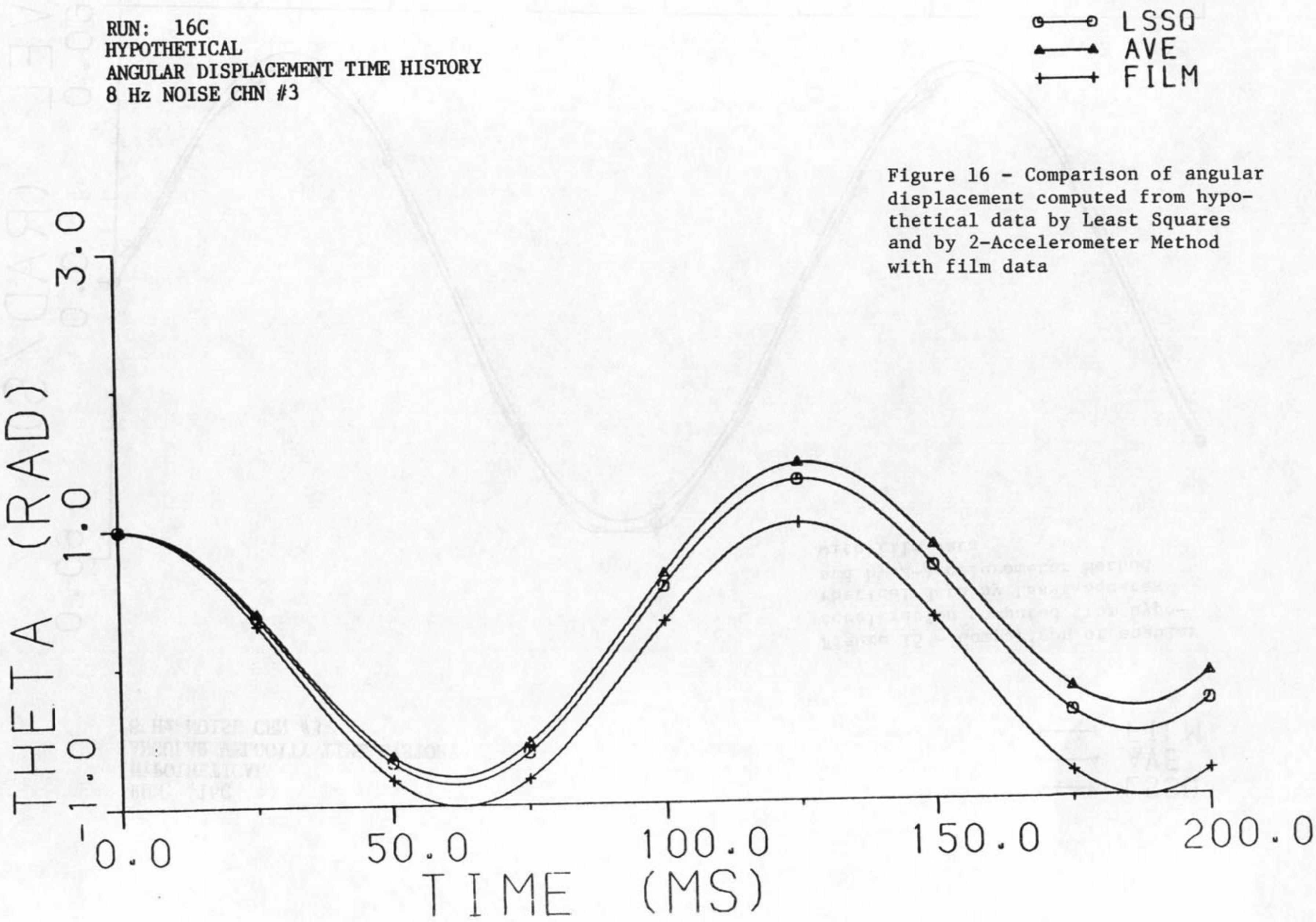


Figure 16 - Comparison of angular displacement computed from hypothetical data by Least Squares and by 2-Accelerometer Method with film data

