FUNDAMENTAL RELATION BETWEEN FIRE TIME AND IMPACT SEVERITY ESTIMATION UNCERTAINTY

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ABSTRACT

The concept of information flow applied to crash sensing in airbag systems introduced in [3] is further developed. An asymptotic formula for the amount of information available to an airbag module is obtained. This formula is applied to study the information flow in a linear spring-mass model and it gives a simple and insightful relationship between the fire time of an airbag and the accuracy of the estimation of the initial impact velocity. This relationship is in the form of an "uncertainty principle" describing the balance between the amount of time needed for the airbag sensor to estimate the severity of an impact and the uncertainty of the estimation itself. An application of the result, based on actual crash data, is presented to illustrate the analytical result.

INTRODUCTION

Airbag systems are tools designed to improve occupant protection when a vehicle is involved in a crash. To achieve this goal, an airbag system must have the ability to detect the severity of an impact in a timely manner. Therefore, based on the estimated severity of the impact, the airbag system decides on whether to deploy the airbag or not before the event is over. As an extreme scenario, at the end of a crash all the information from the sensors has been collected and it is possible to determine if the impact was severe enough to require the deployment of the airbag; however, such a strategy of deployment is far from optimal since the airbag is deployed when the impact is over. In the opposite extreme, an airbag could be deployed right at the beginning of the impact. In this case the airbag will be effective in protecting the occupant, if the crash was severe, but since no information on the severity of the impact is available, the airbag will deploy in all crashes even when it is not needed.

The discussion above suggests that the airbag should not be deployed too early, in order to collect enough information from the sensors to establish, with some level of accuracy, if it is needed or not; and at the same time the deployment should not occur too late in the crash event in order to make the airbag system to perform effectively. One of the factors involved in the tuning of an airbag system is the determination of an optimal time of deployment that balances the two requirements outlined above.

Using concepts from information and estimation theory, this paper establishes and analyzes a fundamental relationship between the uncertainty of estimating the severity of an impact and the amount of time required to collect enough information (from sensors), used to make a firing decision. For a simple impact model, using a probabilistic description of the signals received from the airbag sensors, a relationship is determined that relates the time to fire of an airbag system and the uncertainty in estimating the severity of the crash. In other words, a relationship is found between the uncertainty in determining the vehicle speed and the time needed to collect the data necessary to perform the estimation itself. Finally, a Bayesian analysis of crash tests at 25, 35, and 40 mph into an offset deformable barrier is conducted to illustrate an application of the relationship developed above.

THE ASYMPTOTIC FORMULA

In order to gain an insight into the decision making process of an airbag system, the concept of information flow has been introduced in [3]. The electrical signals coming from transducers placed in a vehicle are regarded as producing a flow of information that is to be processed by an algorithm implemented in the control module, ultimately responsible for the airbag deployment. The idea of information flow has been related to the concept of Fisher information from classical statistical estimation theory [5]. In particular, this theory provides optimal criteria for unbiased estimators of parameters of probability distributions (parametric estimation).

For a vehicle involved in an impact, the barrier equivalent velocity (BEV) is commonly used as a measure of its severity. The BEV is the initial velocity of a vehicle impacting a rigid barrier that results in the same amount of energy absorbed by the vehicle as in the actual impact. The impact model that will be introduced later refers to a rigid barrier event; therefore, in this paper, the initial impact velocity will be regarded as the relevant measure of severity.

An airbag algorithm is expected to make a fire decision based on the impact estimated severity, therefore its main activity can be regarded as an attempt to estimate the impact speed of a vehicle. For notation, v_0 is the actual impact speed of a vehicle (the one to be estimated) and $\tilde{v}_0$ denotes the estimate given by the algorithm on which the firing decision is based. Besides estimating the crash speed, it is also important to know what is the best possible accuracy that can be achieved in determining v_0 as measured by the variance of $\tilde{v}_0$.