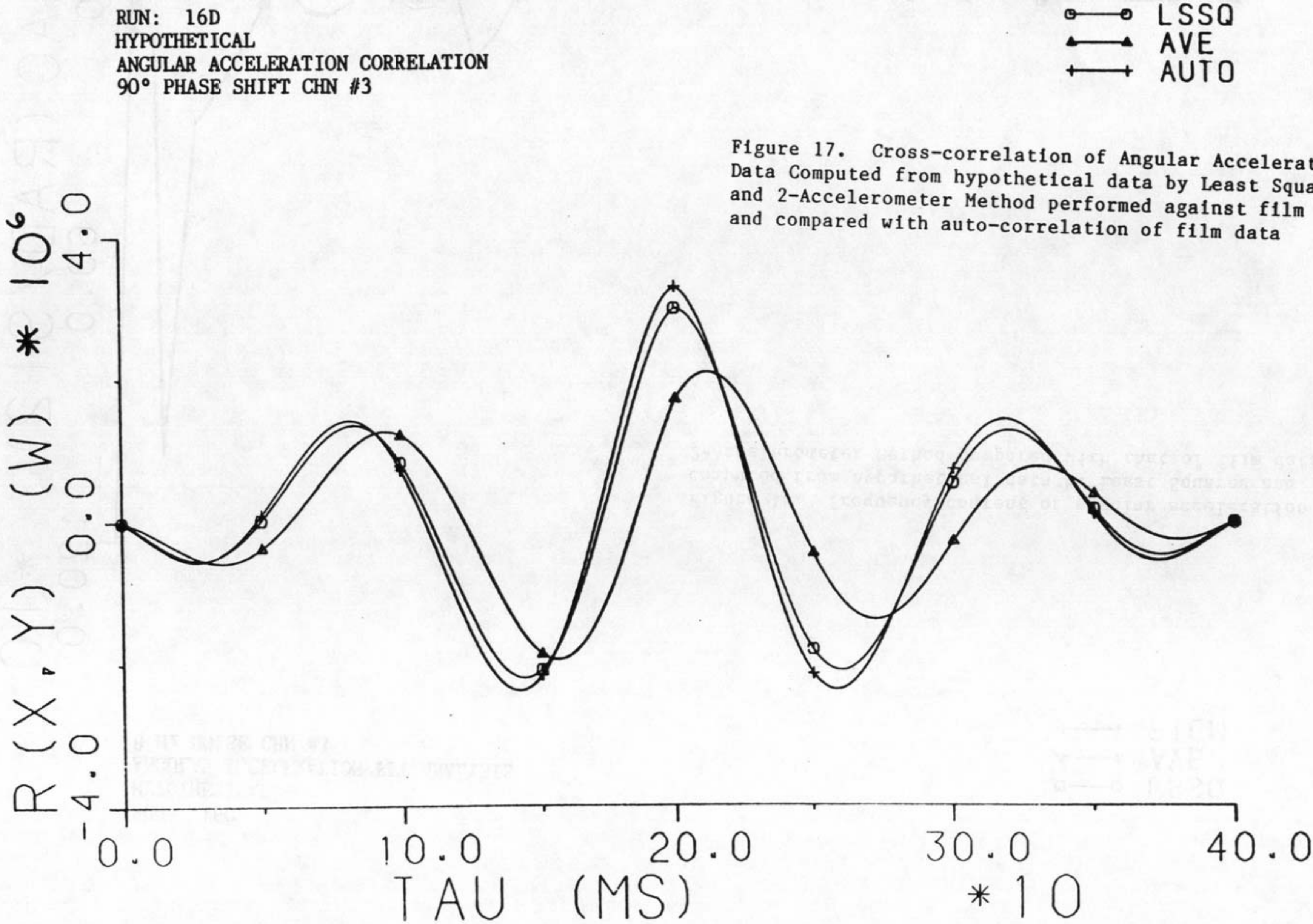


Figure 17. Cross-correlation of Angular Acceleration Data Computed from hypothetical data by Least Squares and 2-Accelerometer Method performed against film data and compared with auto-correlation of film data

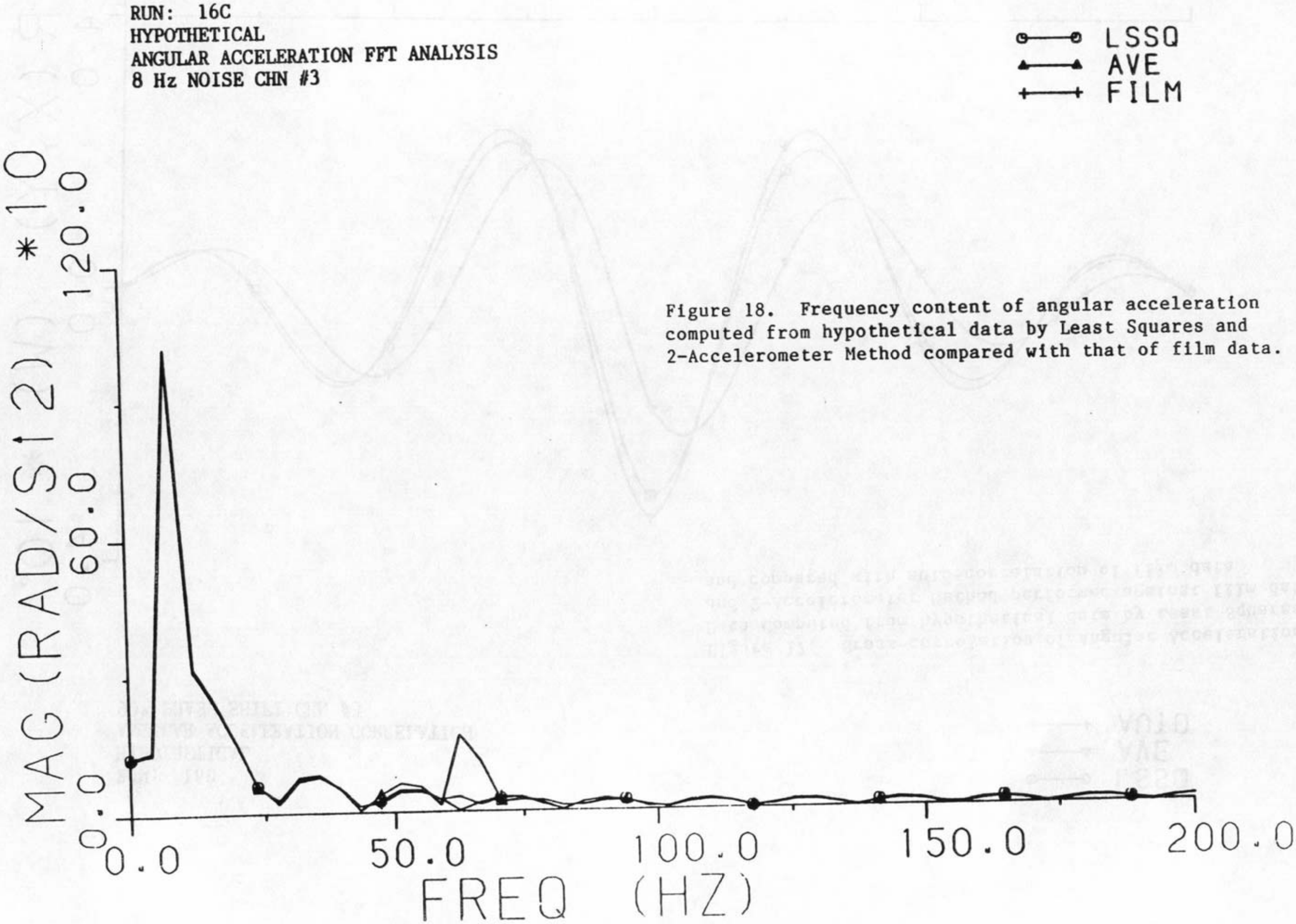


Figure 18. Frequency content of angular acceleration computed from hypothetical data by Least Squares and 2-Accelerometer Method compared with that of film data.

9. The least squares method is not error free.

10.* An electronic delay of about 5 ms exists in the transducer/signal conditioner network.

11.* Time shifts cause large miscalculations of acceleration, velocity, and displacement.

12.* Random noise, as opposed to cyclic noise, creates very unreliable data.

While the least squares method was shown to work very well, it was not shown to be perfect. It is, however, much better than the two-accelerometer method when applied to noisy signals. Interestingly, none of the anticipated sources of error such as vibration, cross-axis sensitivity, low sensitivity, or 60 cycle noise markedly influenced the test results. However, random noise and time shifts cause large errors in calculation. In practice, random noise, or "spikes" can be eliminated, but the time shift of @5ms due to transducer frequency response and signal conditioning delays could create insurmountable problems when trying to compare accelerometer and film data. In general the least squares method should provide a much needed improvement in the calculation of angular accelerations.