The signal is modeled as a discrete time series since the transducer time histories are read by an analog-to-digital device with a pre-determined sampling interval Δt. The signal received from the sensors is then represented by the vector $\mathbf{x}_N = \{x_1, \ldots, x_N\}$, where $N = T/\Delta t$. The signal $\mathbf{x}_N$ and the speed v_0 of the impact are linked by a probabilistic model. This model summarize our knowledge about the crash process and how the signal from the transducers depends on the impact speed v_0. The model is given by

$$p(\mathbf{x}_N | v_0), \tag{1}$$

the probability density of the time history $\mathbf{x}$ given a certain impact speed v_0. A key result of the Fisher information theory is given by the Cramer-Rao information inequality which asserts that, under some regularity assumptions for the distribution density p, the variance of the unbiased estimator $\tilde{v}_0$ of v_0 is bounded from below. This inequality is given in the form

$$I(p(\mathbf{x}_N | v_0))\; var(\tilde{v}_0) \geq 1, \tag{2}$$

where I is called the (Fisher) information which is a function of the probability density p. A key feature of the quantity I is that it is monotonically increasing with N or, in other words, the longer the signal time history, the larger the amount of (Fisher) information. From this point of view the Cramer-Rao inequality can be regarded as a statistical uncertainty principle, that is, shorter signal time history or faster-sought decision time of the airbag algorithm implies a larger variance of $\tilde{v}_0$ or uncertainty in the determination of the impact speed. This uncertainty principle is the tool that describes the balance between the two extreme scenarios discussed in the introduction: either always fire the airbag early or fire it at the end of the impact.

In [3], the Fisher information theory described above has been applied to the case that the distribution density of $\mathbf{x}$ is Gaussian

$$p(\mathbf{x}_N | v_0) = \frac{1}{(2\pi^{N/2}) |\; \Sigma_N \;|^{1/2}} \times e^{-\frac{1}{2}(\mathbf{x}_N - \mathbf{a}_N(v_0))\Sigma_N^{-1}(\mathbf{x}_N - \mathbf{a}_N(v_0))^T}, \tag{3}$$

where Σ_N is the covariance matrix and $\mathbf{a}_N = (a_1, \ldots, a_N)$ is the mean time history which depends on the initial velocity v_0. In the case that $\Sigma_N = \text{diag}(\sigma_1, \ldots, \sigma_N)$ (diagonal matrix), it has been shown that the information I is given by

$$I = \sum_{i=1}^{N} \left(\frac{1}{\sigma_i} \frac{\partial a_i}{\partial v_0} \right)^2. \tag{4}$$

The formula above shows that the information I is monotonically increasing with N, the length of the signal time history, and is "inversely proportional" to Σ_N. It follows that for a given time T, the value of I computed from equation (4), depends on the choice of Δt. While this may look reasonable, for a fixed T the problem arises as $\Delta t \to 0$. In general, assuming that the elements in the diagonal of Σ_N are constant for every N, as $\Delta t \to 0$, I goes to infinity for every value of T; this suggests that an exact determination of v_0, at any time T, is theoretically possible, provided that a continuous reading of the signal is available. This is clearly impossible and shows that the discrete reading assumption is not suited for modeling a continuous reading. In particular, the definition of the matrix Σ_N is not adequate when taking the limit for $\Delta t \to 0$ since this assumption is equivalent to impose that the readings $(x_1, \ldots, x_N)$ are statistically independent with constant variance, no matter how close they are in time. However, in order to obtain a measure of all the information available in the signal, it is necessary to let $\Delta t \to 0$. In order for such a procedure to lead to useful results the independence condition has to be dropped, otherwise I goes to infinity as $\Delta t \to 0$. The scope of this paper is to extend the results obtained in [3] by introducing a probabilistic model for the signals such that taking the limit of I for $\Delta t \to 0$ is meaningful and leads to an expression of I as a function of the time duration of the signal. The quantity I represents the maximum information contained in the signal.

The use of a probabilistic model for the signals that introduces a time correlation between successive readings is discussed. In this case the Σ_N becomes a full matrix whose off-diagonal elements describe the amount of correlation.