CONCLUSION

Although the least squares technique is a major improvement over the two-accelrometer method currently used, it is not entirely without fault. Even though the least square provides displacement data 200 times closer to actual displacements than does original method in some instances, the errors in calculation can still be as high as 100%. In general, however, the least squares approach is able to accommodate errors in frequency, phase, and amplitude of linear acceleration data far better than the two-accelerometer method. A more vigorous comparison using hypothetical data should be performed with more than one channel of noisy data. The effects of all possible types of noise should be examined at the same time, i.e. cyclic noise, random noise, and time shifts should be simulated concurrently. The application of this least squares system should then be extended to generalized three dimensional motion. The lack of numerical analysis and increased number of data points should allow the least squares method to perform well when describing 3D rotational accelerations. If so, this would not only improve the quality of results for current types of testing, but would also open the door for future ventures, such as comparing rigid body (skull) rotations to the response of a viscoelastic system (brain). Thus it is of utmost importance that testing of the least squares method for determining angular accelerations be continued.

ACKNOWLEDGMENT

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DISCUSSION

PAPER: Improved Method for Measurement of Angular Acceleration
SPEAKER: Warren Hardy

Q. Roger Daniel, Ford Motor Company
What would all of this information tell you if you were trying to measure a magnitude of angular acceleration that was 100 to 1000 times higher than what you have done.

A. Do you mean, if you had a case where the angular accelerations were much larger than our test situation, how would it perform?

Q. Roger Daniels
Yes. Substantially larger, 100 to 1000 times.

A. Presumably there would be a direct correlation in the increase in accuracy of the least squares method over the current method that we're using, which corresponds to the 3-2-2-2 method. Also, there would be a direct correlation if the order of magnitude of angular acceleration were much larger; the least squares method would still provide a much more accurate representation of that angular acceleration than the averaging method.

Q. Forit Bendjellal, APR
I would like to understand if the least squares method is the 3-3-3-3 method or is it completely new? And is the averaging method the 3-2-2-1 method?

A. In addressing the second question, our method corresponds to three accelerometers at the origin of the body-fixed basis with two accelerometers on the end of each axis. So it is a 3-2-2-2 method and, for the planar case, it reduces to an averaging method. Essentially, it is taking the difference between the value of linear acceleration from the outer transducer on the axes and the acceleration experienced by the accelerometer at the origin and dividing it by its distance from the origin. It would amount to an average from whatever values that you have for angular acceleration or angular motion.

The least squares method is not putting accelerometers at the origin. Theoretically, this point could be realizable from the equations, which is significant because it is very difficult to physically create a device or machine a fixture where the accelerometer seismic masses are in the proper alignment to measure accelerations at the origin of your body-fixed basis. What the least squares method would suggest is two accelerometers on either side of an origin and taking the least squares approximation of the slope. The linear regression coefficient is the slope of the line of the linear accelerations versus the displacement of a transducer from the origin.

Q. You mentioned that one of your future objectives is to develop a 12-accelerometer array. Could you explain your approach in more detail?

A. What we would like to do is apply it to a 3-dimensional case. The four accelerometers are only good for a planar case. If you were to construct a fixture that had two accelerometers for each plane of motion on either side of the origin of the body-fixed basis, you would be able to achieve that goal. Presumably this would work well for mounting on a cadaver head for impact studies. The orientation of this could be flexible and, through a transformation matrix, you could get the numbers that you're interested in regardless of the orientation of this device.

Q. Guy Nusholtz, UMTRI
What sort of angular accelerations were you using?

A. I believe our maximum angular acceleration was on the order of 3,000 radians per second squared.

Q. John Melvin, GM Research
I'd like to comment on the methods that we presented last year using five accelerometers in a row in a least squares fit to that data. It's essentially the same thing that you're doing, except we're doing it in 3-D. In fact, if the accelerometers are symmetrically placed, the fifth one drops out of the equations and there are only four--like you have. We place our accelerometers asymmetrically in an array so that all five are giving us information. This method would appear to be usable in a very general way. The problem in three dimensions has been the coupled equations and we've found, since our paper, a much easier way to solve those equations in a straight-forward computational manner which gives us much greater accuracy. This method would appear to be very good at the high acceleration levels. The problem is proving that. We're working on a fixture that will produce high angular accelerations in a controlled manner. At present, nobody really has a good way of evaluating these. It is very critical in terms of phasing and such things, particularly if you've got vibrations in your system. I think the basic method will prove to be a good way to solve this problem.