In the model proposed in this paper, signal readings that are Δt units of time apart are assumed to have a correlation of

$$\rho(\Delta t) = e^{-\beta \Delta t} \qquad (\beta > 0). \tag{5}$$

Note that the amount of correlation tends to 1 as Δt tends to zero. This can be viewed as a continuity property and from the standpoint of information content expresses the idea that the increment of information gained from a further reading Δt time apart from the current one goes to zero as Δt goes to 0. This suggests that the time correlation introduced in the model is a mechanism that may strike a balance between the fact that more readings (available with a finer sampling of the signal) contribute to more information, and the fact that this added information goes to zero as the sampling gets finer.

Development of a formula

In the probabilistic model considered here, the signal X is a Gaussian process (time continuous) (cfr. [1]). This process is defined such that for any $(t_1, \ldots, t_N)$ the random vector $\mathbf{x}_N = (x(t_1), \ldots, x(t_N))$ has distribution given by (3) where $\mathbf{a}_N = (a(t_1), \ldots, a(t_N))$ and the covariance matrix $\Sigma_N = \{\Sigma_{ij}\}$ is such that $\Sigma_{ij} = \sigma(t_i)\sigma(t_j)\rho(|t_i - t_j|)$. The functions a and σ are the mean and standard deviation of the process (time wise), respectively, and ρ is given by (5). It is important to note that for $\beta \to \infty$ this model approximates the one in which each reading is independent from the others. The parameter β can be intuitively considered as a coefficient that determines the frequency content of a particular signal in such a way that the smaller β the "smoother" the signal.

Following the above model, the information content in N readings of the signal $\mathbf{x}_N$ is given by (cfr. [3])

$$I = \sum_{i,j=1}^{N} \Sigma_{ij}^{-1} \frac{\partial a(t_i)}{\partial v_0} \frac{\partial a(t_j)}{\partial v_0} \tag{6}$$

where Σ_{ij}^{-1} are the elements of Σ_N^{-1}, the inverse of Σ_N. The matrix Σ_N is given by

$$\Sigma_N = V \begin{pmatrix} 1 & \alpha & \cdots & \alpha^N \\ \alpha & 1 & \cdots & \alpha^{N-1} \\ \vdots & & & \\ \alpha^N & \alpha^{N-1} & \cdots & 1 \end{pmatrix} V \tag{7}$$

with $\alpha = e^{-\beta \Delta t}$ and $t_k = k\,\Delta t$ (uniform time spacing), while V is the diagonal matrix $V = \mathrm{diag}(\sigma(t_1), \ldots, \sigma(t_N))$. Computing I using (6) requires computing Σ_N^{-1} and in the general case it is intractable. Instead, when Σ_N has the form given in (7), Σ_N^{-1} can be computed as the tridiagonal matrix $V^{-1}\,L\,V^{-1}$, where

$$L = \frac{1}{1-\alpha^2} \begin{pmatrix} 1 & -\alpha & & & \\ -\alpha & 1+\alpha^2 & -\alpha & & \\ & -\alpha & \ddots & \ddots & \\ & & \ddots & 1+\alpha^2 & -\alpha \\ & & & -\alpha & 1 \end{pmatrix}. \tag{8}$$

This result can be used into (6) so that, after some lengthy algebraic manipulations, the expression for I can be put in a form that allows taking the limit for $\Delta t \to 0$. The following result is obtained

$$I = \frac{1}{2}\beta \int_0^T g^2(\tau)\,d\tau + \frac{1}{2\,\beta}\int_0^T (g'(\tau))^2\,d\tau + \frac{1}{2}\Big(g^2(0) + g^2(T)\Big) \tag{9}$$

where

$$g(t) = \frac{1}{\sigma(t)} \frac{\partial a(t;v_0)}{\partial v_0}. \tag{10}$$

The linear case

In this section, the asymptotic formula (9) developed above, is applied to the case in which the average signal is an acceleration which is linear in the initial velocity v_0

$$a(t;v_0) = v_0\,f(t) + h(t) \tag{11}$$

for any functions f and h. In particular, a linear spring-mass model is considered, for which a simple formula for the information I can be obtained. For such system the acceleration is given by

$$a(t;v_0) = \omega\,v_0 \sin \omega t \tag{12}$$

and follows that

$$g(t) = \frac{1}{\sigma(t)} \frac{\partial a(t;v_0)}{\partial v_0} = \frac{\omega sin(\omega t)}{\sigma_{noise}},$$

assuming $\sigma(t) = \sigma_{noise}$ is a constant. Applying the asymptotic formula (9) for the information we get

$$\begin{aligned} I(T) &= \frac{\beta\,\omega}{2\,\sigma_{noise}^2}\int_0^T \sin^2(\omega t)\,d(\omega t) + \\ &\frac{\omega^3}{2\beta\,\sigma_{noise}^2}\int_0^T \cos^2(\omega t)\,d(\omega t) + \frac{\omega^2}{2\sigma_{noise}^2}\sin^2(\omega t) \\ &= \frac{\omega\,\beta}{8\,\sigma_{noise}^2}\left[\left(\frac{\omega^2}{\beta^2}+1\right)2\omega T + \left(\frac{\omega^2}{\beta^2}-1\right)\sin 2\omega T \right. \\ &\left. + 2\,\frac{\omega}{\beta}\left(1-\cos 2\omega T\right)\right]. \qquad (13) \end{aligned}$$

The term in square brackets is an increasing function of the time T in which information is collected. Another parameter appearing in this term is the dimensionless ratio ω/β; ω is the natural frequency of the oscillator while β is directly related to the highest frequency in the noise spectrum. Figure (1) shows a plot of the information as a

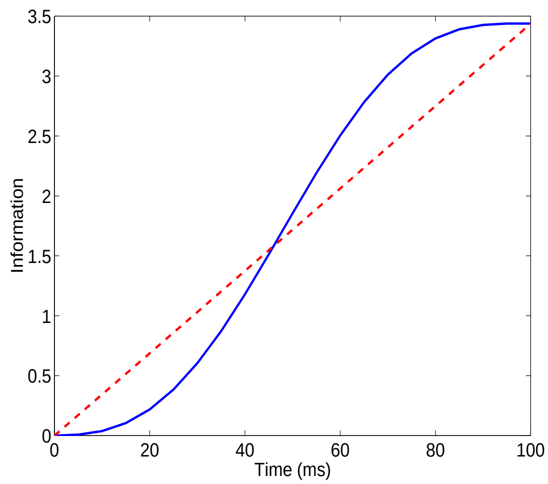

Figure 1: Plot of I(T) (solid line) and and linear approximation (dotted line) from (13) for $\beta = 500$, $\omega = 31.4$, and $\sigma_{\mathbf{noise}} = 60$.

function of the time T. The solid line is the function $I(T)$ while the dotted line is a linear approximation given by keeping only the linear term (in T) in (13). The information content I is an increasing function of T. It is relevant to observe that the rate of increase for the information (information flow) is low at the beginning and at the end of the oscillation cycle while it is high around the peak of the sinusoid a. This implies that more information is available when the acceleration a reaches its peak. The linear approximation of I in figure (1) is given by

$$I(T) \approx \frac{\omega^2\,\beta}{4\,\sigma_{noise}^2}\left(\frac{\omega^2}{\beta^2}+1\right)T. \qquad (14)$$

Later, this approximation will be used to explain the uncertainty principle type of result that derives from the Cramer-Rao inequality (2). By substituting I, as approximated in (14), the following is obtained

$$T\ var(\tilde{v}_0) \geq \frac{4\,\beta\,\sigma_{noise}^2}{\omega^2(\omega^2+\beta^2)}, \qquad (15)$$

for any unbiased estimator $\tilde{v}_0$ of v_0. The quantity on the right is a constant that depends on the physical model (ω) and on the noise characteristics (σ_{noise} and β). In particular, it increases with σ_{noise}, the standard deviation of the noise.

STATISTICAL ANALYSIS OF OFFSET TESTS

From a simplified point of view in which an airbag algorithm estimates the impact velocity from the signals, the algorithm is calibrated by fixing a “threshold” velocity v_F with which the estimated velocity $\tilde{v}_0$ is compared. It is conceivable that many automotive companies in their effort to address the issue of inadvertent airbag deployment have increased the firing “threshold” v_F in the airbag algorithm. The effect of raising v_F has been discussed in [3]. For a given impact velocity v_0 that warrants the use of an airbag ($v_0 > v_F$), raising the threshold velocity v_F requires a more accurate estimate of v_0, in order to maintain the same system performance. Therefore, a smaller variance for the estimate of v_0, $var(\tilde{v}_0)$, is required. From formula (15) it is concluded that T, the airbag fire time, must increase. Summing up, if auto companies after 1998 have raised the “threshold velocity” in airbag systems, an increase in airbag fire time should be observed in the field data. To evaluate this hypothesis the relationship between the airbag fire time for offset tests and the calendar year in which the data has been collected has been studied. The study has been performed only for offset tests since it is known (see [3]) that the information flow for offset tests is lower than for rigid barrier tests. This implies that an increase in the threshold velocity will lead to a larger increase in the airbag fire time for the offset tests than for the rigid barrier tests. Thus if there is a trend between airbag fire time and period of data collection, this trend should be more evident for offset tests than for rigid barrier tests.

Data

The data set consists of crash tests collected during the calendar year 1997 through 2001. The analysis was restricted to crash tests in 25, 35 and 40 mph events into an offset deformable barrier. The data were collected from various sources. High speed offset tests data came from the Insurance Institute for Highway Safety (IIHS). The lower speed offset test data were obtained from Transport Canada. Some data were gathered from various experiments conducted by several different companies. Moreover, for each of the three crash test velocities considered, three periods of data collections were used: up to April 1998, from April 1998 to November 2000, and from November 2000 to October 2001. The specific time frame subdivision used has been decided by the authors as a convenient method to organize the data.

Data Analysis

Before describing the statistical method that has been used, some notations and symbol definitions will be introduced. The following notation will be used: z_{ij} is the random variable denoting the fire time at the i^{th} period of data collection, $i = 1, 2, 3$, and at speed j, $j = 0, 1, 2$. In particular $j = 0$ is for the 25 mph offset tests, $j = 1$ is for the 35 mph offset tests while $j = 2$ is for 40 mph offset tests.
From a preliminary analysis the airbag fire times appeared to follow a Log-normal distribution as it was already noticed in [4]. Thus, it was assumed that $ln(z_{ij})$ are independently and normally distributed with mean θ_{ij} and unequal variance σ^2_{ij} with $i = 1, 2, 3$ and $j = 0, 1, 2$.

An unequal variance assumption has been considered since the sample data sizes from each of the nine speed-time collections (3 periods for each of the three speeds considered) differ.

The data was analyzed using a Bayesian approach ([5]). The Bayes analysis allows introduction of prior information (prior distribution) into the model. This is information about the unknown means (θ_{ij}) of the logarithmic of the fire time. Once a prior distribution is given, the whole inference process can be summed up according to this approach: an observed result changes our degrees of belief in different parameter values by changing a prior mean and variance into a posterior mean and variance. In this study, the (prior) means of the logarithm of the airbag fire-time θ_{ij} were assumed to arise from a regression model:

$$\theta_{i,0} = \gamma_0 + \gamma_1 * i + \epsilon_{i,0} \quad (16)$$
$$\theta_{i,1} = (\gamma_0 + \gamma_2) + \gamma_1 * i + \epsilon_{i,1} \quad (17)$$
$$\theta_{i,2} = (\gamma_0 + \gamma_3) + \gamma_1 * i + \epsilon_{i,2} \quad (18)$$

Where $\vec{\gamma} = (\gamma_0, \gamma_1, \gamma_2, \gamma_3)^t$ being an unknown vector of regression coefficients and the ϵ_{ij} being independent random variables normally distributed with zero mean and variance σ^2_π. The expression introduced above can concisely be rewritten as:

$$\vec{\theta} = Y * \vec{\gamma} + \vec{\epsilon} \quad (19)$$

where Y is a (9x4) matrix.
The model introduced amounts to assuming that the prior distribution of $\vec{\theta}$ is a normal with mean $Y * \vec{\gamma}$ and variance σ^2_π. In particular, we are assuming that on average the (prior) mean of the logarithm of the air bag fire time is changing at the same rate for all the three offset crash test speeds considered (γ_1).

Generally, in the Bayesian analysis, the parameters of the prior distribution are known; In our study the prior parameters, $\vec{\gamma}$, are unknown thus we will perform an Empirical Bayes analysis. The Empirical Bayes analysis entails estimating the hyperparameters $\vec{\gamma}$ and σ^2_π and using the estimates in the prior distribution for $\vec{\theta}$. The estimation of the hyperparameters have been performed following the parametric empirical Bayes procedure developed by Morris ([2]).
The performed analysis (Empirical parametric Bayes) can be seen as a compromise between the model where the mean logarithm of the airbag fire time, $\vec{\theta}$, are completely unrestricted (unrelated to each other) and the standard linear regression model where the mean airbag fire time is linearly increasing, without error, that is $\theta = Y * \vec{\gamma}$ is non random.

The estimated mean fire times and their 90% credible sets are plotted in Fig. 2. A 90% credible set for θ is a subset C such that the probability that θ belongs to C is at least 0.9.
This analysis shows that the airbag fire time has been significantly increasing since 1998 (see Fig.2). The increasing trend is observed for all the three crash test speeds considered. The uncertainty principle discussed in this paper, implies that this trend is consistent with the hypothesis that velocity thresholds in airbag algorithms have been functionally raised after 1998.

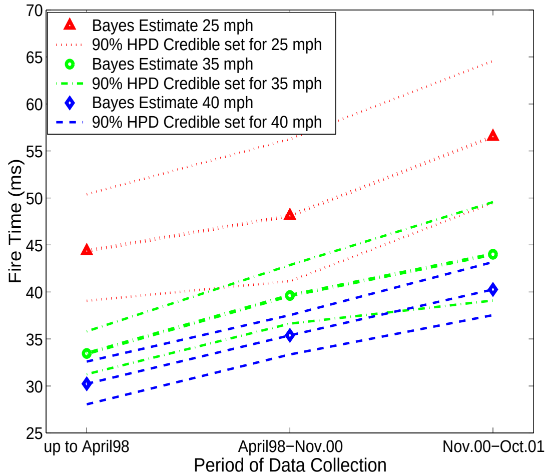

Figure 2: Bayes estimate of the Air-Bag Fire Time for crash tests into an Offset Deformable Barrier.

DISCUSSION

The main result of this paper is the derivation of (9), that expresses the maximum amount of information available in a continuous reading of a signal up to time T. For each time t, the quantity $g(t)$ in (9) is a measure of the ability to separate the means of the signals at different values of v_0, weighted by the standard deviation of the reading itself. The information I increases as a function of T and depends on the "smoothing" parameter β in such a way that as $\beta \to \infty$, then $I \to \infty$. This is expected since as $\beta \to \infty$ the model of the signals approximates the discrete model in which each reading is independent of the others. The first term in (9) is directly proportional to β while the second is inversely proportional to it. In particular, this first term can be regarded as a continuous version of (4) and it expresses the fact that the higher the values of β (small time correlation of the signals) the more information is available in the signal. This is analogous to the discrete case where the information increases as the number of readings increases. On the other side, for low values of β, the second term can be higher than the first one. This term implies that some information is stored in the time variation of $g(t)$ for low values of β (high time correlation). The second term can be regarded as added information in the signal coming from the possibility to infer the error in a reading of the signal from the error of the previous reading, if they are highly correlated (small β).

Using (9) in the Cramer-Rao inequality, an explicit relationship between the time T, regarded as the fire time, and the variance (accuracy) of the estimator of the impact velocity $\tilde{v}_0$ is obtained. This relationship has been approximated as the inequality (15), which describes the balance of all the factors that are involved in the airbag firing decision for a simple linear model. If T is regarded as the time at which the airbag is fired, the inequality expresses the fact that the error in estimating the actual value of v_0, as measured by $var(\tilde{v}_0)$, at time T, cannot be smaller than some known value that depends on the characteristics of the system and the noise. The inequality implies that a more accurate estimate of v_0 is possible if more time is allowed for making a decision on firing an airbag, that is, if T is larger. As the noise in the signal is higher, the constant on the right of the information inequality increases, so that to maintain the same accuracy in estimating v_0 more time is necessary. In other words, if the time cannot be increased a less accurate estimate will result. This can be summarized by saying that, in this simple case, airbag fire time and the minimum error in estimating v_0 (necessary to decide if an airbag is needed) are inversely proportional.

In order to illustrate some of the implications of the principle expressed above, an airbag algorithm can be regarded as a process that estimates the impact velocity of a crash. As discussed at the beginning of the the paper, the impact velocity is taken as a measure of the severity of the impact, on which the deployment decision is based. The algorithm is calibrated by fixing a "threshold" velocity v_F with which the estimated velocity $\tilde{v}_0$ is compared: the algorithm decides to fire the airbag if the estimated velocity is above the value of v_F. Under the assumption that the estimator $\tilde{v}_0$ is normally distributed it is possible to compute confidence intervals of the operation of the algorithm and to relate these to the airbag fire time. More specifically, if v_0 is the actual value of the impact velocity of a firing event ($v_0 > v_F$), being $\tilde{v}_0$ an unbiased estimator, a 99% reliability of the airbag algorithm, for example, can be expressed as

$$p(\tilde{v}_0 > v_F) = 0.99. \tag{20}$$

That is, 99% of the time the algorithm correctly guesses that it is observing an impact that requires the use of the airbag. Assuming that $\tilde{v}_0$ is normally distributed, the condition above is satisfied if

$$|v_0 - v_F| = z_{0.99}\, \sigma_{\tilde{v}_0}, \tag{21}$$

where $\sigma_{\tilde{v}_0}$ is the standard deviation of $\tilde{v}_0$ and z_α is defined by

$$\mathcal{P}(X < z_\alpha) = \alpha,$$

where $\mathcal{P}$ denotes probability and X is a random variable normally distributed with mean 0 and variance 1. Using this identity in the approximated information inequality for the linear case, equation (15), it is concluded that

$$T \geq \frac{4\,\beta\,\sigma^2_{noise}\,z^2_\alpha}{\omega^2(\omega^2+\beta^2)}\frac{1}{|v_0 - v_F|^2}. \qquad (22)$$

The quantity on the right hand side is the minimum amount of time necessary to collect enough information to correctly estimate the airbag outcome to a α confidence level. In figure 3, a plot of the minimum time

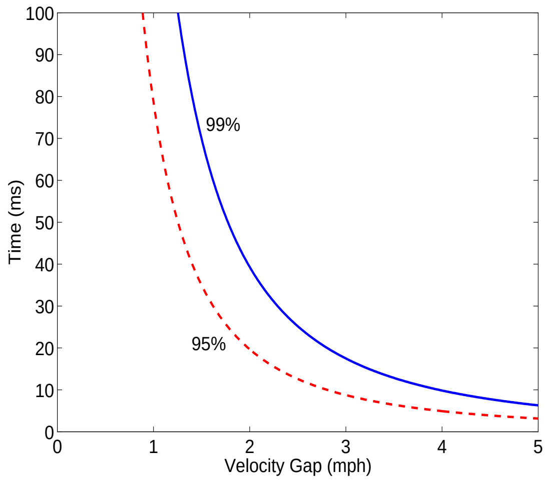

Figure 3: Plot of T from (22) assuming the equal sign for values of $\alpha = 0.99 \quad (99\%)$ and $\alpha = 0.95 \quad (95\%)$.

as a function of the velocity gap $|v_0 - v_F|$ is presented (the distance of the actual impact velocity from the velocity threshold). Note that the smaller the velocity gap the higher the time that is required to make a fire decision with a fixed level of reliability. Faster firing decisions can be accomplished for the same velocity gap at the expense of the system reliability.

Inequality (22) can also be regarded as describing the firing, that is the reliability of the firing decision process. For example, it is assumed that the requirement is set that for impacts on a rigid barrier with initial velocities (or BEV) above 14 mph the algorithm must always fire the airbag with a 95% success rate. In the actual calibration process of an airbag algorithm such speed (14 mph) might be called the "all fire" speed. Furthermore, as an additional requirement, the decision on firing has to be made within 10 ms from the beginning of the crash. Inequality (22) can be used to evaluate the performance of the algorithm under these conditions. To take care of the "all fire" condition, it is observed that setting v_F at about 11 mph, the corresponding equality in (22) is turned into an equality for $T = 0.01$ s, $v_0 = 14$ mph, and $\alpha = 0.95$ (also, $\beta = 500$, $\sigma_{noise} = 60$, and $\omega = 3.14$). This step is the "calibration" step. The fire rate of this set-up of the "algorithm" is obtained by solving the correspondent equality in (22) (corresponding to best use of information) for z_α, obtaining

$$z_\alpha = (v_0 - v_F)\sqrt{T\,\frac{\omega^2(\omega^2+\beta^2)}{4\,\beta\,\sigma^2_{noise}}}. \qquad (23)$$

In this context, α is the probability that a decision to fire is taken by the algorithm for an impact at velocity v_0. A plot of α as a function of v_0 is shown in figure 4 with the solid line, where v_F is set from the above "calibration" step, $T = 0.01$ s, and the other parameters are chosen as in the previous example. The plot shows that by allowing 10 ms of data collection, the algorithm ensures a no-fire condition (with a 95% confidence rate) only below about 8 mph. A gray zone for this algorithm is defined as the region between 8 and 14 mph. This is the critical velocity region in which the algorithm successfully guesses the firing strategy with less than 95% confidence rate. If instead, 50 ms of data collection is allowed to the algorithm, its performance is expected to increase, as more information becomes available. The same computation as before can be carried, resulting in a threshold velocity of $v_F \approx$ 13 mph, in order to satisfy the "all fire" condition. The fire rate for this case is shown also in figure 4 as the dashed line. The fire rate characteristic of this algorithm is steeper suggesting a better performance of the airbag system. As a consequence the gray zone is considerably smaller, 11 to 14 mph.

CONCLUSIONS

The concept of information flow has been used to measure the performance of an airbag firing system. A simple expression for the maximum amount of (Fisher) information available to the algorithm as a function of fire time has been developed for a model of the signal in which the noise is correlated in time as an exponential function. This quantity is used in the Cramer-Rao (information) inequality that relates Fisher information and variance of an unbiased estimator of the impact velocity that an airbag algorithm produces. This information inequality analytically establishes a kind of uncertainty principle for which more accurate determination of the severity of the crash is achieved at the expense of having higher fire times; while, imposing lower fire time to achieve better airbag

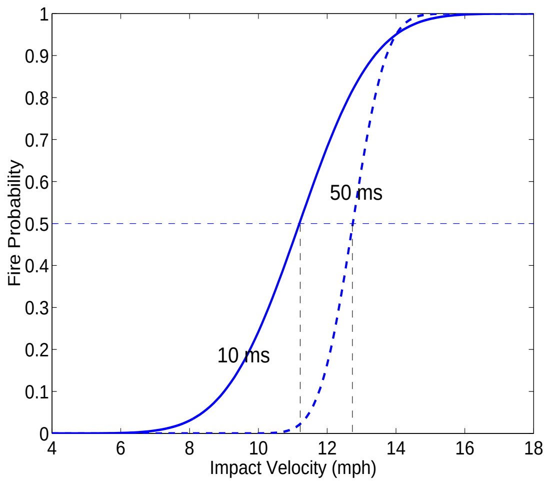

Figure 4: Plot of fire probability α as function of impact velocity $\mathbf{v_0}$ for $\mathbf{10}$ and $\mathbf{50}$ ms maximum time for fire decision. ($\mathbf{T = 0.01}$ s, $\mathbf{v_F \approx 11}$ mph)

performance leads to a less accurate determination of the severity of the crash. In this paper this principle has been used to explain the observed increase of fire time in ODB tests as a function of the calendar year, by assuming that velocity thresholds in airbag algorithms have in time been effectively raised to avoid inadvertent firing of airbags.

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